Computational Sensorimotor Systems Lab

Neuro-Current Response Functions: A Unified Approach to MEG Source Analysis Under Continuous Stimuli Paradigm

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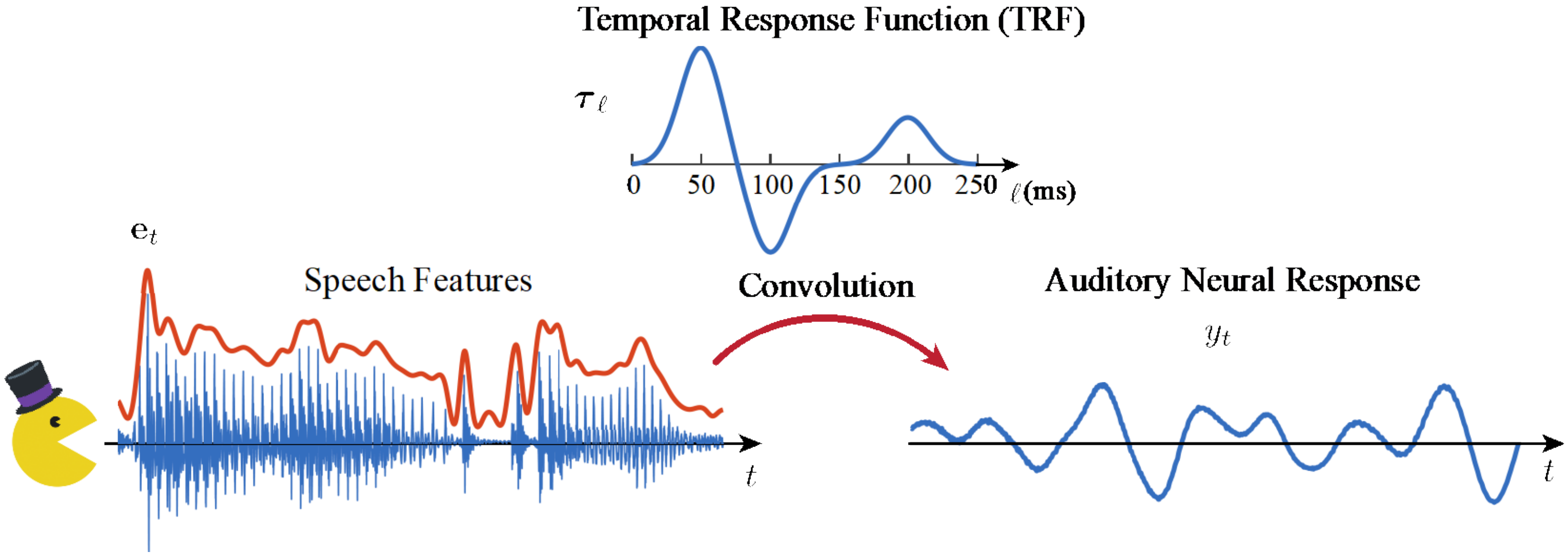
Presentation available at

Encoding Model of Speech Processing



Linear filter model:

- predict the M/EEG response from continuous stimulus

Lalor et. al., 2006, 2009, 2010

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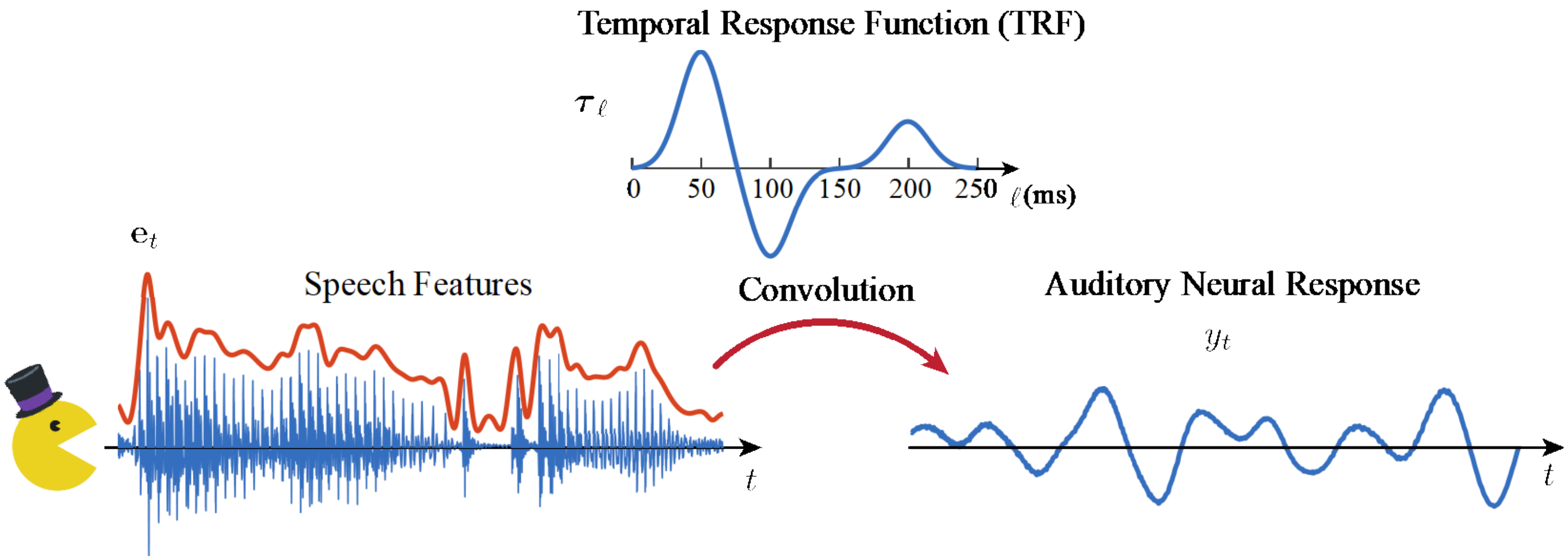

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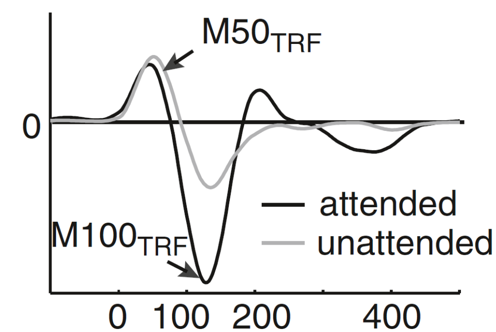
Functional Role:

- Different peaks are associated with different modes of processing
- M50: encoding of acoustic-level features
- M100: encoding of attentional state



Ding and Simon, 2012

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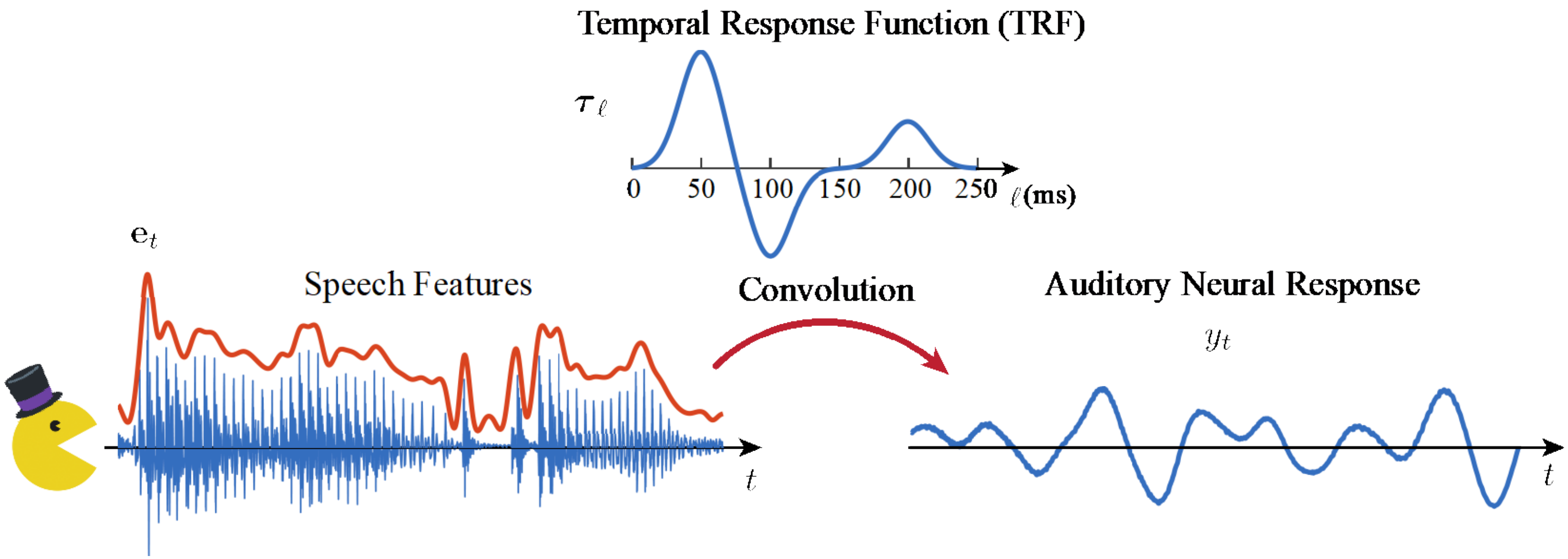

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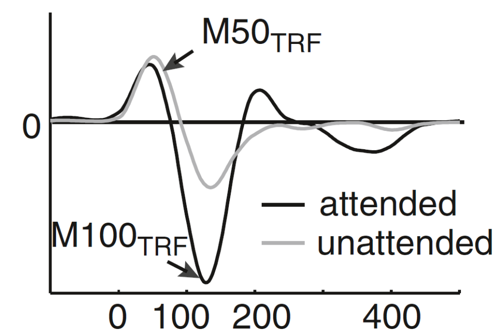
Beyond acoustic processing:

- Phoneme level processing
- Lexical processing
- semantic processing

Lalor et. al., 2006, 2009, 2010



Ding and Simon, 2012

Di Liberto et. al., 2015

Brodbeck et. al., 2018

Boderick et. al., 2018

Cortical Origins of TRFs?

Insight from intracranial recordings

- Electrophysiology, electrocorticography
- Limited spatial range

Mesgarani et. al., 2008, 2014

Existing M/EEG approaches works in two-stage:

- estimate the TRFs for each sensor → localize them on cortical mantle
- Decompose MEG signals to source time courses → estimate TRF for each source

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Challenges:

Bias propagation, spatio-temporal leakage, sensitivity to forward model

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MEG observations

Lead-field matrix

Source time courses

$$\mathbf{Y}_{N\times T} = \mathbf{L}_{N\times 3M} \boldsymbol{J}_{3M\times T} + \mathbf{W}_{N\times T}$$

Quasi-static solution to Maxwell's equations

N : sensors

T : time points

M : sources

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Quasi-static solution to Maxwell's equations

$$\mathbf{j}_{m,t} := [j_{m,t,R}, j_{m,t,A}, j_{m,t,S}]^\top \in \mathbb{R}^3$$

$$i \in \{R, A, S\}$$

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$$\mathbf{j}_{m,t} := [j_{m,t,R}, j_{m,t,A}, j_{m,t,S}]^\top \in \mathbb{R}^3$$

$$j_{m,t,i} = (\boldsymbol{\tau}_{m,i})^\top \mathbf{e}_t + v_{m,t,i} \qquad i \in \{R, A, S\}$$

$$\boldsymbol{\Phi} := [\boldsymbol{\tau}_1, \boldsymbol{\tau}_2, \cdots, \boldsymbol{\tau}_M]^\top \qquad \mathbf{S} := [\mathbf{e}_1, \mathbf{e}_2, \cdots, \mathbf{e}_T]$$

N : sensors T : time points M : sources L : lags

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Source time courses

$$\mathbf{Y}_{N \times T} = \mathbf{L}_{N \times 3M} \boldsymbol{J}_{3M \times T} + \mathbf{W}_{N \times T}$$

Quasi-static solution to Maxwell's equations

$$\boldsymbol{J}_{3M \times T} = \boldsymbol{\Phi}_{3M \times L} \mathbf{S}_{L \times T} + \mathbf{V}_{3M \times T}$$

Neuro-current response function

Auditory covariates

N : sensors T : time points M : sources L : lags

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$$\mathbf{Y}_{N\times T} = \mathbf{L}_{N\times 3M}\boldsymbol{J}_{3M\times T} + \mathbf{W}_{N\times T}$$

Measurement noise $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_w)$

Quasi-static solution to Maxwell's equations

$$\boldsymbol{J}_{3M\times T} = \mathbf{\Phi}_{3M\times L}\mathbf{S}_{L\times T} + \mathbf{V}_{3M\times T}$$

Background nuisance $\mathcal{N}(\mathbf{0}, \mathbf{\Gamma})$

Neuro-current response function

Auditory covariates

N : sensors

T : time points

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NCRF Learning: Bayesian Estimation

Measurement noise: Gaussian $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_w)$

$$p\left(\mathbf{Y}|\mathbf{J}\right) = |(2\pi)\mathbf{\Sigma}_w|^{-T/2} \exp\left(-\frac{1}{2}\|\mathbf{Y} - \mathbf{L}\mathbf{J}\|^2_{\mathbf{\Sigma}_w^{-1}}\right)$$

Background activity: Gaussian $\mathcal{N}(\mathbf{0}, \mathbf{\Gamma})$

$$p(\mathbf{V}|\mathbf{\Gamma}) = \left(\prod_{m=1}^{M} |(2\pi)\mathbf{\Gamma}_m|^{-T/2}\right) \exp\left(-\frac{1}{2}\sum_{m=1}^{M} \|\mathbf{V}_m\|^2_{\mathbf{\Gamma}_m^{-1}}\right)$$

$$p\left(\mathbf{J}|\mathbf{\Phi}, \mathbf{\Gamma}\right) = |(2\pi)\mathbf{\Gamma}|^{-T/2} \exp\left(-\frac{1}{2}\|\mathbf{J} - \mathbf{\Phi}\mathbf{S}\|^2_{\mathbf{\Gamma}^{-1}}\right)$$

Joint density of measurements and current dipoles:

$$p\left(\mathbf{Y}, \mathbf{J}|\mathbf{\Phi}, \mathbf{\Gamma}\right) = |(2\pi)\mathbf{\Sigma}_w|^{-T/2}|(2\pi)\mathbf{\Gamma}|^{-T/2} \exp\left(-\frac{1}{2}\|\mathbf{Y} - \mathbf{L}\mathbf{J}\|^2_{\mathbf{\Sigma}_w^{-1}} - \frac{1}{2}\|\mathbf{J} - \mathbf{\Phi}\mathbf{S}\|^2_{\mathbf{\Gamma}^{-1}}\right)$$

P. Das, C. Brodbeck, J. Z. Simon, and B. Babadi, NeuroImage, 2020

NCRF Learning: Bayesian Estimation

Measurement noise: Gaussian $\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_w)$

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Background activity: Gaussian $\mathcal{N}(\mathbf{0}, \boldsymbol{\Gamma})$

$$p(\mathbf{V}|\boldsymbol{\Gamma}) = \left(\prod_{m=1}^{M} |(2\pi)\boldsymbol{\Gamma}_m|^{-T/2}\right) \exp\left(-\frac{1}{2}\sum_{m=1}^{M} \|\mathbf{V}_m\|^2_{\boldsymbol{\Gamma}_m^{-1}}\right)$$

$$p\left(\mathbf{J}|\boldsymbol{\Phi}, \boldsymbol{\Gamma}\right) = |(2\pi)\boldsymbol{\Gamma}|^{-T/2} \exp\left(-\frac{1}{2}\|\mathbf{J} - \boldsymbol{\Phi}\mathbf{S}\|^2_{\boldsymbol{\Gamma}^{-1}}\right)$$

Joint density of measurements and current dipoles:

not observed

$$p\left(\mathbf{Y}, \mathbf{J}|\boldsymbol{\Phi}, \boldsymbol{\Gamma}\right) = |(2\pi)\boldsymbol{\Sigma}_w|^{-T/2}|(2\pi)\boldsymbol{\Gamma}|^{-T/2} \exp\left(-\frac{1}{2}\|\mathbf{Y} - \mathbf{L}\mathbf{J}\|^2_{\boldsymbol{\Sigma}_w^{-1}} - \frac{1}{2}\|\mathbf{J} - \boldsymbol{\Phi}\mathbf{S}\|^2_{\boldsymbol{\Gamma}^{-1}}\right)$$

Marginal density of the measurements given NCRFs:

$$p\left(\mathbf{Y}|\boldsymbol{\Phi}, \boldsymbol{\Gamma}\right) = \left|(2\pi)\left(\boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^\top\right)\right|^{-T/2} \exp\left(-\frac{1}{2}\|\mathbf{Y} - \mathbf{L}\boldsymbol{\Phi}\mathbf{S}\|^2_{(\boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^\top)^{-1}}\right)$$

NCRF Learning: Bayesian Estimation

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Background activity: Gaussian $\mathcal{N}(\mathbf{0}, \mathbf{\Gamma})$

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$$p\left(\mathbf{J}|\mathbf{\Phi}, \mathbf{\Gamma}\right) = |(2\pi)\mathbf{\Gamma}|^{-T/2} \exp\left(-\frac{1}{2}\|\mathbf{J} - \mathbf{\Phi}\mathbf{S}\|^2_{\mathbf{\Gamma}^{-1}}\right)$$

Joint density of measurements and current dipoles:

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Maximum likelihood estimate:

$$\min_{\mathbf{\Phi}} \quad \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\mathbf{\Phi}\mathbf{S}\|^2_{(\mathbf{\Sigma}_w + \mathbf{L}\mathbf{\Gamma}\mathbf{L}^\top)^{-1}}$$

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NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

$$\min_{\mathbf{\Phi}} \qquad \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\mathbf{\Phi}\mathbf{S}\|^2_{(\mathbf{\Sigma}_w + \mathbf{L}\mathbf{\Gamma}\mathbf{L}^\top)^{-1}}$$

But

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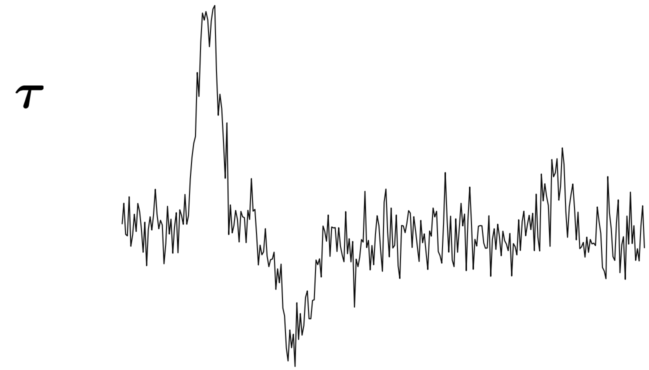
But

- NCRFs may not be smooth



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But

- NCRFs may not be smooth

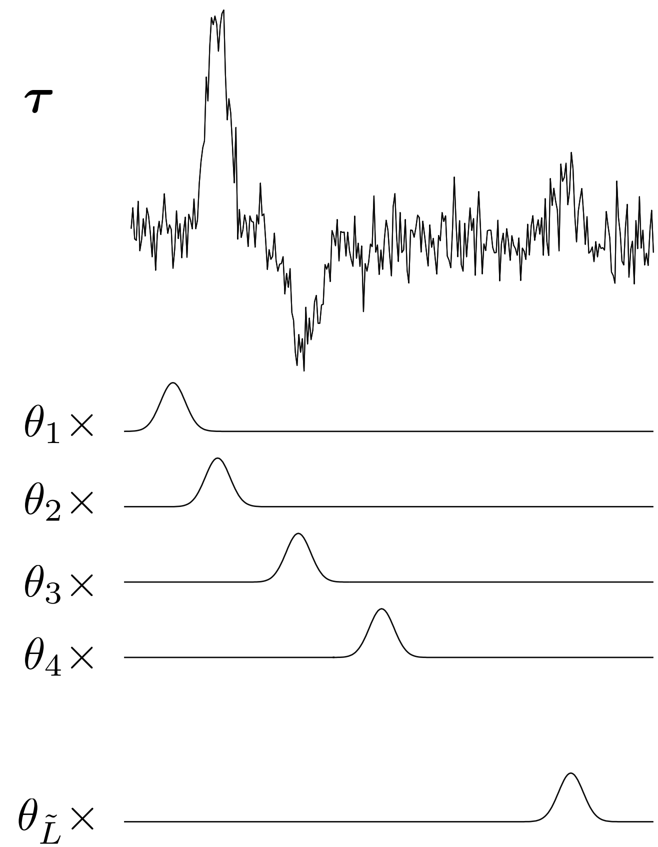
Sol:

Represent $\boldsymbol{\Phi}$ using a Gabor dictionary, $\mathbf{G}$

$$\boldsymbol{\Phi} = \boldsymbol{\Theta}\mathbf{G}^\top$$

$$\widetilde{\mathbf{S}} := \mathbf{G}^\top\mathbf{S}$$

$$\min_{\boldsymbol{\Theta}} \quad \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}\widetilde{\mathbf{S}}\|_{\boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^{\top -1}}$$



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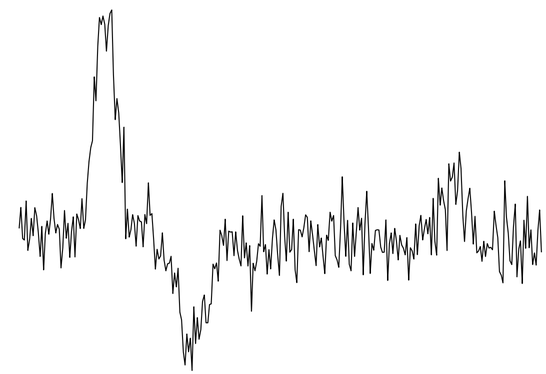
Sol:

Represent $\boldsymbol{\Phi}$ using a Gabor dictionary, $\mathbf{G}$

$\boldsymbol{\tau}$



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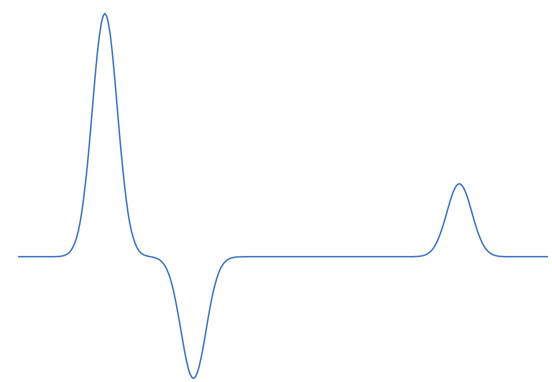

$$\min_{\Theta} \qquad \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}\widetilde{\mathbf{S}}\|_{\boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^\top{}^{-1}}$$

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NCRF Learning: Bayesian Estimation (more modeling)

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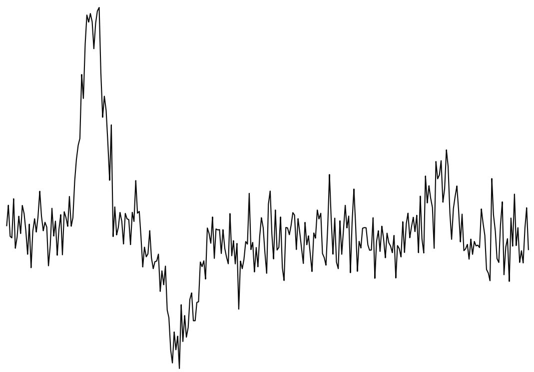

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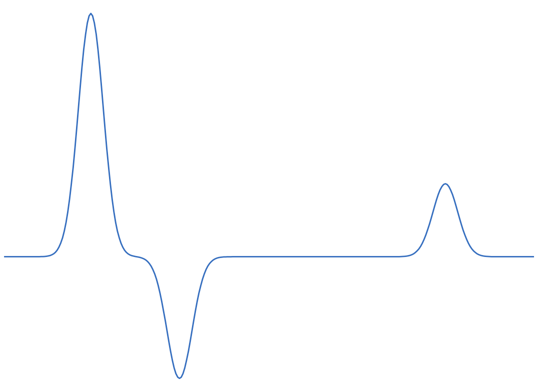

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NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

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But

- NCRFs may not be smooth ⟶ Consider Gabor dictionary representation
- It is an ill-posed problem

$$N \sim 10^2 \ll M \sim 10^4$$

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Maximum likelihood estimate:

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$$N \sim 10^2 \ll M \sim 10^4$$

Incorporate prior knowledge:

- Not all regions are processing same stimulus
- TRFs exhibits peaks and troughs

NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

$$\min_{\Theta} \quad \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}\widetilde{\mathbf{S}}\|_{\boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^{\top}{}^{-1}}$$

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Enforce spatial and lag-domain sparsity

P. Das, C. Brodbeck, J. Z. Simon, and B. Babadi, NeuroImage, 2020

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Maximum likelihood estimate:

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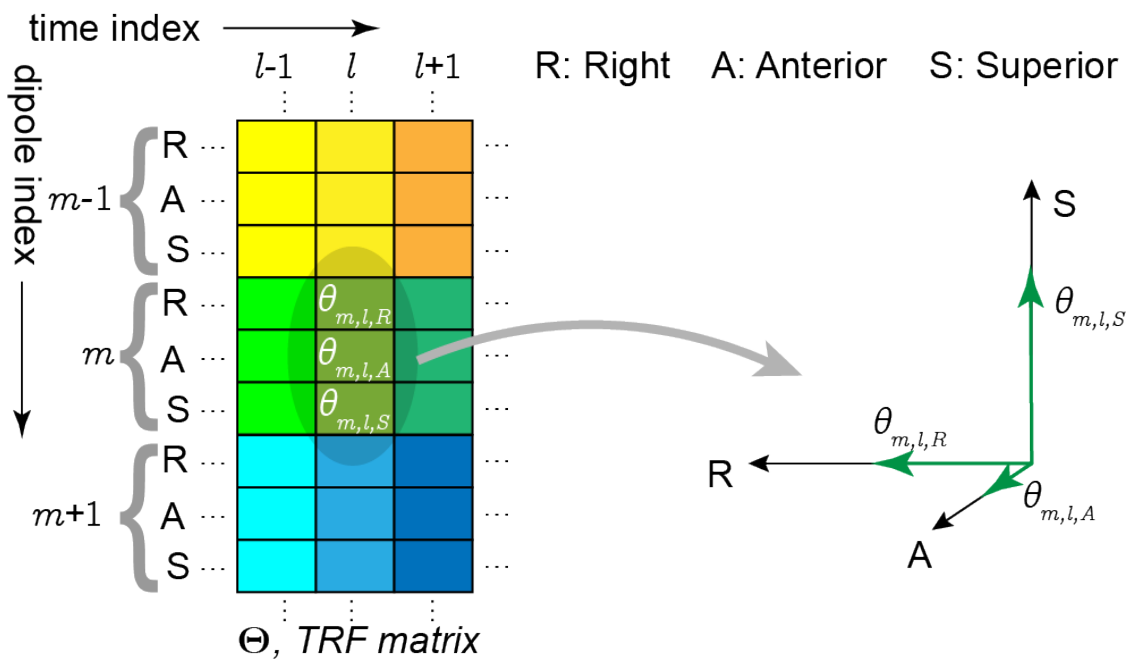

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Maximum likelihood estimate:

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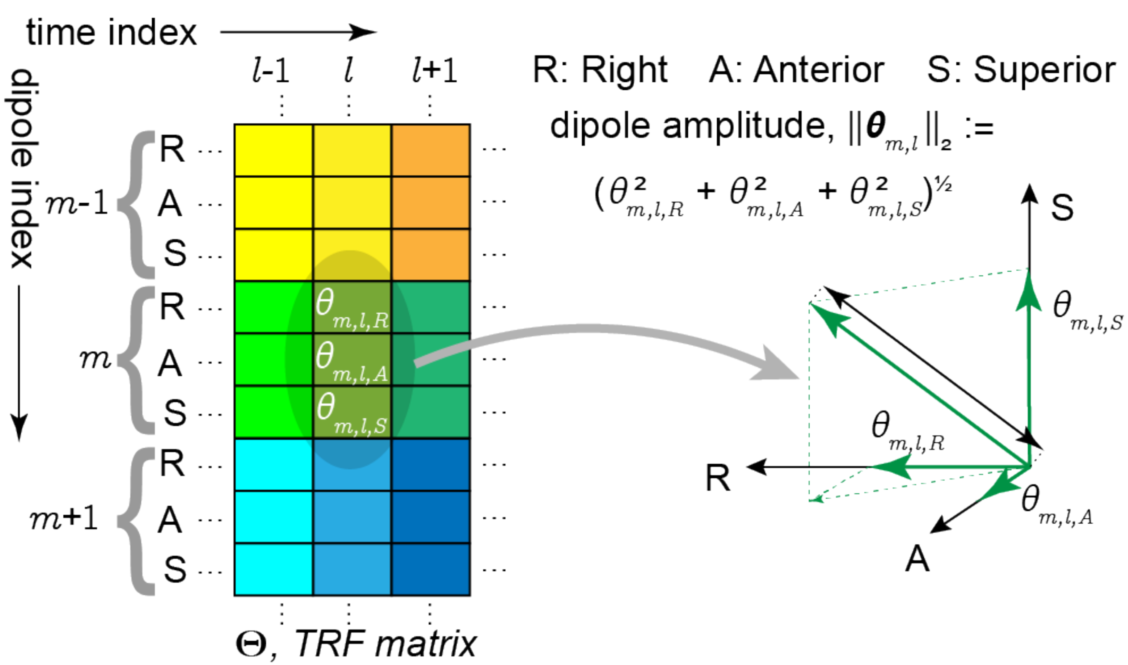
Rotational Invariance



$\mathcal{P}_{2,1,1}(\Theta) = \sum_m \sum_l \|\boldsymbol{\theta}_{m,l}\|_2$

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NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

$$\min_{\boldsymbol{\Theta}} \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}\widetilde{\mathbf{S}}\|^2_{(\boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^\top)^{-1}} + \eta\mathcal{P}_{2,1,1}(\boldsymbol{\Theta})$$

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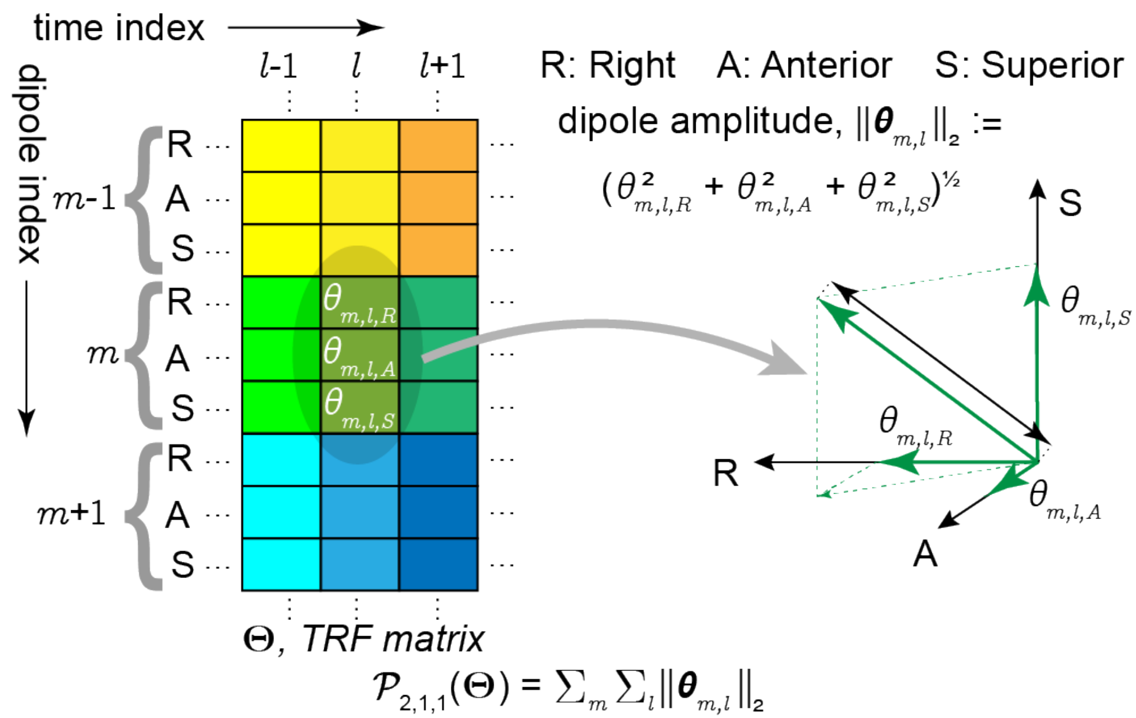

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NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

Regularization parameter

$$\min_{\mathbf{\Theta}} \ \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\mathbf{\Theta}\widetilde{\mathbf{S}}\|^2_{(\mathbf{\Sigma}_w + \mathbf{L}\mathbf{\Gamma}\mathbf{L}^\top)^{-1}} + \eta \mathcal{P}_{2,1,1}(\mathbf{\Theta})$$

But

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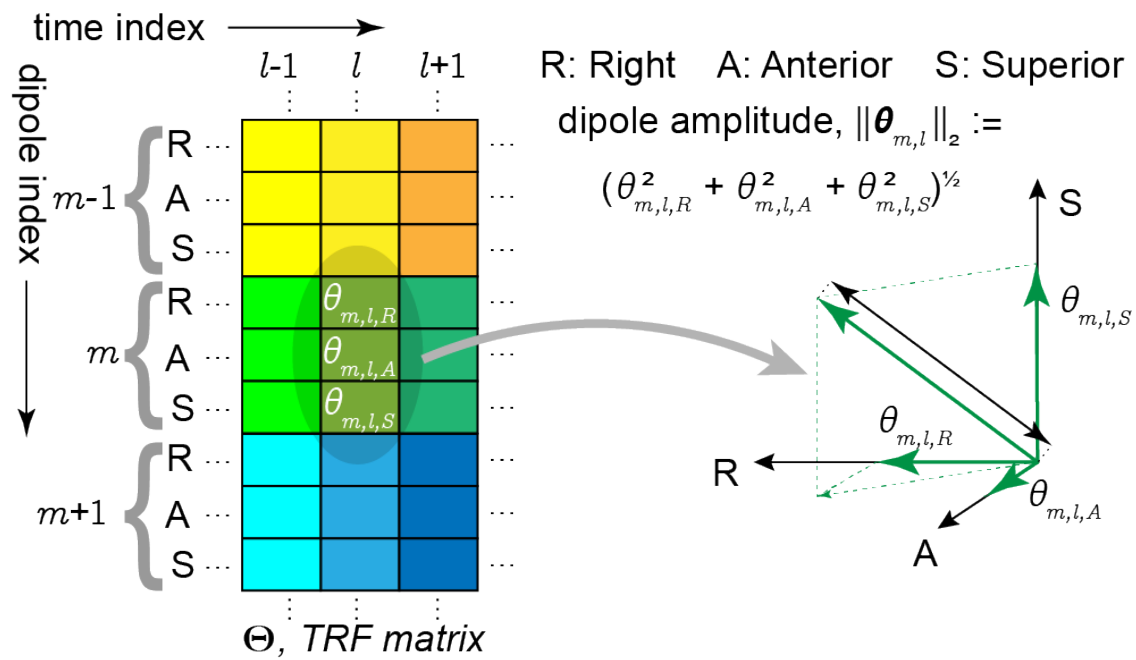

$$\mathcal{P}_{2,1,1}(\mathbf{\Theta}) = \sum_m \sum_l \|\boldsymbol{\theta}_{m,l}\|_2$$

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NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

$$\min_{\boldsymbol{\Theta}} \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}\widetilde{\mathbf{S}}\|^2_{(\boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^\top)^{-1}} + \eta\mathcal{P}_{2,1,1}(\boldsymbol{\Theta})$$

But

- NCRFs may not be smooth → Consider Gabor dictionary representation
- It is an ill-posed problem → Enforce spatial and lag-domain sparsity
- Nuisance covariance, $\boldsymbol{\Gamma}$ is unknown

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NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

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But

- NCRFs may not be smooth → Consider Gabor dictionary representation
- It is an ill-posed problem → Enforce spatial and lag-domain sparsity
- Nuisance covariance, $\boldsymbol{\Gamma}$ is unknown → Estimate $\boldsymbol{\Gamma}$ jointly with $\boldsymbol{\Theta}$

Estimate $\boldsymbol{\Gamma}$ jointly with $\boldsymbol{\Theta}$:

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P. Das, C. Brodbeck, J. Z. Simon, and B. Babadi, NeuroImage, 2020

NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

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$$\boldsymbol{\Gamma}^{(r+1)} = \operatorname*{argmin}_{\boldsymbol{\Gamma}} \quad \text{tr}\left(\boldsymbol{\Sigma}_v^{-1}\mathbf{C}_v^{(r)}\right) + \log|\boldsymbol{\Sigma}_v|$$
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Champagne
Wipf et. al, 2010

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Forward-Backward splitting
Nesterov, 2005
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Champ-Lasso Algorithm

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Application to MEG

MEG Data:

Brodbeck & Simon, 2018

- 17 young-adult participants.
- Two 60-second segments from 'The Legend of Sleepy Hollow' by W. Irving.
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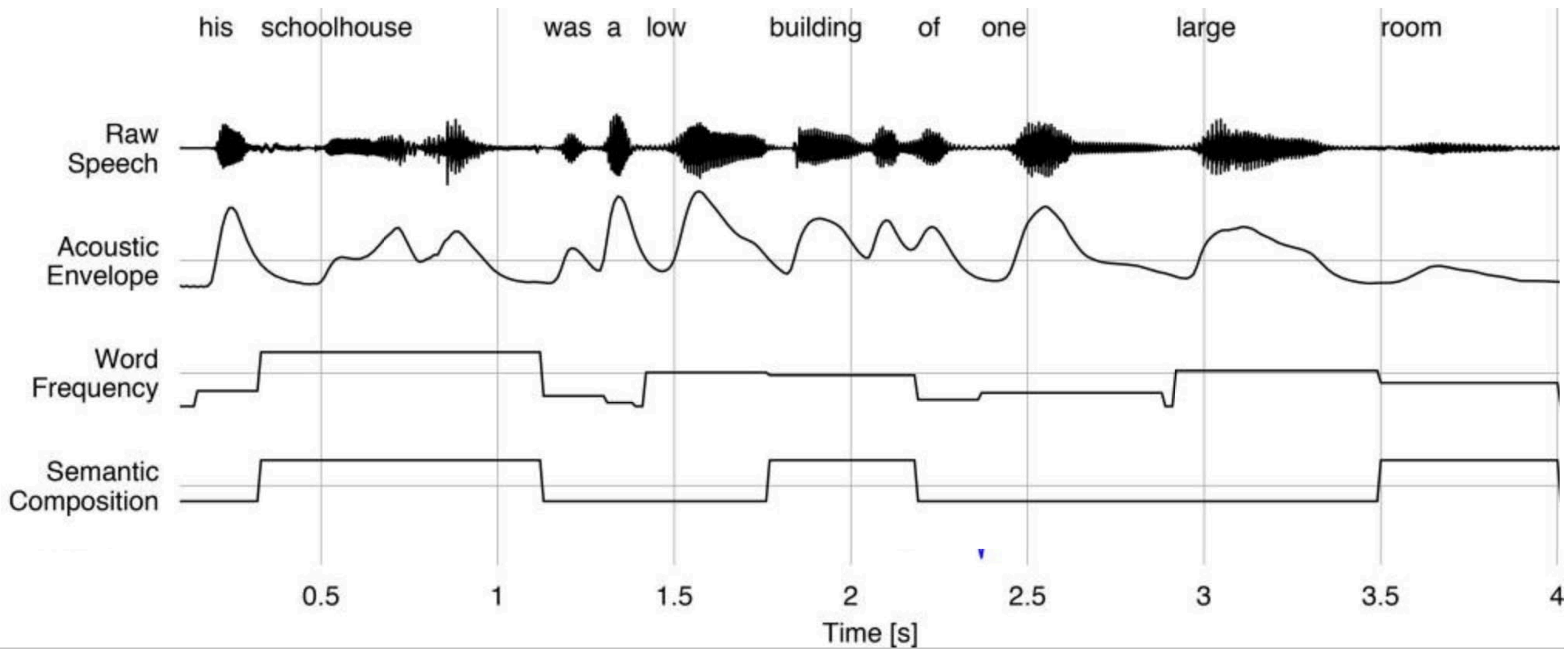

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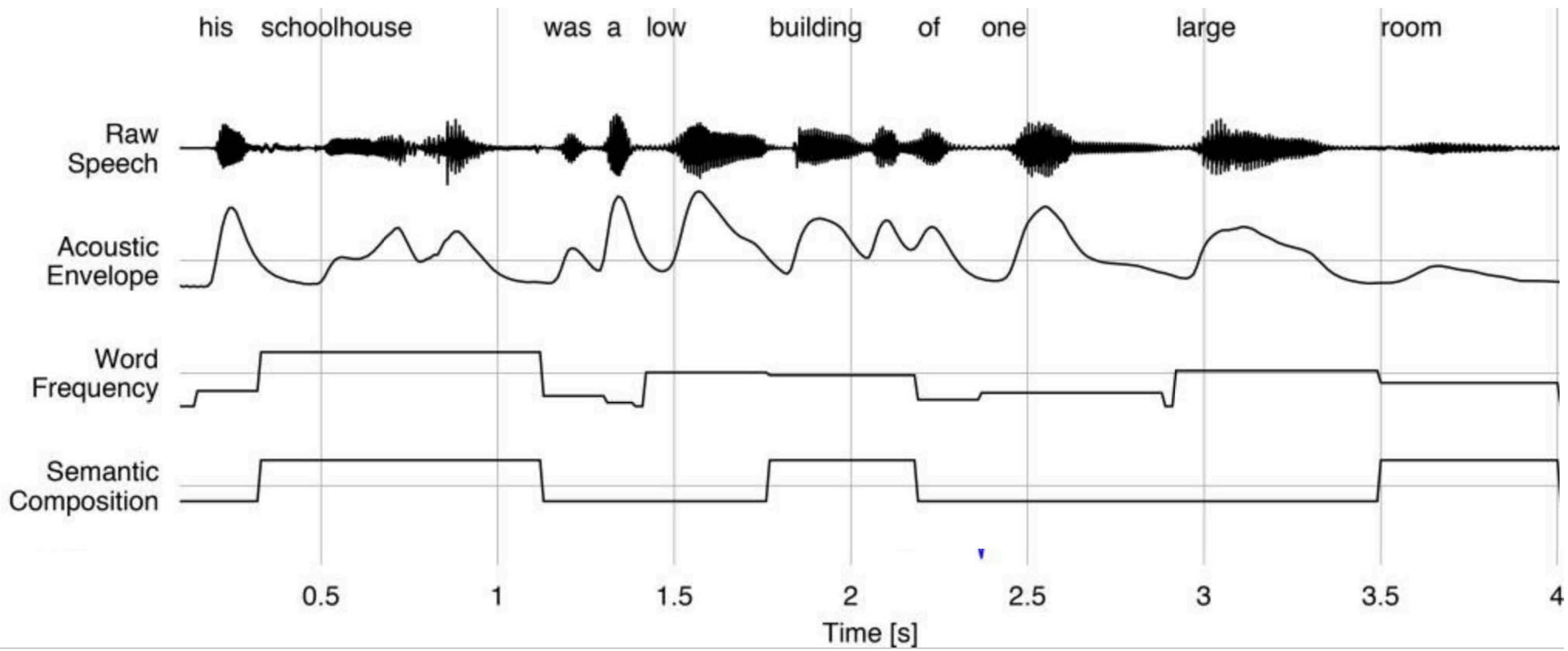

- No MRI was available:
- 'fsaverage' morphed in individual head shape

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Application to MEG

Semantic composition
Westerlund et. al., 2015

Word frequency
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Acoustic envelope
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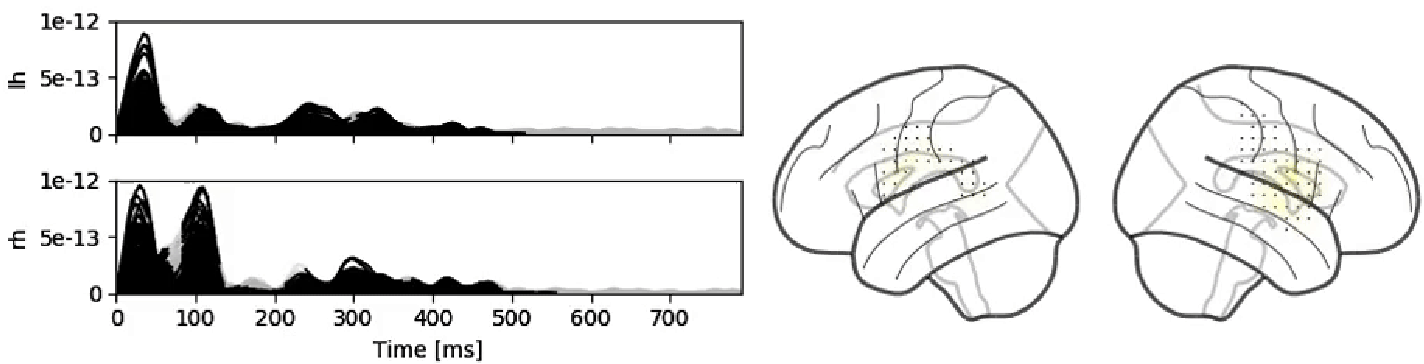

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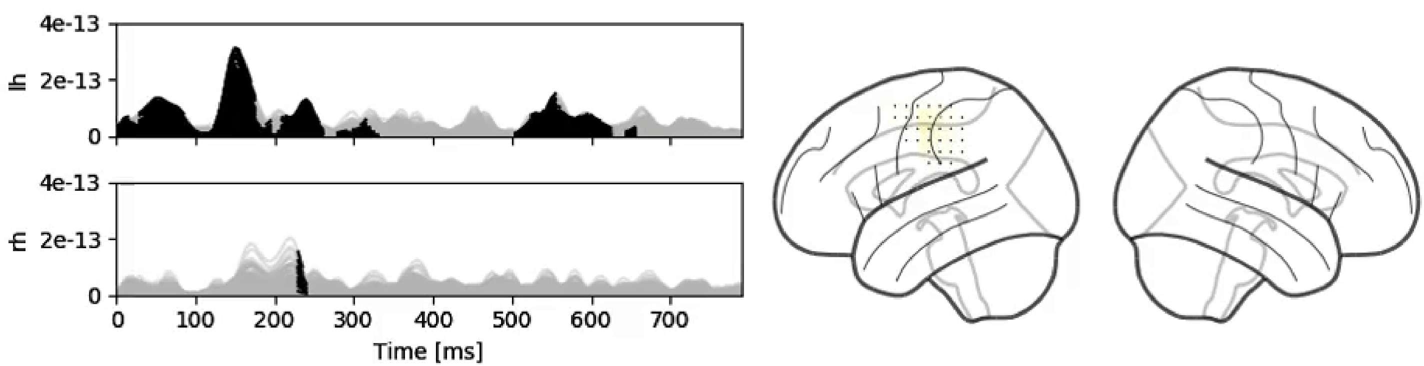

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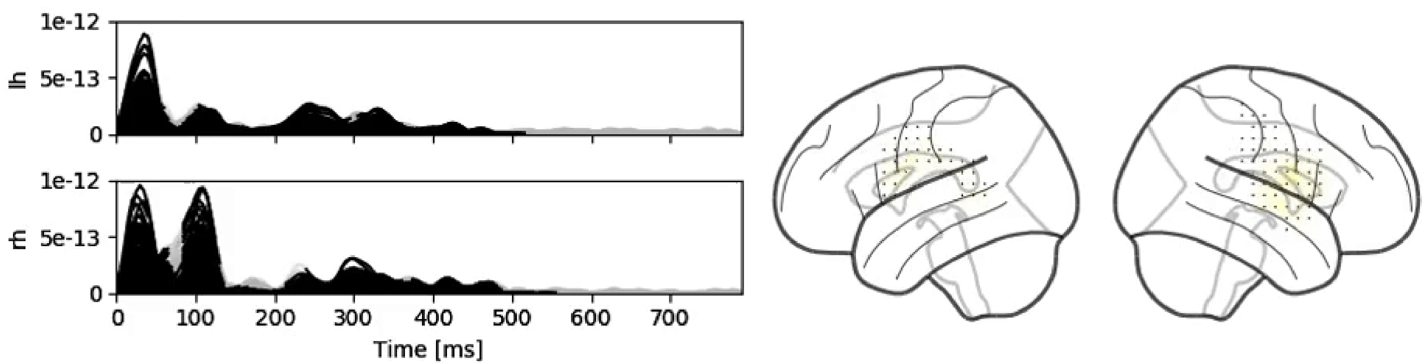

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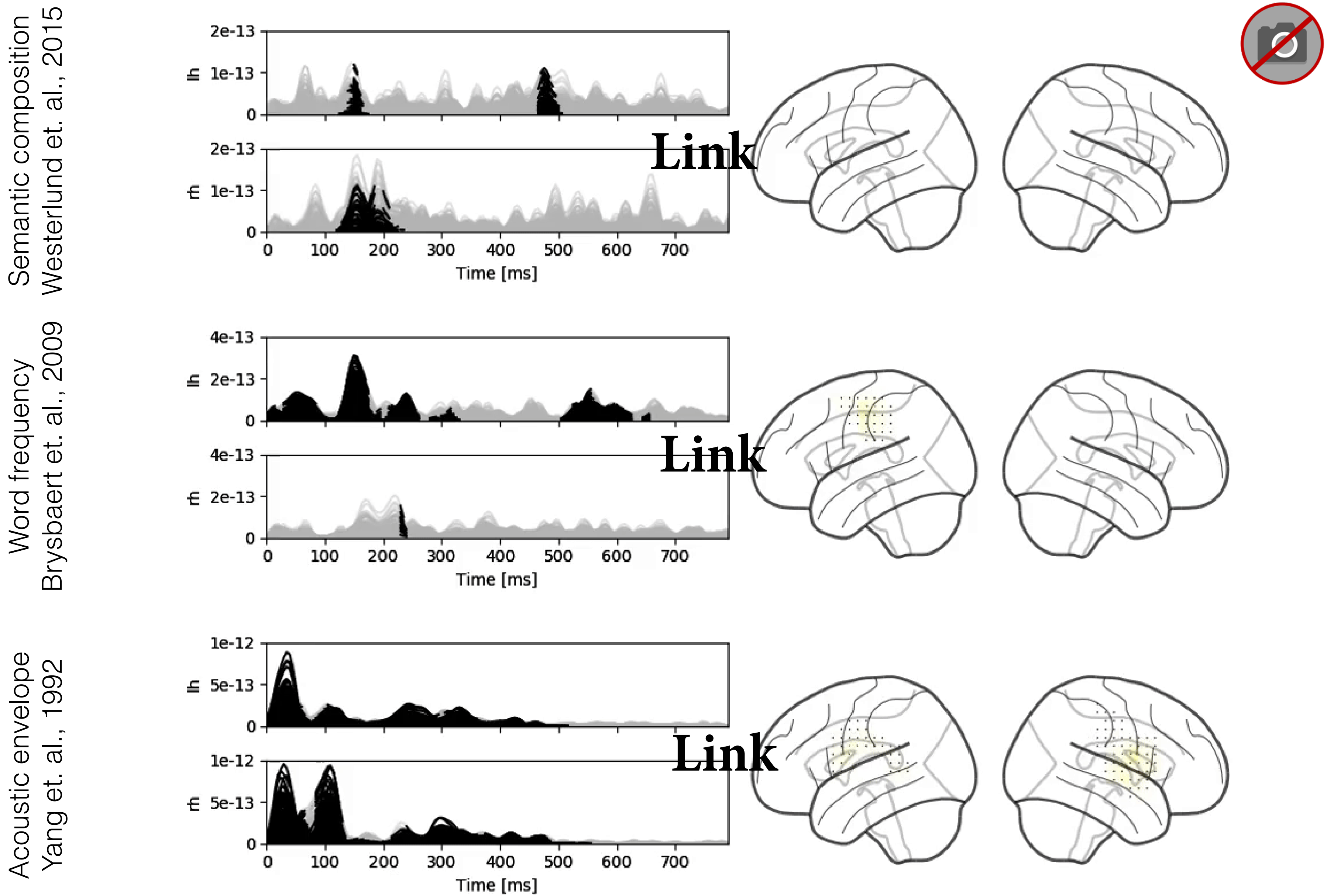

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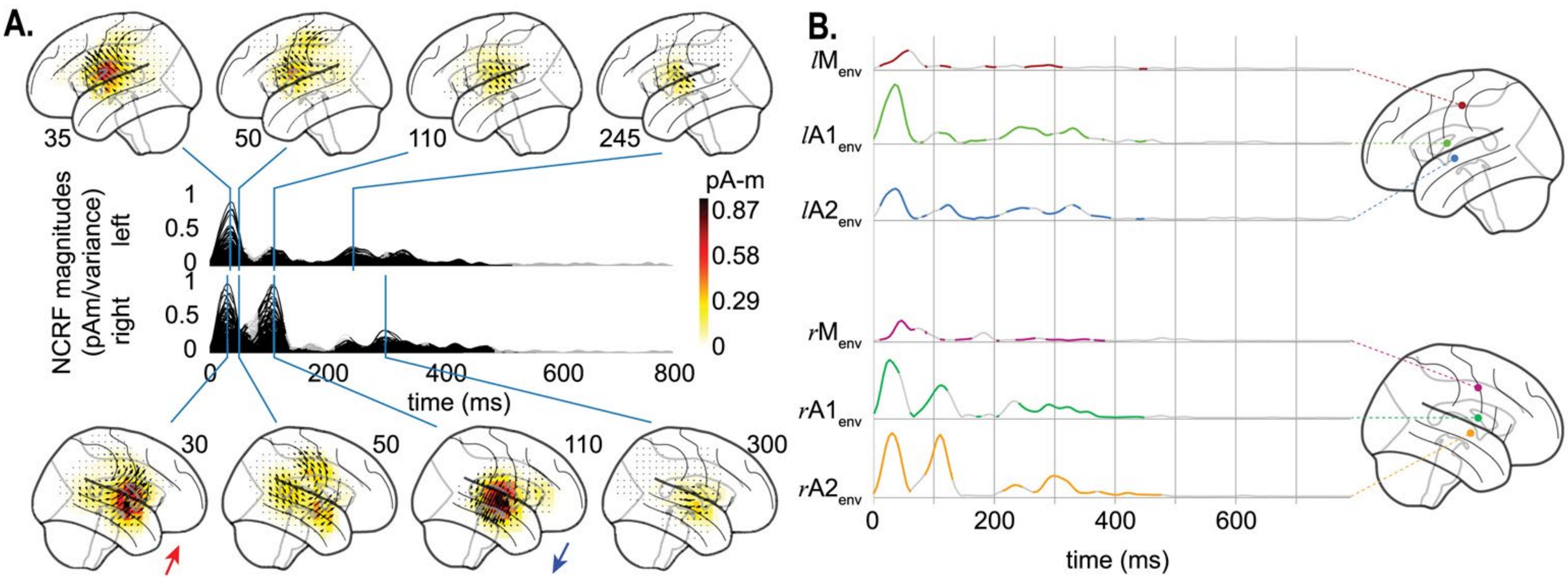

- M50 at ~ 30-35 ms, M100 at ~ 110 ms (reversed polarity, stronger in r.h.)
- Bi-lateral motor activity at ~ 50 ms

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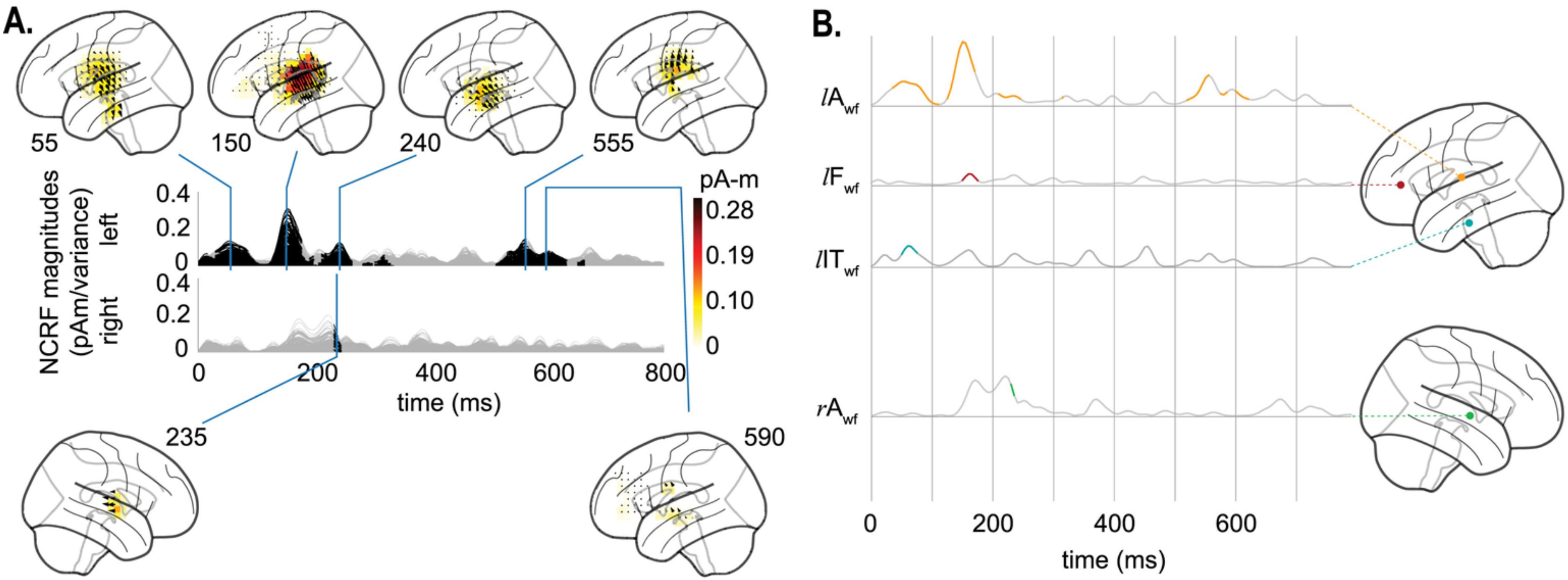
Application to MEG

Word frequency:



- Strong auditory component in the l.h. at ~ 150 ms.
- Weak frontal and inferior temporal components in l.h.

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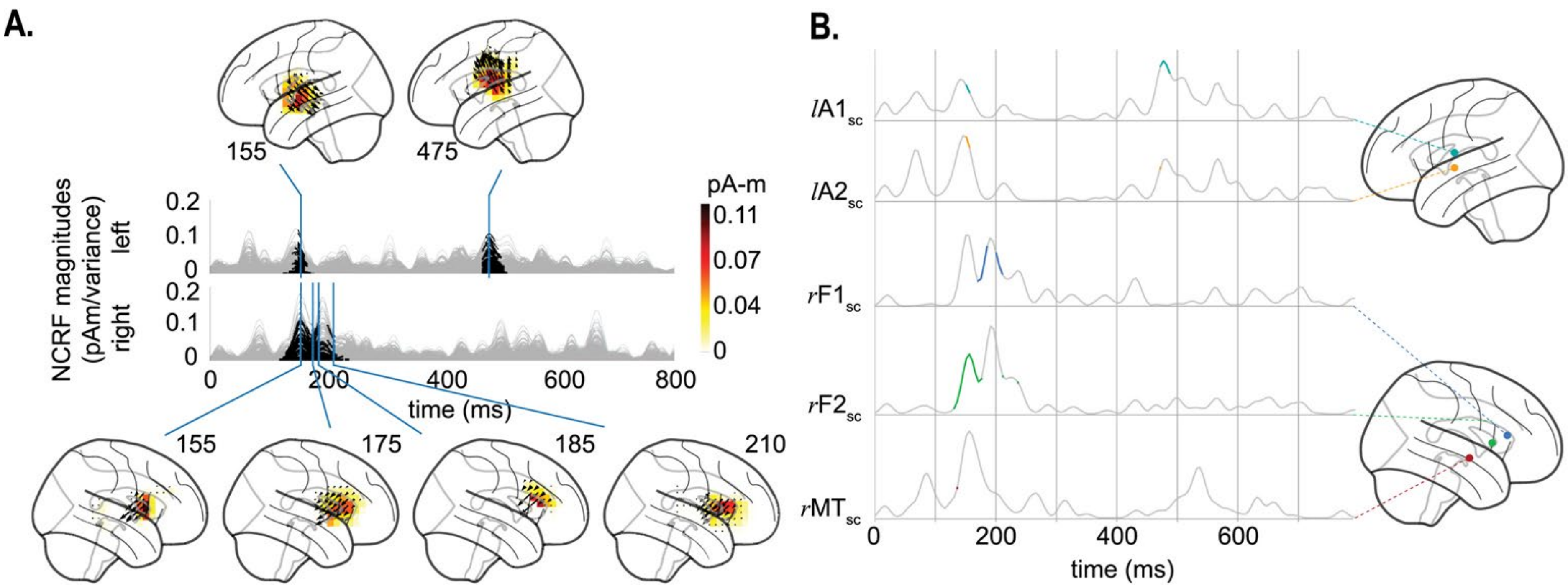
Application to MEG

Semantic composition:



- Bilateral auditory component at ~ 155 ms, late auditory component at ~ 475 ms.
- Auditory-frontal dynamics at ~ 175-210 ms ($r\mathrm{MT}_{\mathrm{sc}} \rightarrow r\mathrm{F2}_{\mathrm{sc}} \rightarrow r\mathrm{F1}_{\mathrm{sc}} \rightarrow r\mathrm{F2}_{\mathrm{sc}}$).

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Summary

- A tool for directly extracting the cortical dynamics that underlie continuous stimuli processing from MEG.
- Novel spatiotemporal prior that not only combats overfitting and spatio-temporal dispersion but also handles lack of MR scans.
- The NCRFs are readily interpretable in a meaningful fashion without any recourse to post-hoc processing.

NCRFs as a powerful source localization tool for continuous stimuli experiments.

Acknowledgements

Advisor:

- Behtash Babadi

Collaborators:

- Christian Brodbeck
- Jonathan Z. Simon

Funding:

- National Science Foundation (Award No: 1552946 and 1734892)
- Defense Advanced Research Projects Agency (Award No:N66001182240224)
- National Institutes of Health (Award No: R01-DC014085

Github Repo Link

Thank you!

Presentation available at