Integrating Temporal Response Function Estimation with Sparse Source Localization

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Funding:
• National Science Foundation (Award No: 1552946 and 1734892)
• Defense Advanced Research Projects Agency (Award: N6600118240224)
• National Institutes of Health (Award No: R01-DC014085)
Encoding Model of Speech Processing

Linear filter model:
- Predict the M/EEG response from continuous stimulus

Functional Role:
- Different peaks are associated with different modes of processing
- M50 (c.f. EEG P1): encoding of acoustic-level features
- M100 (c.f. EEG N1): encoding modulated by attention

Beyond acoustic processing:
- Phoneme level processing
- Lexical processing
- Semantic processing

Cortical Origins?
Cortical Origins of TRFs?

Insight from intracranial recordings
› Electrophysiology, electrocorticography
› Limited spatial range

Existing M/EEG approaches work in two-stage:
› Estimate the TRFs for each sensor → localize them within cortex
› Decompose MEG signals to source time courses → estimate TRF for each source

Challenges:
Bias propagation, spatio-temporal leakage, sensitivity to forward model mismatch

Mesgarani et. al., 2008, 2014
Lalor et. al., 2009
Brodbeck et. al., 2018
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- Estimate the TRFs for each sensor → localize them on cortical mantle
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\[ Y_{N \times T} = L_{N \times 3M} J_{3M \times T} + W_{N \times T} \]
\[ J_{3M \times T} = \Phi_{3M \times L} S_{L \times T} + V_{3M \times T} \]

Quasi-static solution to Maxwell’s equations
Neuro-current response function “NCRF”
Auditory covariates

\( N \): sensors \hspace{2cm} \( T \): time points \hspace{2cm} \( M \): sources \hspace{2cm} \( L \): lags
Cortical Origins of TRFs?

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MEG observations

\[ \mathbf{Y}_{N \times T} = \mathbf{L}_{N \times 3M} \mathbf{J}_{3M \times T} + \mathbf{W}_{N \times T} \]

Measurement noise \( \mathcal{N}(0, \Sigma_w) \)

Source time courses

Quasi-static solution to Maxwell’s equations

\[ \mathbf{J}_{3M \times T} = \Phi_{3M \times L} \mathbf{S}_{L \times T} + \mathbf{V}_{3M \times T} \]

Background nuisance \( \mathcal{N}(0, \Gamma) \)

Neuro-current response function “NCRF”

Auditory covariates

\( N \): sensors \quad \mathbf{T} \): time points \quad \mathbf{M} \): sources \quad \mathbf{L} \): lags
Measurement noise: Gaussian $\mathcal{N}(0, \Sigma_w)$

$$p(Y|J) = |(2\pi)\Sigma_w|^{-T/2} \exp \left(-\frac{1}{2} \|Y - LJ\|_{\Sigma_w^{-1}}^2 \right)$$

Background activity: Gaussian $\mathcal{N}(0, \Gamma)$

$$p(V|\Gamma) = \left(\prod_{m=1}^M |(2\pi)\Gamma_m|^{-T/2}\right) \exp \left(-\frac{1}{2} \sum_{m=1}^M \|V_m\|_{\Gamma_m^{-1}}^2 \right)$$

$$p(J|\Phi, \Gamma) = |(2\pi)\Gamma|^{-T/2} \exp \left(-\frac{1}{2} \|J - \Phi S\|_{\Gamma^{-1}}^2 \right)$$

not observed

Joint density of measurements ($Y$) and current dipoles ($J$):

$$p(Y, J|\Phi, \Gamma) = |(2\pi)\Sigma_w|^{-T/2}|(2\pi)\Gamma|^{-T/2} \exp \left(-\frac{1}{2} \|Y - LJ\|_{\Sigma_w^{-1}}^2 - \frac{1}{2} \|J - \Phi S\|_{\Gamma^{-1}}^2 \right)$$

Marginal density of the measurements given NCRFs ($\Phi$):

$$p(Y|\Phi, \Gamma) = |(2\pi) (\Sigma_w + L\Gamma L^\top)|^{-T/2} \exp \left(-\frac{1}{2} \|Y - L\Phi S\|_{(\Sigma_w + L\Gamma L^\top)^{-1}}^2 \right)$$

Maximum likelihood estimate of NCRFs ($\Phi$):

$$\min_{\Phi} \frac{1}{2} \|Y - L\Phi S\|_{(\Sigma_w + L\Gamma L^\top)^{-1}}^2$$
Measurement noise: Gaussian \( \mathcal{N}(0, \Sigma_w) \)
\[
p(Y|J) = \frac{1}{(2\pi)^\frac{N}{2} |\Sigma_w|^{\frac{N}{2}}} \exp \left( -\frac{1}{2} \| Y - LJ \|_{\Sigma_w^{-1}}^2 \right)
\]

Background activity: Gaussian \( \mathcal{N}(0, \Gamma) \)
\[
p(V|\Gamma) = \frac{1}{(2\pi)^\frac{M}{2} |\Gamma|^{\frac{M}{2}}} \exp \left( -\frac{1}{2} \sum_{m=1}^{M} \| V_m \|_{\Gamma^{-1}}^2 \right)
\]
\[
p(J|\Phi, \Gamma) = \frac{1}{(2\pi)^\frac{N}{2} |\Gamma|^{\frac{N}{2}}} \exp \left( -\frac{1}{2} \| J - \Phi S \|_{\Gamma^{-1}}^2 \right)
\]

Joint density of measurements (\( Y \)) and current dipoles (\( J \)):
\[
p(Y, J|\Phi, \Gamma) = \frac{1}{(2\pi)^\frac{N+M}{2} |\Sigma_w \Gamma|^{\frac{N+M}{2}}} \exp \left( -\frac{1}{2} \| Y - LJ \|_{\Sigma_w^{-1}}^2 - \frac{1}{2} \| J - \Phi S \|_{\Gamma^{-1}}^2 \right)
\]

Marginal density of the measurements given NCRFs (\( \Phi \)):
\[
p(Y|\Phi, \Gamma) = \frac{1}{(2\pi)^\frac{N}{2} (\Sigma_w + LGL^T)^{\frac{N}{2}}} \exp \left( -\frac{1}{2} \| Y - L\Phi S \|_{(\Sigma_w + LGL^T)^{-1}}^2 \right)
\]

Maximum likelihood estimate of NCRFs (\( \Phi \)):
\[
\min_{\Phi} \frac{1}{2} \| Y - L\Phi S \|_{(\Sigma_w + LGL^T)^{-1}}^2
\]
Maximum likelihood estimate:

$$\min_{\Phi} \frac{1}{2} \left\| Y - L\Phi S \right\|^2_{(\Sigma_w + L\Gamma L^\top)^{-1}}$$

But

- NCRF estimates contain high frequency noise
Maximum likelihood estimate:

\[
\min_{\Phi} \frac{1}{2} \| Y - L\Phi S \|^2 (\Sigma_w + LL^T)^{-1}
\]

But

- NCRF estimates contain high frequency noise

Solution:

Represent \( \Phi \) using a Gabor dictionary, \( G \)

\[
\Phi = \Theta G^T
\]

\[
\tilde{S} := G^T S
\]

\[
\min_{\Theta} \frac{1}{2} \| Y - L\Theta \tilde{S} \| \Sigma_w + LL^T^{-1}
\]

NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

$$\min_{\Phi} \frac{1}{2} \| Y - L\Phi S \|_2^2 (\Sigma_w + L\Gamma L^\top)^{-1}$$

But

- NCRF estimates contain high frequency noise → Use Gabor dictionary representation

Solution:

Represent $\Phi$ using a Gabor dictionary, $G$

$$\Phi = \Theta G^\top$$

$$\tilde{S} := G^\top S$$

$$\min_{\Theta} \frac{1}{2} \| Y - L\Theta \tilde{S} \|_{\Sigma_w + L\Gamma L^\top}^{-1}$$

Maximum likelihood estimate:

$$\min_{\Theta} \frac{1}{2} \| Y - L\Theta \tilde{S} \|_{\Sigma_w + L\Gamma L^T}^{-1}$$

But

- NCRF estimates contain high frequency noise $\rightarrow$ Use Gabor dictionary representation
- It is an ill-posed problem
  \[ N \sim 10^2 \ll M \sim 10^4 \]

Incorporate prior knowledge
Maximum likelihood estimate:

$$\min_\Theta \frac{1}{2} \| Y - L\Theta S \|^2_{\Sigma_w + L\Gamma L^T}^{-1}$$

But

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Maximum likelihood estimate:

\[
\min_{\Theta} \quad \frac{1}{2} \| Y - L\Theta S \|_{\Sigma_w + L\Gamma L^T}^{-1}
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But

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\[ N \sim 10^2 \ll M \sim 10^4 \]

Incorporate prior knowledge:

- Most of brain does not process this stimulus
  
  Enforce spatial sparsity

- TRFs dominated by peaks and troughs
  
  Enforce lag-domain sparsity

- Dipole currents are vectors
  
  Enforce coordinate rotational invariance
NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

\[
\min_{\Theta} \frac{1}{2} \| Y - L\Theta S \|_{\Sigma + LL^T}^{-1}
\]

But
- NCRF estimates contain high frequency noise \( \rightarrow \) Use Gabor dictionary representation
- It is an ill-posed problem \( \rightarrow \) Enforce spatial and lag-domain sparsity

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\[ P_{2,1,1}(\Theta) = \sum_m \sum_l \| \theta_{m,l} \|_2 \]

NCRF Learning: Bayesian Estimation (more modeling)

Maximal likelihood estimate:

$$\min_{\Theta} \frac{1}{2} \| Y - L\tilde{S}\|_2^2 (\Sigma + LGL^\top)^{-1} + \eta P_{2,1,1}(\Theta)$$

But

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$$N \sim 10^2 \ll M \sim 10^4$$

Incorporate prior knowledge:

- Most of the brain does not process this stimulus
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- Dipole currents are vectors
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Use Gabor dictionary representation

$$P_{2,1,1}(\Theta) = \sum_m \sum_l \| \Theta_{m,l} \|_2$$

Maximum likelihood estimate:

\[
\min_{\Theta} \frac{1}{2} \| \mathbf{Y} - L\Theta \tilde{S} \|_{(\Sigma_w + LL^\top)^{-1}}^2 + \eta P_{2,1,1}(\Theta)
\]

But

- NCRF estimates contain high frequency noise \(\longrightarrow\) Use Gabor dictionary representation
- It is an ill-posed problem \(\longrightarrow\) Enforce spatial and lag-domain sparsity
- Nuisance covariance \((\Gamma)\) is unknown \(\longrightarrow\) Estimate \(\Gamma\) jointly with \(\Theta\)

Estimate \(\Gamma\) jointly with \(\Theta\):

\[
\min_{\Theta, \Gamma} \frac{T}{2} \log |\Sigma_w + LL^\top| + \frac{1}{2} \| \mathbf{Y} - L\Theta \tilde{S} \|_{(\Sigma_w + LL^\top)^{-1}}^2 + \eta P_{2,1,1}(\Theta)
\]
Maximum likelihood estimate:

$$\min_{\Theta} \frac{1}{2}\|Y - L\Theta \tilde{S}\|^2_{(\Sigma_w + L\Gamma L^\top)^{-1}} + \eta P_{2,1,1}(\Theta)$$

But

- NCRF estimates contain high frequency noise $\implies$ Use Gabor dictionary representation
- It is an ill-posed problem $\implies$ Enforce spatial and lag-domain sparsity
- Nuisance covariance ($\Gamma$) is unknown $\implies$ Estimate $\Gamma$ jointly with $\Theta$

Estimate $\Gamma$ jointly with $\Theta$:

$$\min_{\Theta,\Gamma} \frac{T}{2} \log |\Sigma_w + L\Gamma L^\top| + \frac{1}{2}\|Y - L\Theta \tilde{S}\|^2_{(\Sigma_w + L\Gamma L^\top)^{-1}} + \eta P_{2,1,1}(\Theta)$$

Maximum likelihood estimate:

\[
\min_{\Theta} \frac{1}{2} \left\| Y - L\Theta\tilde{S}\right\|^2_{(\Sigma_w + L\Gamma L^\top)^{-1}} + \eta \mathcal{P}_{2,1,1}(\Theta)
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But

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Estimate \(\Gamma\) jointly with \(\Theta\):

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\min_{\Theta, \Gamma} \frac{T}{2} \log |\Sigma_w + L\Gamma L^\top| + \frac{1}{2} \left\| Y - L\Theta\tilde{S}\right\|^2_{(\Sigma_w + L\Gamma L^\top)^{-1}} + \eta \mathcal{P}_{2,1,1}(\Theta)
\]
NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

\[
\min_{\Theta, \Gamma} \frac{T}{2} \log \left| \Sigma_w + \mathbf{L} \Gamma \mathbf{L}^\top \right| + \frac{1}{2} \| \mathbf{Y} - \mathbf{L} \Theta \tilde{\mathbf{S}} \|_2^2 (\Sigma_w + \mathbf{L} \Gamma \mathbf{L}^\top)^{-1} + \eta \mathcal{P}_{2,1,1}(\Theta)
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- Hard to solve the non-convex problem
NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

$$\min_{\Theta, \Gamma} \frac{T}{2} \log \left| \Sigma_w + L \Gamma L^\top \right| + \frac{1}{2} \| Y - L \Theta \tilde{S} \|_{(\Sigma_w + L \Gamma L^\top)^{-1}}^2 + \eta \mathcal{P}_{2,1,1}(\Theta)$$

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Champ-Lasso Algorithm

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Maximum likelihood estimate:

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- NCRF estimates contain high frequency noise  \rightarrow Use Gabor dictionary representation
- It is an ill-posed problem  \rightarrow Enforce spatial and lag-domain sparsity
- Nuisance covariance ($\Gamma$) is unknown  \rightarrow Estimate $\Gamma$ jointly with $\Theta$
- Hard to solve the non-convex problem  \rightarrow Champ Lasso algorithm

Idea: coordinate descent approach

Initialize $\Theta^{(0)} = 0$

$\Theta^{(r+1)} = \arg\min_{\Theta} \quad \frac{1}{2} \|L\Theta \tilde{S} - Y\|^2_{\Sigma_v^{(r+1)}^{-1}} + \eta P_{2,1,1}(\Theta)$

$\Gamma^{(r+1)} = \arg\min_{\Gamma} \quad \text{tr} \left( \Sigma_v^{-1} C_v^{(r)} \right) + \log |\Sigma_v|$

s.t. $\Sigma_v = \Sigma_w + L\Gamma L^\top$

Champagne
Wipf et. al, 2010

Forward-Backward splitting
Nesterov, 2005
Beck & Teboulle, 2009

Code available on Github

MEG Data:

- 17 young-adult participants.
- Two 60-second segments from ‘The Legend of Sleepy Hollow’ by W. Irving.
- 3 repetitions for each segment.

No MRI was available: ‘fsaverage’ morphed in individual head shape
MEG Application to Cortical Processing of Speech

Semantics composition
Westerlund et al., 2015

Word frequency
Brysbaert et al., 2009

Acoustic envelope
Yang et al., 1992

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MEG Application to Cortical Processing of Speech

- Acoustic envelope: Yang et al., 1992
- Word frequency: Brysbaert et al., 2009
- Semantic composition: Westerlund et al., 2015

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MEG Application to Cortical Processing of Speech

- Acoustic envelope
  - Yang et. al., 1992

- Semantic composition
  - Westerlund et al., 2015

- Word frequency
  - Brysbaert et. al., 2009
Semantic composition:

- Bilateral auditory component at ~ 155 ms, late auditory component at ~ 475 ms.
- Auditory-frontal dynamics at ~ 175-210 ms (\( rMT_{sc} \rightarrow rF2_{sc} \rightarrow rF1_{sc} \rightarrow rF2_{sc} \)).
Summary

- NCRFs: A tool for directly extracting the cortical dynamics that underlie **continuous stimulus processing** from MEG.

- Novel spatiotemporal prior that not only combats overfitting and spatio-temporal dispersion but is also robust in absence of MR scan.

- The NCRFs are readily interpretable in a without post-hoc processing.

NCRFs as a powerful source localization tool for continuous stimulus experiments.

Thank you!