Computational
Sensorimotor
Systems Lab

Integrating Temporal Response Function Estimation with Sparse Source Localization

Proloy Das[1], Christian Brodbeck[2], Jonathan Z Simon[3], Behtash Babadi[3]
[1]Massachusetts General Hospital
[2]University of Connecticut
[3]University of Maryland

these slides
available at

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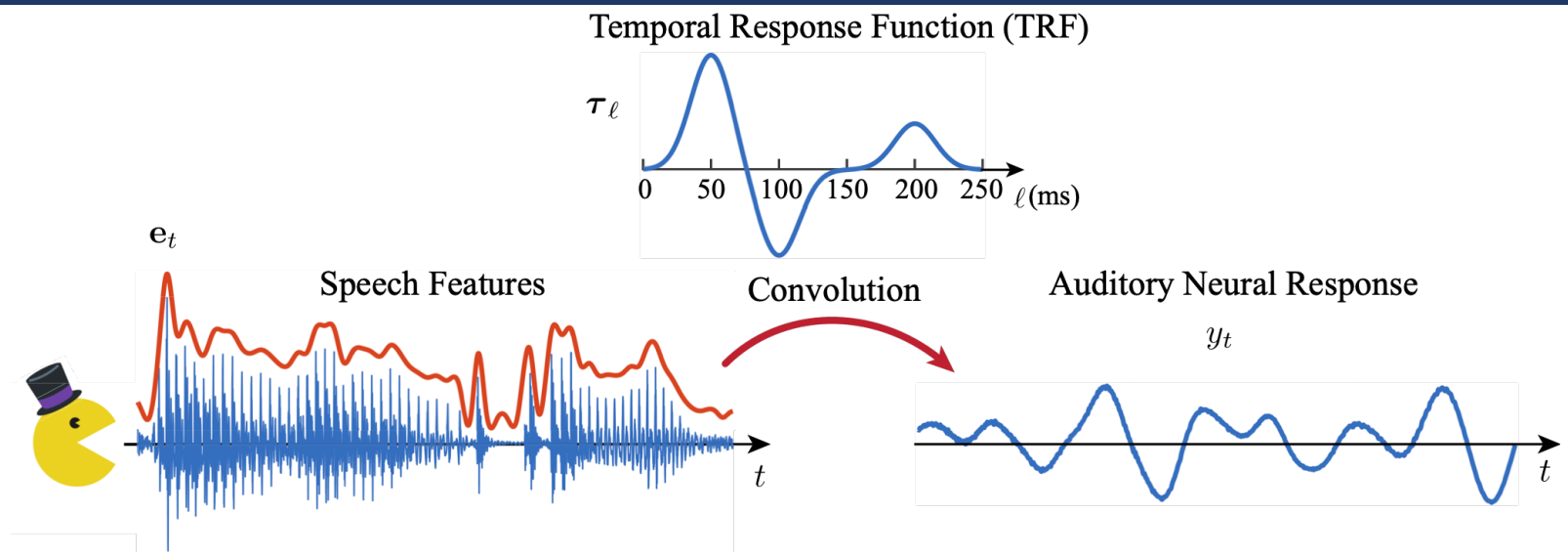
Encoding Model of Speech Processing



Linear filter model:

- predict the M/EEG response from continuous stimulus

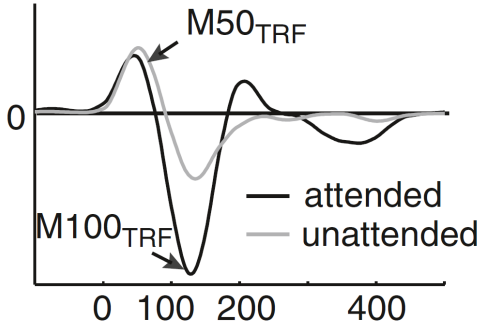
Functional Role:

- Different peaks are associated with different modes of processing
- M50 (c.f. EEG P1): encoding of acoustic-level features
- M100 (c.f. EEG N1): encoding modulated by attention

Beyond acoustic processing:

- phoneme level processing — Di Liberto et. al., 2015
- lexical processing — Brodbeck et. al., 2018
- semantic processing — Broderick et. al., 2018

Lalor et. al., 2006, 2009, 2010



Ding and Simon, 2012

Cortical Origins?

Cortical Origins of TRFs?

Insight from intracranial recordings

- Electrophysiology, electrocorticography
- Limited spatial range

Mesgarani et. al., 2008, 2014

Existing M/EEG approaches works in two-stage:

- Estimate the TRFs for each sensor → localize them within cortex
- Decompose MEG signals to source time courses → estimate TRF for each source

Lalor et. al., 2009

Brodbeck et. al., 2018

Challenges:

Bias propagation, spatio-temporal leakage, sensitivity to forward model mismatch

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MEG observations

Lead-field matrix

Source time courses

$$\mathbf{Y}_{N\times T} = \mathbf{L}_{N\times 3M}\boldsymbol{J}_{3M\times T} + \mathbf{W}_{N\times T}$$

Measurement noise

$\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_w)$

Quasi-static solution to Maxwell's equations

$$\boldsymbol{J}_{3M\times T} = \boldsymbol{\Phi}_{3M\times L}\mathbf{S}_{L\times T} + \mathbf{V}_{3M\times T}$$

Background nuisance

$\mathcal{N}(\mathbf{0}, \boldsymbol{\Gamma})$

Neuro-current response function "NCRF"

Auditory covariates

N : sensors T : time points M : sources L : lags

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NCRF Learning: Bayesian Estimation

Measurement noise: Gaussian $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_w)$

$$p\left(\mathbf{Y}|\mathbf{J}\right) = \left|(2\pi)\mathbf{\Sigma}_w\right|^{-T/2} \exp\left(-\frac{1}{2}\|\mathbf{Y} - \mathbf{L}\mathbf{J}\|^2_{\mathbf{\Sigma}_w^{-1}}\right)$$

Background activity: Gaussian $\mathcal{N}(\mathbf{0}, \mathbf{\Gamma})$

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not observed

Joint density of measurements ($\mathbf{Y}$) and current dipoles ($\mathbf{J}$):

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Marginal density of the measurements given NCRFs (Φ):

$$p\left(\mathbf{Y}|\mathbf{\Phi}, \mathbf{\Gamma}\right) = \left|(2\pi)\left(\mathbf{\Sigma}_w + \mathbf{L}\mathbf{\Gamma}\mathbf{L}^\top\right)\right|^{-T/2} \exp\left(-\frac{1}{2}\|\mathbf{Y} - \mathbf{L}\mathbf{\Phi}\mathbf{S}\|^2_{(\mathbf{\Sigma}_w + \mathbf{L}\mathbf{\Gamma}\mathbf{L}^\top)^{-1}}\right)$$

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P. Das, C. Brodbeck, J. Z. Simon, and B. Babadi, NeuroImage, 2020

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Maximum likelihood estimate:

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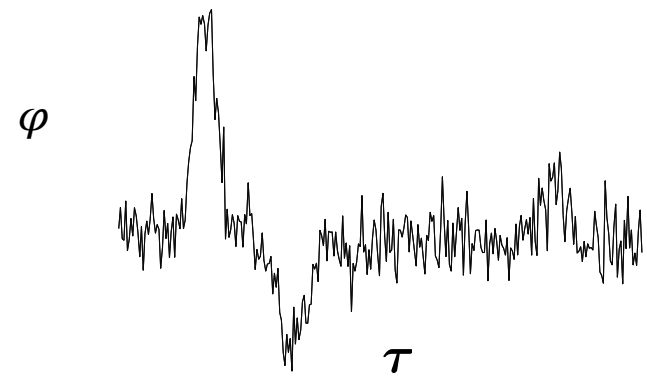

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Solution:

Represent $\mathbf{\Phi}$ using a Gabor dictionary, $\mathbf{G}$

NCRF dictionary coefficients

$$\mathbf{\Phi} = \mathbf{\Theta}\mathbf{G}^\top$$
$$\widetilde{\mathbf{S}} := \mathbf{G}^\top\mathbf{S}$$

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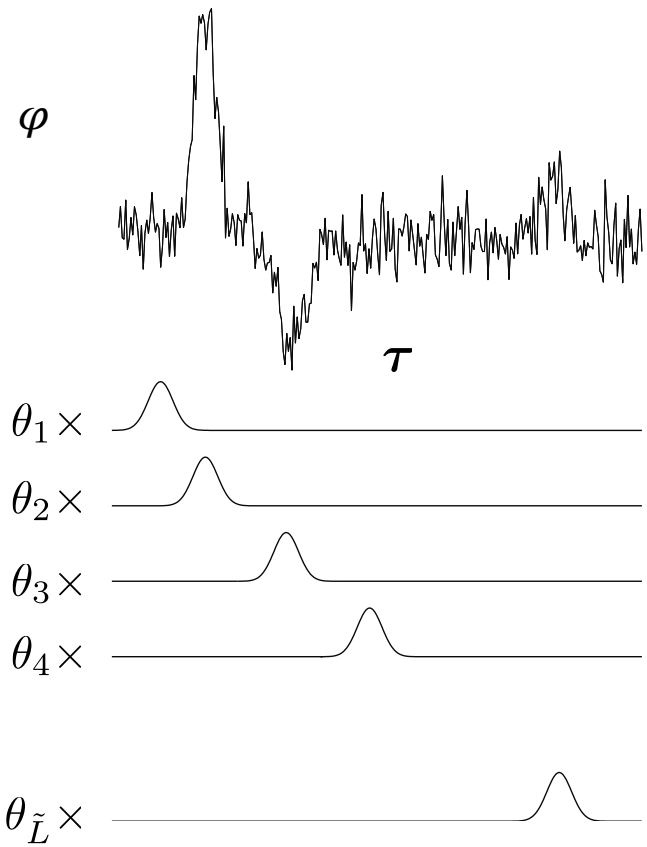

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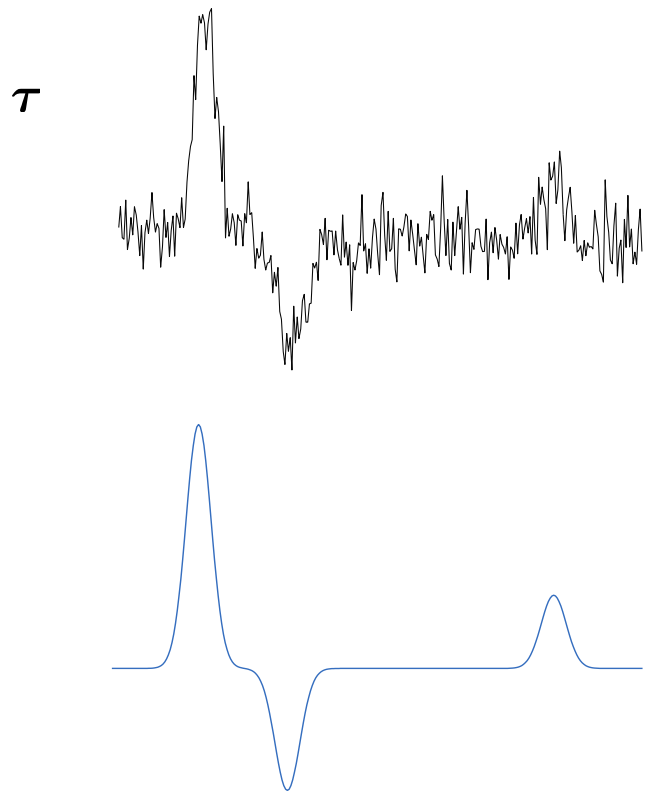

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- NCRF estimates contain high frequency noise $\longrightarrow$ Use Gabor dictionary representation
- It is an ill-posed problem

  $N \sim 10^2 \ll M \sim 10^4$

Incorporate prior knowledge

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Incorporate prior knowledge:

- Most of brain does not process this stimulus

  Enforce spatial sparsity

- TRFs dominated by peaks and troughs

  Enforce lag-domain sparsity

- Dipole currents are vectors

  Enforce coordinate rotational invariance

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$$\mathcal{P}_{2,1,1}(\boldsymbol{\Theta}) = \sum_m \sum_l \|\boldsymbol{\theta}_{m,l}\|_2$$

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NCRF Learning: Bayesian Estimation (more modeling)

Maximum likelihood estimate:

Regularization parameter

$$\min_{\boldsymbol{\Theta}} \ \frac{1}{2}\|\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}\widetilde{\mathbf{S}}\|^2_{(\boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^\top)^{-1}} + \eta\mathcal{P}_{2,1,1}(\boldsymbol{\Theta})$$

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- Hard to solve the non-convex problem $\longrightarrow$ Champ Lasso algorithm

Idea: coordinate descent approach

Initialize $\boldsymbol{\Theta}^{(0)} = \mathbf{0}$

Champ-Lasso Algorithm

$$\mathbf{C}_v{}^{(r)} = \frac{1}{T}(\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}^{(r)}\widetilde{\mathbf{S}})(\mathbf{Y} - \mathbf{L}\boldsymbol{\Theta}^{(r)}\widetilde{\mathbf{S}})^\top$$

- Update $\boldsymbol{\Gamma}$

$$\boldsymbol{\Gamma}^{(r+1)} = \underset{\boldsymbol{\Gamma}}{\operatorname{argmin}} \quad \operatorname{tr}\left(\boldsymbol{\Sigma}_v^{-1}\mathbf{C}_v^{(r)}\right) + \log|\boldsymbol{\Sigma}_v|$$
$$\text{s.t. } \boldsymbol{\Sigma}_v = \boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}\mathbf{L}^\top$$

Champagne
Wipf et. al, 2010

- Update $\boldsymbol{\Theta}$

$$\boldsymbol{\Theta}^{(r+1)} = \underset{\boldsymbol{\Theta}}{\operatorname{argmin}} \quad \frac{1}{2}\|\mathbf{L}\boldsymbol{\Theta}\widetilde{\mathbf{S}} - \mathbf{Y}\|^2_{\boldsymbol{\Sigma}_v{}^{(r+1)^{-1}}} + \eta\mathcal{P}_{2,1,1}(\boldsymbol{\Theta})$$

Forward-Backward splitting
Nesterov, 2005
Beck & Teboulle, 2009

$$\boldsymbol{\Sigma}_v{}^{(r+1)} = \boldsymbol{\Sigma}_w + \mathbf{L}\boldsymbol{\Gamma}^{(r+1)}\mathbf{L}^\top$$

Code available on Github

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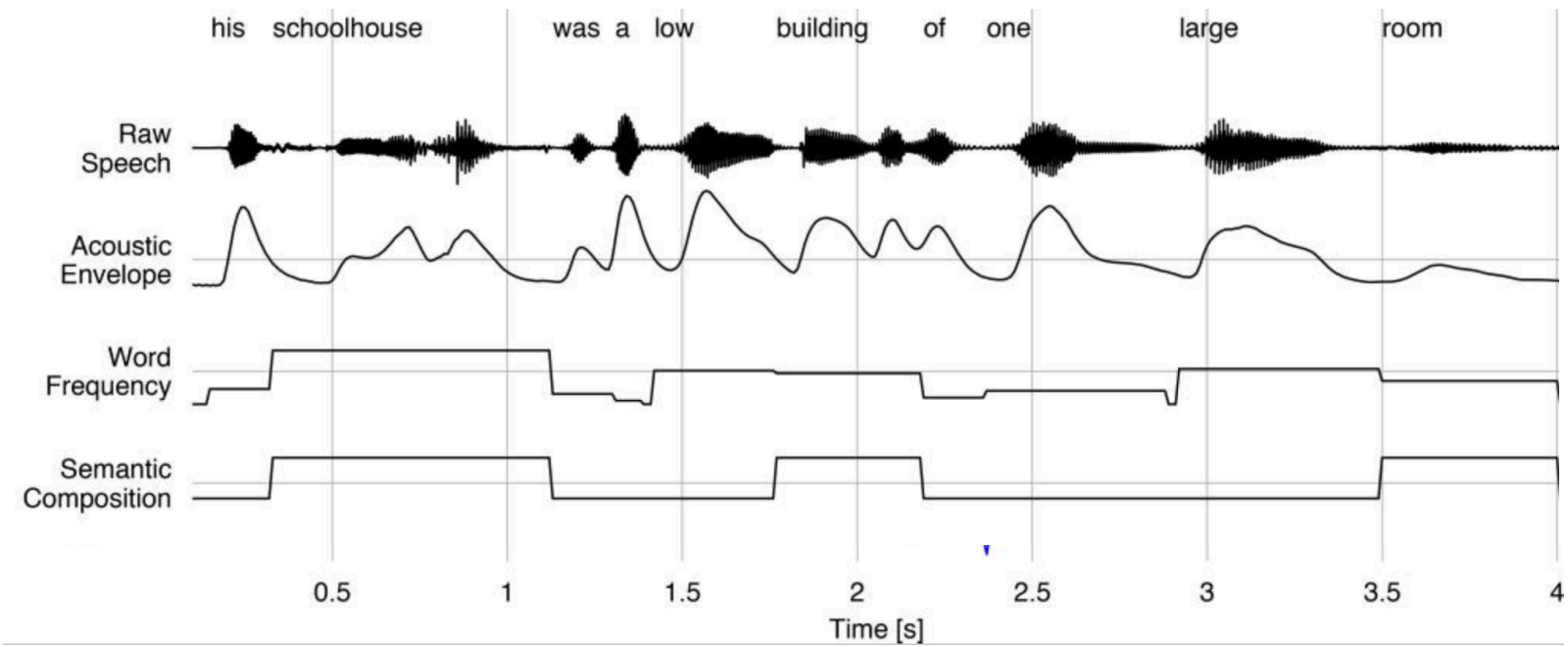
MEG Application to Cortical Processing of Speech

MEG Data:

Brodbeck & Simon, 2018

- 17 young-adult participants.
- Two 60-second segments from 'The Legend of Sleepy Hollow' by W. Irving.
- 3 repetitions for each segment.



- No MRI was available:
  'fsaverage' morphed in individual head shape

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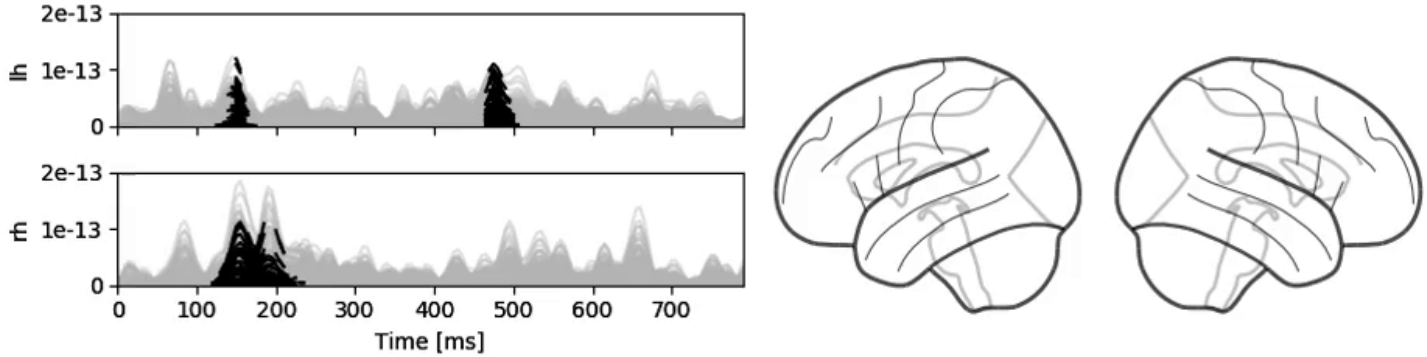
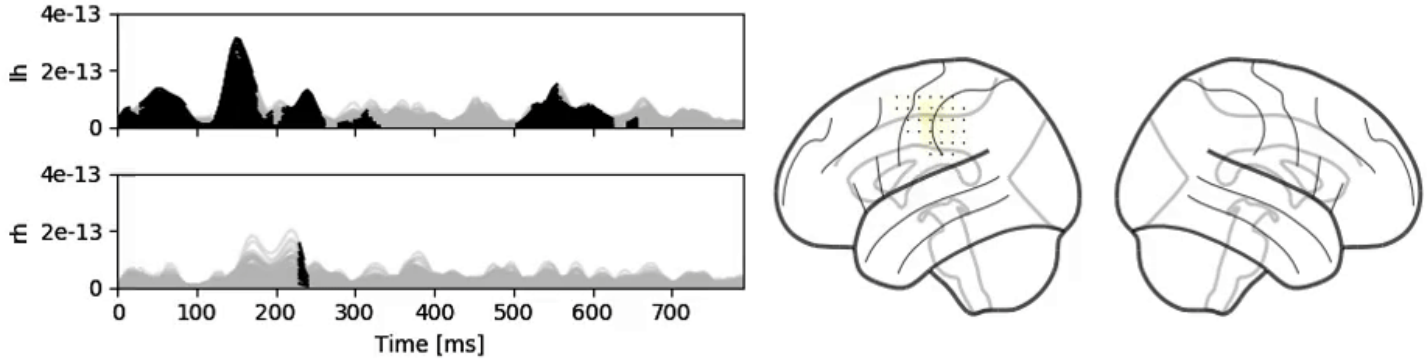
MEG Application to Cortical Processing of Speech

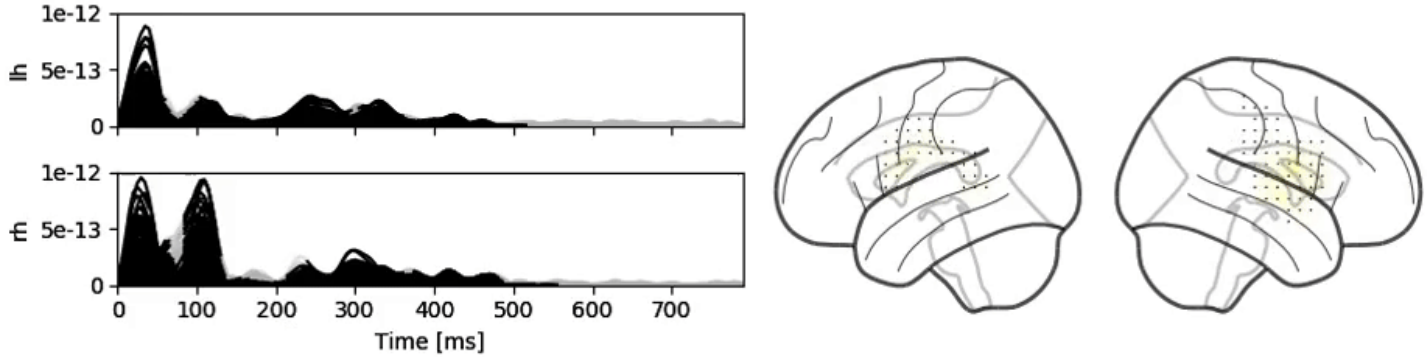
Semantic composition
Westerlund et. al., 2015



Word frequency
Brysbaert et. al., 2009



Acoustic envelope
Yang et. al., 1992



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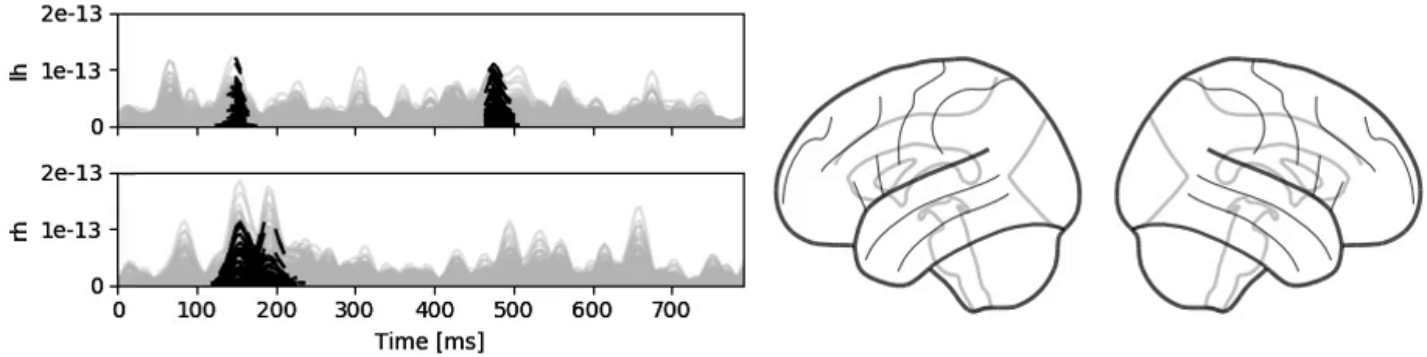

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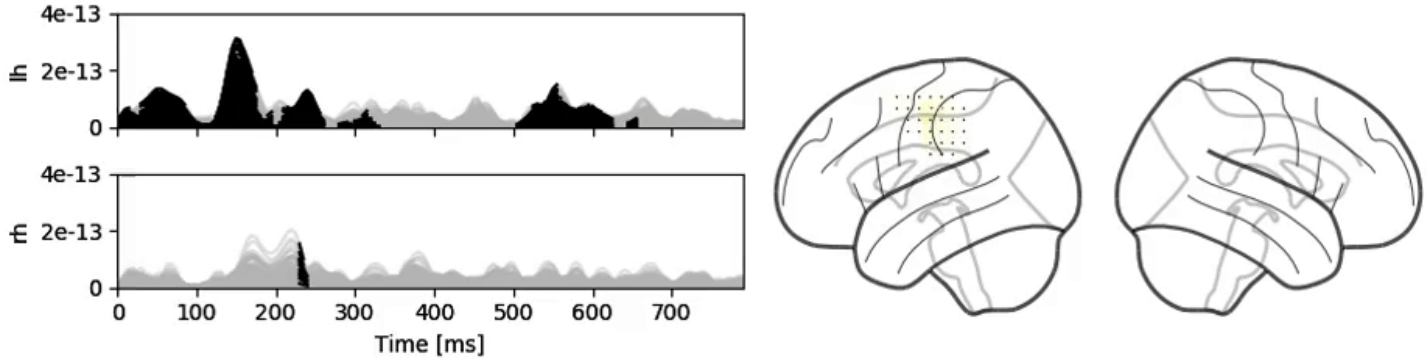

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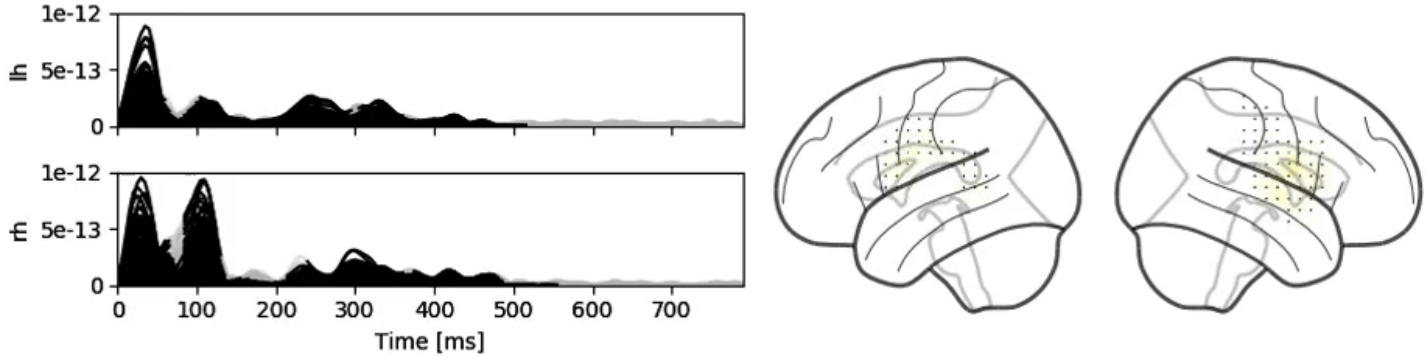
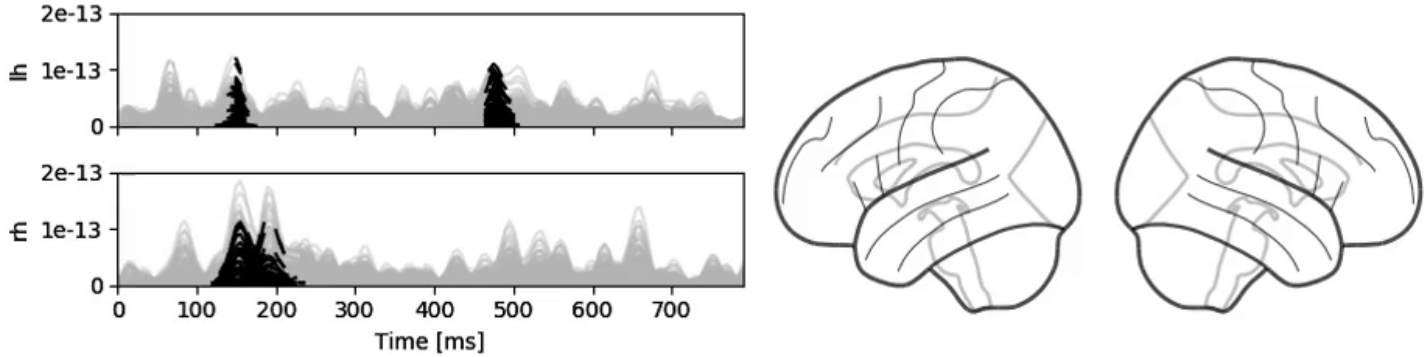


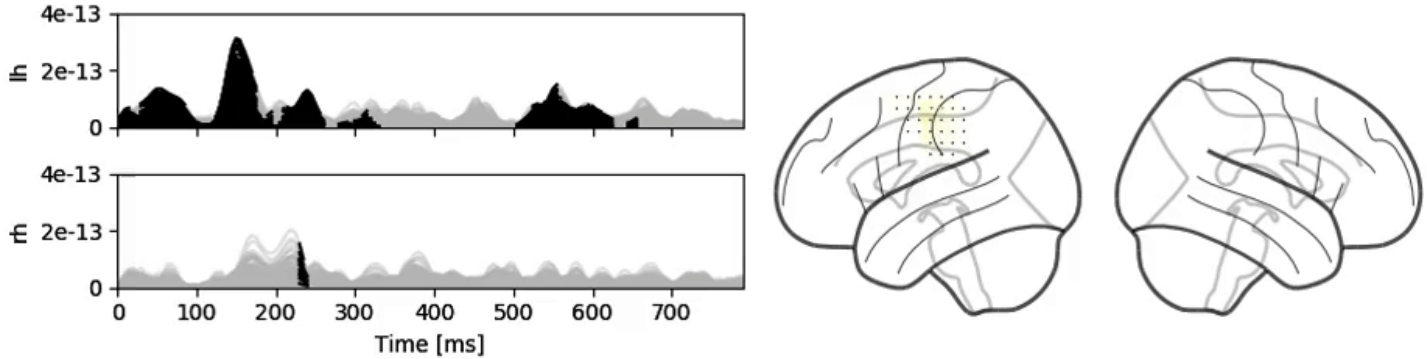
P. Das, C. Brodbeck, J. Z. Simon, and B. Babadi, NeuroImage, 2020

MEG Application to Cortical Processing of Speech

Semantic composition
Westerlund et. al., 2015



Word frequency
Brysbaert et. al., 2009



Acoustic envelope
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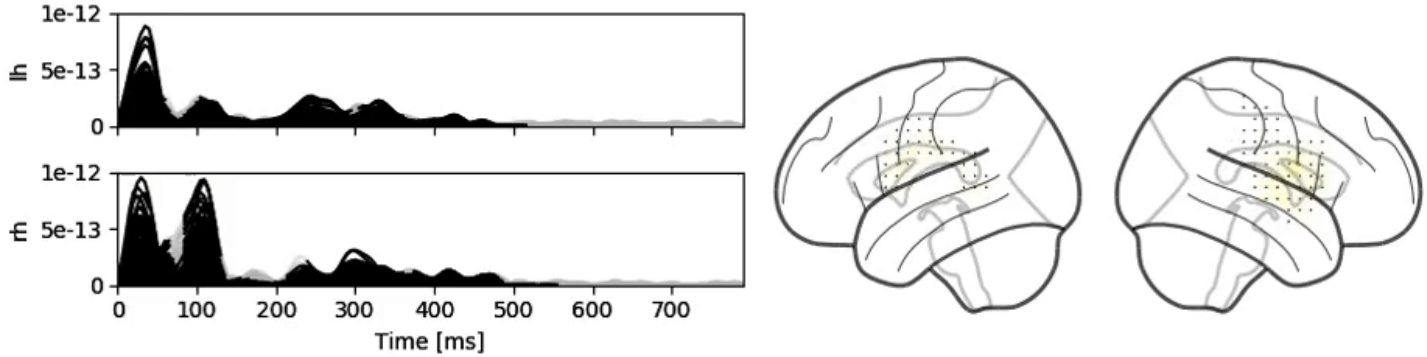

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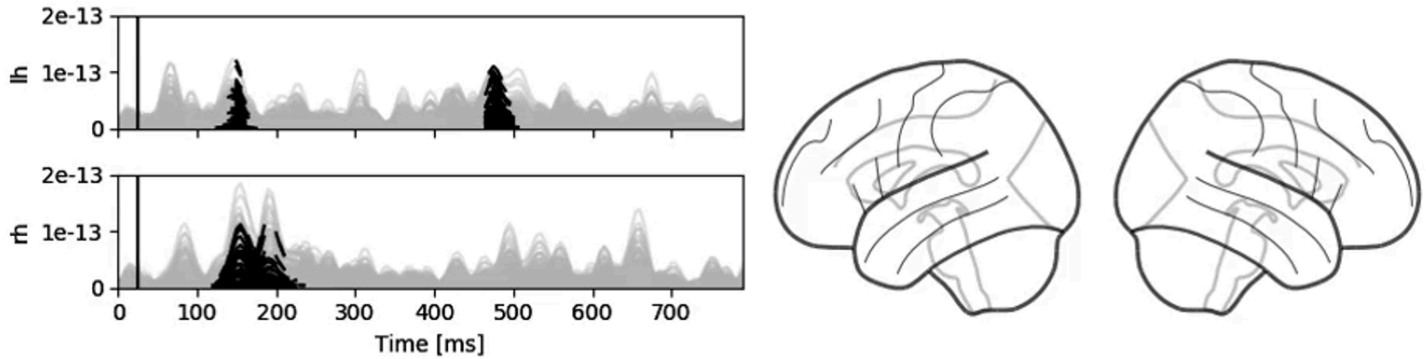
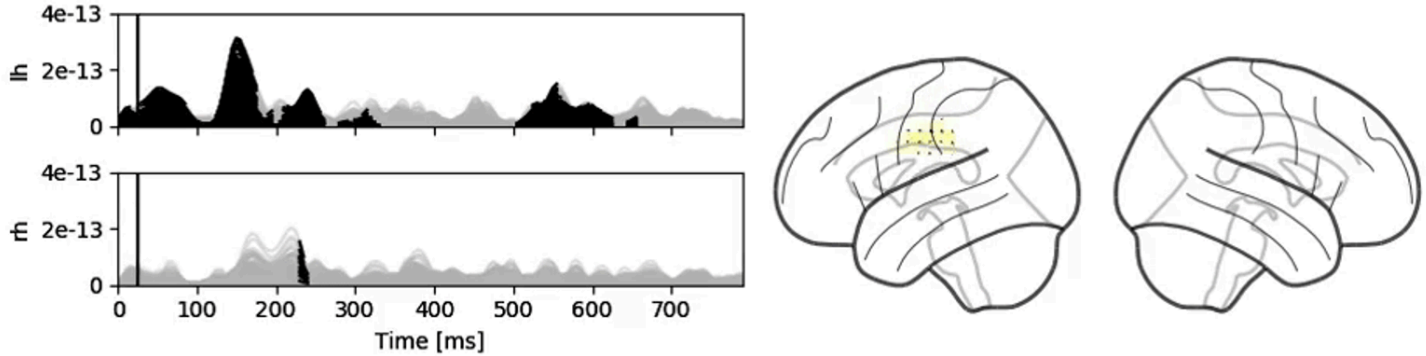
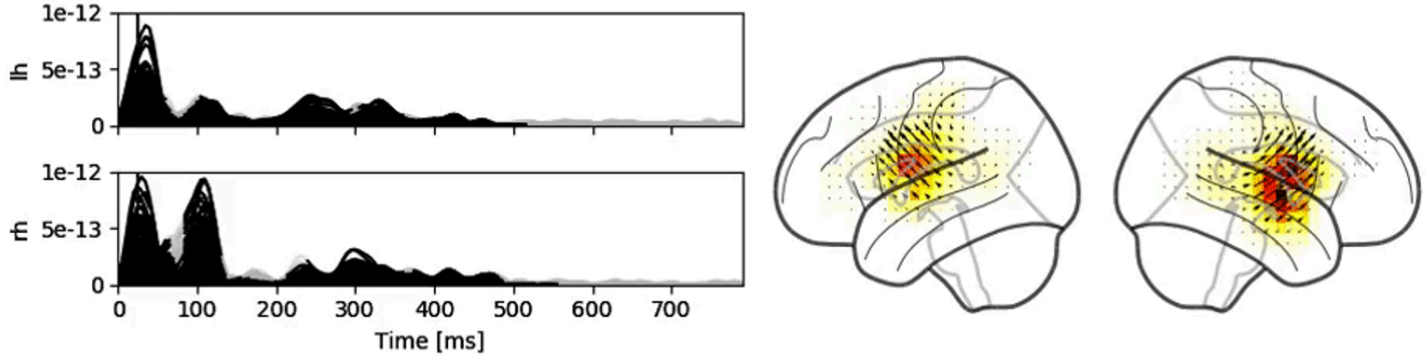


Word frequency
Brysbaert et. al., 2009



Acoustic envelope
Yang et. al., 1992



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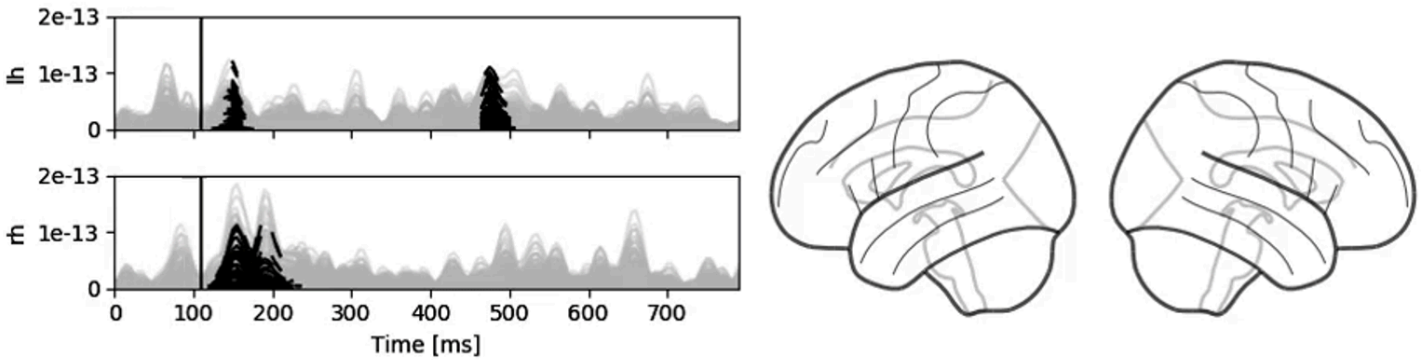

Word frequency
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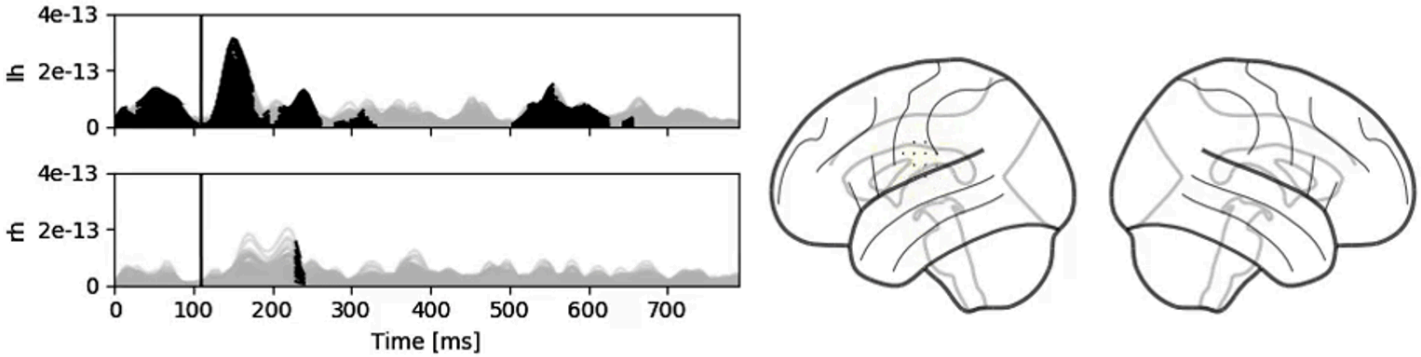

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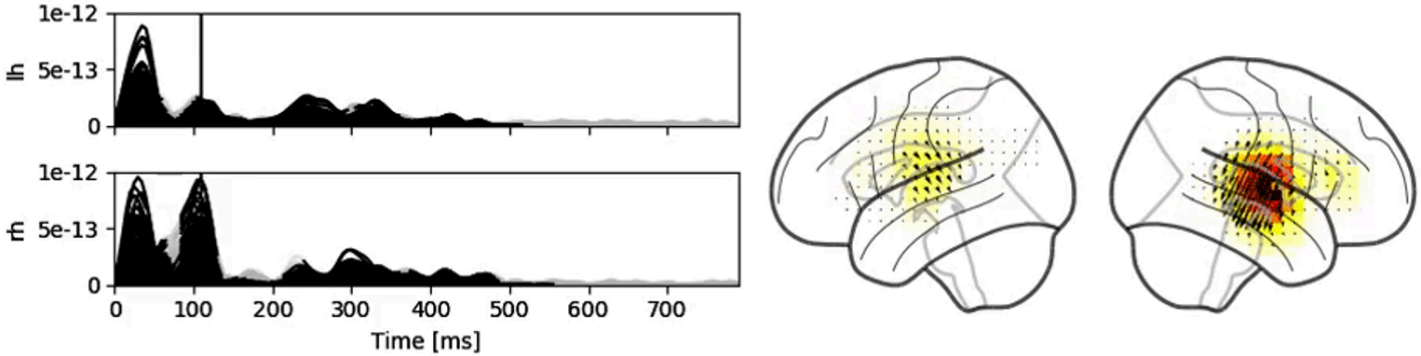

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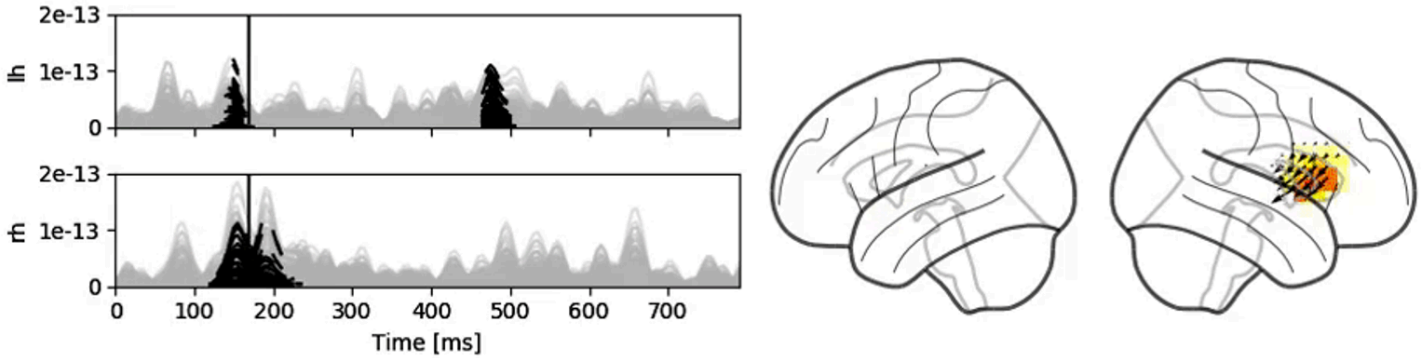

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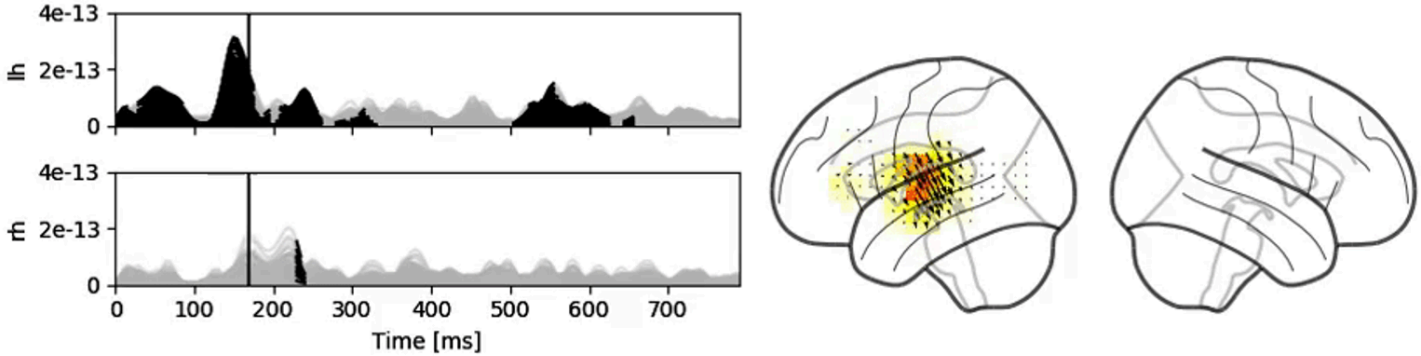

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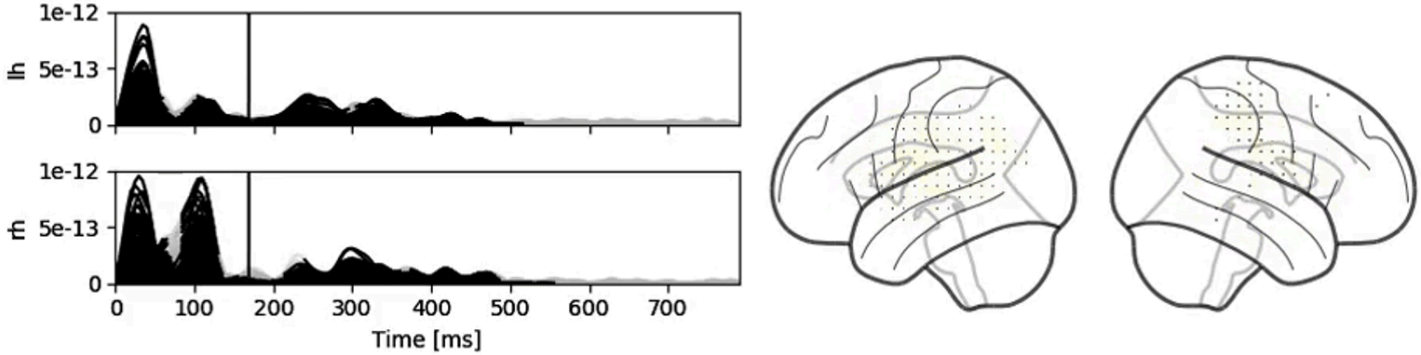

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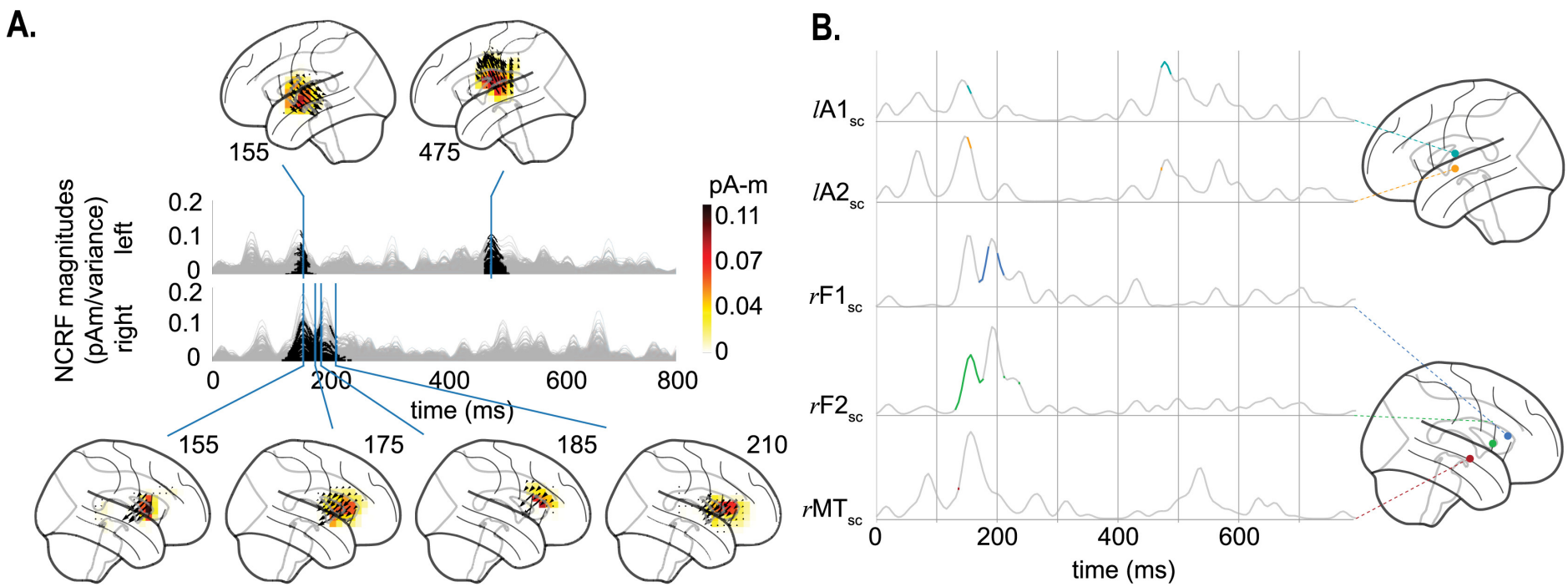
MEG Application to Cortical Processing of Speech

Semantic composition:



- Bilateral auditory component at ~ 155 ms, late auditory component at ~ 475 ms.
- Auditory-frontal dynamics at ~ 175-210 ms ($r\mathrm{MT}_{\mathrm{sc}} \rightarrow r\mathrm{F2}_{\mathrm{sc}} \rightarrow r\mathrm{F1}_{\mathrm{sc}} \rightarrow r\mathrm{F2}_{\mathrm{sc}}$).

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Summary

- NCRFs: A tool for directly extracting the cortical dynamics that underlie **continuous stimulus processing** from MEG.
- Novel spatiotemporal prior that not only combats overfitting and spatio-temporal dispersion but is also robust in absence of MR scan.
- The NCRFs are readily interpretable in a without post-hoc processing.

NCRFs as a powerful source localization tool for continuous stimulus experiments.

Thank you!

slides available at

github repo at