Cortical Localization of the Auditory Temporal Response Function from MEG via non-convex Optimization

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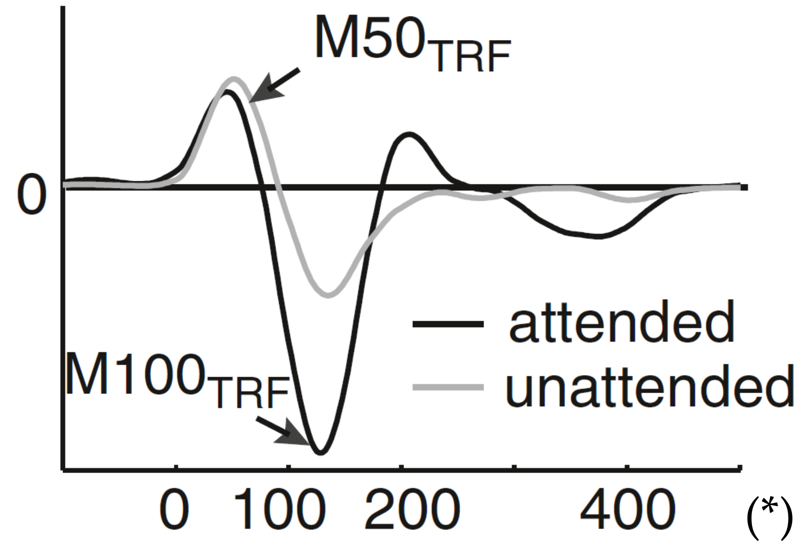
- Background
- Motivation
- Unified Computational Framework
- Proposed Algorithm
- Results:
  - A Simulation Study
  - Analysis of MEG from story listening
- Summary

MEG in speech: TRFs

- Linear filter model
- Predicts MEG response from continuous stimulus.
- “Best” Kernel : temporal response function



M100
Attentional state

Cortical origin???

(*adapted from Ding & Simon (2013). Robust cortical encoding of slow temporal modulations of speech. In Basic Aspects of Hearing Springer, New York, NY.)

Existing methods:

Two stage operation

Problems:

- High bias
- Leakage
- Limited spatial resolution

Full neural source localization power of MEG

Distributed source model

- N sensors over the scalp
- *T* time points
- MEG matrix, **Y** *(N* x *T)*

$$\boldsymbol{y}_t := \begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \\ \\ y_t^{(N)} \end{bmatrix}$$

$$\begin{array}{c c} \text{Sensor space} \uparrow & \boldsymbol{Y} := \begin{bmatrix} y_1^{(1)} & y_2^{(1)} & \cdots & y_t^{(1)} & \cdots & y_T^{(1)} \\ y_1^{(2)} & y_2^{(2)} & \cdots & y_t^{(2)} & \cdots & y_T^{(2)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ y_1^{(N)} & y_2^{(N)} & \cdots & y_t^{(N)} & \cdots & y_T^{(N)} \end{bmatrix} \\ & \xrightarrow[\text{Time}]{} \end{array}$$

- M voxels
- Primary current ~ virtual current dipole
- Each dipole ~ 3 components, $j_{m,R,t}, j_{m,A,t}, j_{m,S,t}$
- Neural current matrix, ***J*** *(3M* x *T)*

Distributed source model

$$\boldsymbol{Y} = \mathbf{L}\boldsymbol{J} + \mathbf{W}$$

- **L:** lead-field matrix *($N \times 3M$)*

  Maxwell's equations

- **W:** measurement noise matrix

Typically $N \sim 10^2$, $M \sim 10^4$

TRF model

- Assumption: neural sources process the stimulus linearly

$$j_{m,d,t} = (\boldsymbol{\tau}_{m,d})^\top \mathbf{e}_t + v_{m,d,t}$$

- $\boldsymbol{\tau}_{m,d}$: dth component of 3D vector TRFs
- $\mathbf{e}_t$: Stimulus history
- In matrix form:

$$\boldsymbol{J} = \mathbf{\Phi}\mathbf{S} + \mathbf{V}$$

- $\mathbf{\Phi}$: TRF matrix
- $\mathbf{S}$: stimulus covariate matrix
- $\mathbf{V}$: background activity matrix

Distributed Source Model $\quad \boldsymbol{Y} = \mathbf{L}\boldsymbol{J} + \mathbf{W}$

TRF model $\quad \boldsymbol{J} = \mathbf{\Phi}\mathbf{S} + \mathbf{V}$

Our Aim: directly estimate the TRF matrix, $\mathbf{\Phi}$ given
- MEG measurement matrix, $\mathbf{Y}$
- stimulus covariate matrix, $\mathbf{S}$

Modelling Assumptions:

$$p\left(\mathbf{W}\right) \propto \left|\mathbf{\Sigma}_w\right|^{-T/2} \exp -\frac{1}{2}\|\mathbf{W}\|^2_{\mathbf{\Sigma}_w^{-1}}$$

$$p(\mathbf{V}|\mathbf{\Gamma}) \propto \prod_{m=1}^{M} |\Gamma_m|^{-T/2} \;\exp -\frac{1}{2}\sum_{m=1}^{M} \|V_m\|^2_{\Gamma_m^{-1}},$$

$$\|A\|_B := \sqrt{\text{tr}\left\{A^\top B A\right\}}$$

Joint distribution:

$$p\left(\mathbf{Y},\boldsymbol{J}|\mathbf{\Phi},\mathbf{\Gamma}\right)\propto|\mathbf{\Sigma}_w|^{-T/2}\exp-\frac{1}{2}\|\mathbf{Y}-\mathbf{L}\boldsymbol{J}\|^2_{\mathbf{\Sigma}_w^{-1}}\times|\mathbf{\Gamma}|^{-T/2}\exp-\frac{1}{2}\|\mathbf{J}-\mathbf{\Phi}\mathbf{S}\|_{\mathbf{\Gamma}^{-1}}$$

Eliminate *J* by marginalization:

$$p\left(\mathbf{Y}|\mathbf{\Phi},\mathbf{\Gamma}\right)\propto|\mathbf{\Sigma}_w+\mathbf{L}\mathbf{\Gamma}\mathbf{L}^\top|^{-T/2}\times\exp\left(-\frac{1}{2}\|\mathbf{Y}-\mathbf{L}\mathbf{\Phi}\mathbf{S}\|^2_{(\mathbf{\Sigma}_w+\mathbf{L}\mathbf{\Gamma}\mathbf{L}^\top)^{-1}}\right)$$

$$(P0)\qquad\qquad \min_{\mathbf{\Phi}}\ \frac{1}{2}\|\mathbf{Y}-\mathbf{L}\mathbf{\Phi}\mathbf{S}\|^2_{(\mathbf{\Sigma}_w+\mathbf{L}\mathbf{\Gamma}\mathbf{L}^\top)^{-1}}$$

- Temporal smoothness: Gabor basis

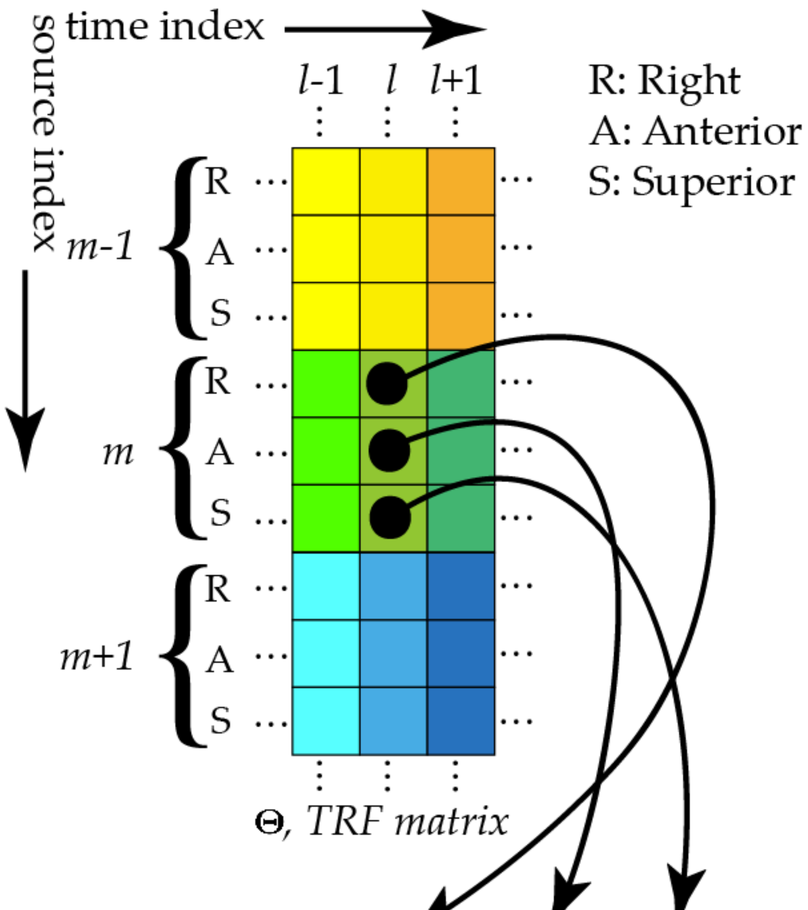
$$\boldsymbol{\tau}_{m,d}=\mathbf{G}\boldsymbol{\theta}_{m,d}$$

- Sparsity: Add norm penalty

$$(P1)\qquad\qquad \min_{\mathbf{\Theta}}\ \frac{1}{2}\|\mathbf{Y}-\mathbf{L}\mathbf{\Theta}\widehat{\mathbf{S}}\|^2_{(\mathbf{\Sigma}_w+\mathbf{L}\mathbf{\Gamma}\mathbf{L}^\top)^{-1}}+\eta\|\mathbf{\Theta}\|_{2,1,1}$$

2,1,1-Mixed norm

- Direction invariant
- Penalizes only the length of 3D TRF vector
- No prior assumptions on the source activity directions!!!



$$\|\Theta\|_{2,1,1} = \sum_{m=1}^{M} \sum_{l=1}^{L} \left(\theta_{m,R,l}^2 + \theta_{m,A,l}^2 + \theta_{m,S,l}^2\right)^{1/2}$$

$$(P1) \qquad \min_{\boldsymbol{\Theta}} \ \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}\widehat{\mathbf{S}}\|^2_{(\boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^\top)^{-1}} + \eta\|\boldsymbol{\Theta}\|_{2,1,1}$$

- Easy to solve for $\boldsymbol{\Theta}$
- But requires knowledge of $\boldsymbol{\Gamma}$
- Not available.
- Idea: Solve for both TRF matrix, and source variance

$$(P2) \qquad \min_{\boldsymbol{\Theta},\boldsymbol{\Gamma}} \ \frac{T}{2}\log\left(\boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^\top\right) + \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}\widehat{\mathbf{S}}\|^2_{(\boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^\top)^{-1}} + \eta\|\boldsymbol{\Theta}\|_{2,1,1}$$

$$(P2) \qquad \min_{\mathbf{\Theta},\mathbf{\Gamma}} \; \frac{T}{2}\log\left(\mathbf{\Sigma}_w + \mathbf{L}\mathbf{\Gamma}\mathbf{L}^\top\right) + \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\mathbf{\Theta}\widehat{\mathbf{S}}\|^2_{(\mathbf{\Sigma}_w+\mathbf{L}\mathbf{\Gamma}\mathbf{L}^\top)^{-1}} + \eta\|\mathbf{\Theta}\|_{2,1,1}$$

- Non-convex in $\mathbf{\Gamma}$
- Direct optimization hard
- Update $\mathbf{\Gamma}$ and $\mathbf{\Theta}$ alternatingly
- Co-ordinate descent algorithm

Algorithm

Initialize $\boldsymbol{\Theta}^0 = \mathbf{0}$

$$\mathbf{C}_v{}^{(r)} = \frac{1}{T}(\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}^{(r)}\widetilde{\mathbf{S}})(\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}^{(r)}\widetilde{\mathbf{S}})^\top$$

- Update $\boldsymbol{\Gamma}$

$$\boldsymbol{\Gamma}^{(r+1)} = \arg\min_{\boldsymbol{\Gamma}} \mathrm{tr}\left(\boldsymbol{\Sigma}_v^{-1}\mathbf{C}_v^{(r)}\right) + \log|\boldsymbol{\Sigma}_v|$$
$$\text{s.t. } \boldsymbol{\Sigma}_v = \boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^\top$$

- Update $\boldsymbol{\Theta}$

$$\boldsymbol{\Theta}^{(r+1)} = \arg\min_{\boldsymbol{\Theta}} \frac{1}{2}\|\mathbf{L}\boldsymbol{\Theta}\widetilde{\mathbf{E}} - \mathbf{Y}\|^2_{\boldsymbol{\Sigma}_v^{(r+1)\,-1}} + \eta\|\boldsymbol{\Theta}\|_{2,1,1}$$

$$\boldsymbol{\Sigma}_v{}^{(r+1)} = \boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}^{(r+1)}\mathbf{L}^\top$$

- non-convex problem
- ‘Champagne’ (Wipf et al., 2010)
- each pass is guaranteed to reduce cost function

- smooth + non-smooth
- forward-backward splitting
- ‘FASTA’ (Goldstein et al., 2014)

Dataset

- **MEG Data**
  – 17 participants.
  – Two 60-second segments from ‘The Legend of Sleepy Hollow’ by Washington Irving.
  – 3 repetitions for each segment.

- **Average “brain model”**
  – ‘fsaverage’, FreeSurfer.
  – scaled and coregistered to each subject's head.
  – volume source space.
  – free orientation lead-field matrix.

Dataset (Continued…)

- **Stimulus representation:**

  – acoustic envelope, average over frequency band of an auditory spectrogram representation

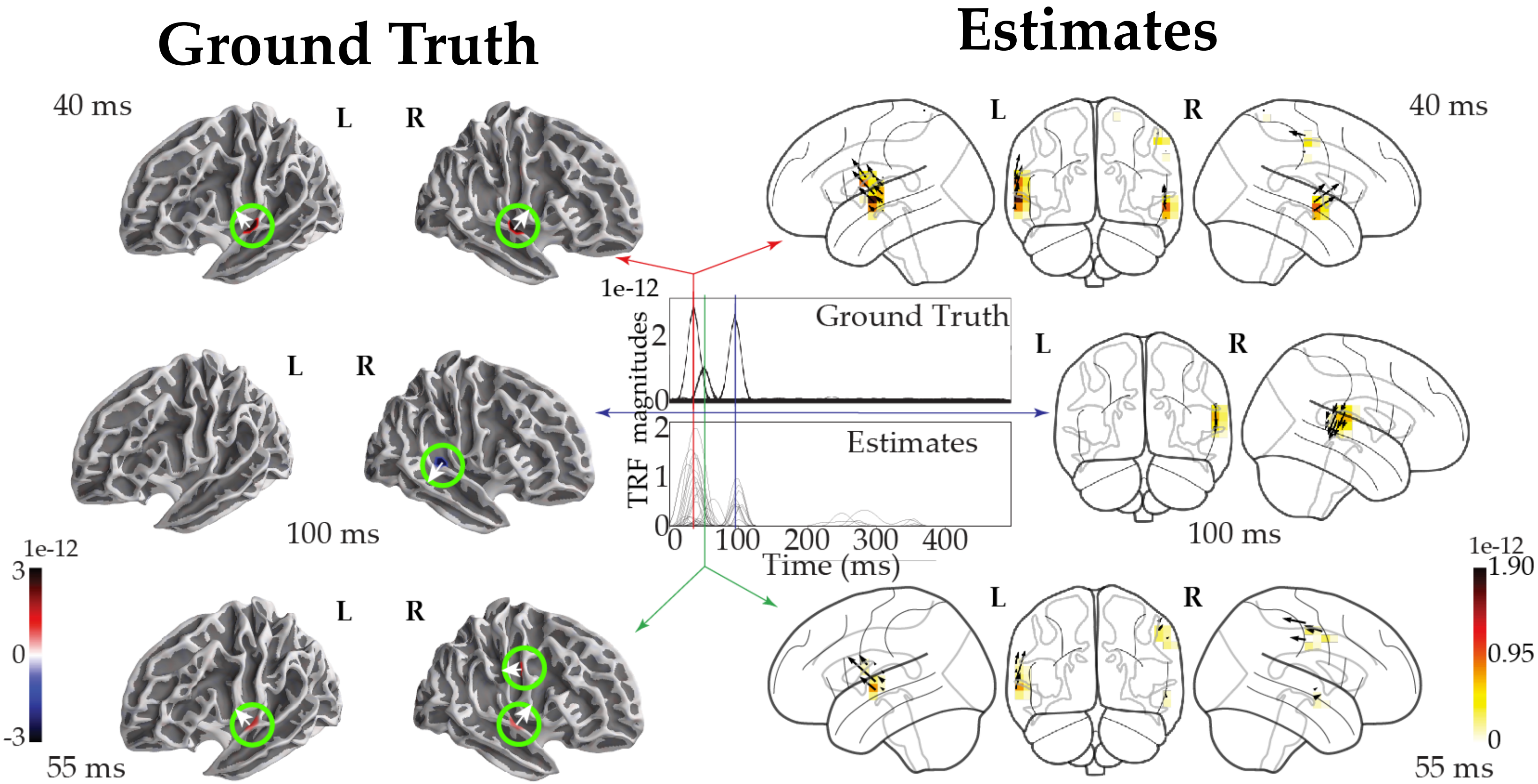
- **Statistical Tests:**

  – spatial smoothing w/ Gaussian kernel (s.d. 10 mm).

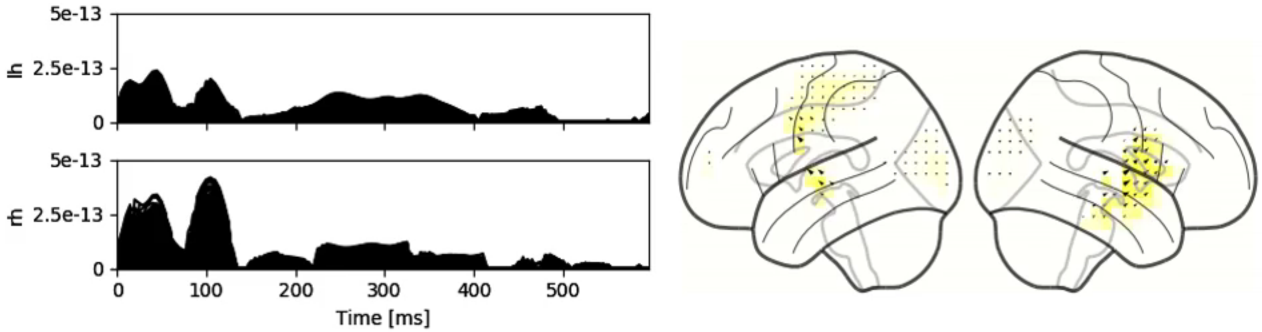
  – tested for consistent directionality vs. uniformity (Mardia, 2009) using permutation test.

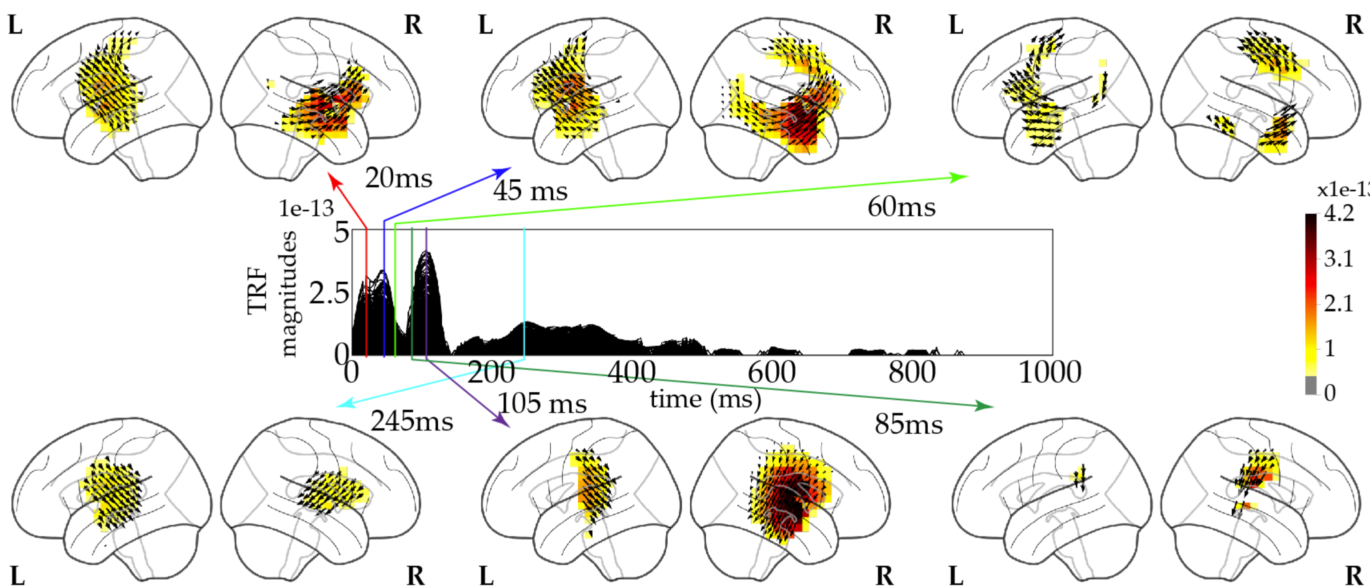
Simulation Results:

– finer source space
– direction constrained



Auditory response functions





Only significant (p <.05) values are shown.

- a new framework for direct TRF localization
- integrates the TRF and distributed forward source models
- alternatingly update the TRFs and Source variances to optimize the objective.
- Improves estimates + their location

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Some of this work has been accepted for presenatation at 2018 annual meeting, Society for Neuroscience, San Diego, CA

Thank you!
Questions?

Follow on Github:

Model Schematic