Failure of unitarity for interacting fields on spacetimes with closed timelike curves

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The scattering of free quantum fields is well defined on a class of asymptotically flat spacetimes with closed timelike curves (CTC's), and, at least on these spacetimes, the S matrix is unitary as well. For interacting fields, however, the preceding paper has obtained a set of unitarity relations that must be satisfied by the Feynman propagator if the scattering is to be unitary to each order in perturbation theory. In a globally hyperbolic spacetime, the causal form of the propagator guarantees that the relations are satisfied, but for spacetimes with CTC's, the form of the propagator is altered, and we show that the unitarity relations are not satisfied for interacting fields. We consider the $\lambda\phi^4$ theory in detail, but the results appear to hold for a wide class of fields. Although a conventional interpretation of quantum mechanics leads to inconsistency, a path-integral interpretation appears to allow a consistent assignment of probabilities to histories.

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I. INTRODUCTION

If the notion of spacetime, of manifold and Lorentz metric, retains its meaning at scales where quantum fluctuations of the metric are of order unity, closed timelike curves (CTC's) seem likely to pervade a microscopic quantum geometry. Local fluctuations of the metric would be roughly independent of fluctuations several Planck volumes away, and the resulting randomly oriented field of light cones would give rise to a sea of small CTC's. One apparently avoids microscopic CTC's only by adopting a theory in which causal structure is fundamental or one in which the metric is not meaningful or not Lorentzian at small scales.

A number of authors have recently challenged the assumption that one cannot construct a consistent physical theory on spacetimes with CTC's [2–11]. In particular, Morris, Thorne, and Yurtsever [3] give a compelling argument that on a class of asymptotically flat spacetimes, in which all CTC's are confined to a finite spatial region, the Cauchy problem for free fields is well defined. This was confirmed by Friedman and Morris for a class of static spacetimes in which CTC's are forever present: given arbitrary data on $\mathcal{J}^-$, they showed the existence of a smooth, asymptotically regular solution to the massless Klein-Gordon equation. In addition to the existence theorem, a simple uniqueness theorem implies that no smooth free field can be trapped in a finite region. Although an isolated null ray can remain forever trapped in a finite region, circling a closed null geodesic, any smooth field leaks out.

Free-field unitarity is essentially this last result, that in the spacetimes where the Cauchy problem is known to be well defined, solutions to the free-field wave equation are not trapped by closed geodesics. That is, as we observed in the preceding paper [1], for spacetimes on which the scattering matrix exists, unitarity is a consequence of a conserved inner product on the one-particle Hilbert space: conservation of the classical symplectic product implies conservation of probability in the Fock space. Thus, for free fields, unitarity does not rely on the existence of a global causal structure.

At first sight it appears that interacting fields are similarly unitary. One can again make the intuitive argument that interacting fields are not trapped by the geometry; and, although the classical Cauchy problem for interacting fields is not yet understood, simple systems of interacting particles appear generically to have classical solutions for arbitrary data on a class of spacetimes with CTC's [4,8,11]. Moreover, the initial steps in constructing a perturbative scattering theory for interacting fields appear to go through without change in a spacetime with CTC's. A formal path-integral reduction of S-matrix elements to products of Feynman propagators can be carried through in a way that is independent of the causal structure [1]. Since the evolution of both free and interacting fields is determined by the propagator alone, the fact that free-field scattering is unitary suggests that the scattering of interacting fields will be unitary as well.

Unfortunately, perturbative unitarity of interacting field theories rests on an additional property of the propagator. As we noted in [1], it relies on a series of relations, all of which are satisfied if the Feynman propagator Δ_F has a form reflecting the causal structure of the spacetime:

$$i\Delta_F(x,y)=\theta(x^0-y^0)D(x,y)+\theta(y^0-x^0)\bar{D}(x,y)\ , \tag{1}$$

where $D(x,y)$ is the Wightman function. When there are CTC's, one cannot assign a time ordering to spacetime points x and y, and this causal form is lost. The difficulty

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appears to be fatal: the scattering of interacting fields does not satisfy the unitarity relations on spacetimes with closed timelike curves. This implies (as we discuss in Sec. IV), that a Copenhagen interpretation cannot consistently describe observations both before and after a region of CTC's. A sum-over-histories interpretation, on the other hand, may allow one to make sense of quantum field theory. However, to obtain a consistent assignment of probabilities one can consider only paths that start to the past of *any* region of CTC's and terminate to the future of every such region. Experiments at any intermediate time can, as usual, be described by including measuring instruments in the quantum system, but there is an unexpected loss of causality even *before* any CTC's form. That is, one can set up experiments whose outcomes depend on whether CTC's form to the future of the experiment. Measurement in theories where unitarity is violated has also been discussed by Hartle [12], who reaches conclusions similar to ours.

Boulware [13] independently has investigated the $\lambda\phi^4$ theory on one of the Gott spacetimes [14] in which there are CTC's in the vacuum surrounding two infinite cosmic strings that move past each other. He has explicitly computed the propagator, and it again fails to satisfy the unitarity relations of Ref. [1]. (See also Gerbert and Jackiw [34].) (He considers tadpole diagrams, and we have strengthened an earlier version of our paper by including an analogous treatment here for time-tunnel spacetimes.)

Our work appears to be closely related to a recent investigation by Klinkhammer and Thorne on the quantum mechanics of billiard balls on time-tunnel spacetimes and subsequent work by Politzer [18]. Using a WKB approximation, they find a loss of unitarity that arises because the number of classical solutions depends on the initial data for the billiard balls. The norm of the final WKB state depends on how many classical solutions there are. As a result, initial WKB states peaked about different initial classical trajectories have different final norms.

Certainly some spacetimes with CTC's do not allow solutions for most initial data of interacting classical systems. Deutsch [19] has assumed that this is a generic property of spacetimes with CTC's, and he suggests a substantially different approach to quantum mechanics on such spacetimes. We will briefly comment on his work in Sec. III.

This paper proceeds as follows. In Sec. II we review work on the Cauchy problem for free fields on spacetimes with CTC's. On spacetimes for which the Cauchy problem is well defined (and for which the evolution of the vacuum has finite norm) we show that the evolution of free fields is unitary between two spacelike hypersurfaces that do not intersect any CTC's. In Sec. III we examine the structure of the propagator for a class of wormhole spacetimes with CTC's. We show that the propagator fails to satisfy the unitarity relations governing interacting fields in curved spacetime. Finally, in Sec. IV we discuss the implications of the loss of unitarity, asking whether a sum-over-histories approach might allow one to recover a meaningful quantum field theory.

Lower case Latin letters serve as spacetime indices, and our signature is $-+++$.

II. UNITARITY OF FREE FIELDS

As in paper [1], let M,g_{ab} be an asymptotically flat spacetime, with static regions in the past and future containing spacelike hypersurfaces $\Sigma_{\rm in}$ and $\Sigma_{\rm out}$, respectively. Let $\mathcal{H}^{\rm in}$ and $\mathcal{H}^{\rm out}$ be the spaces of positive-frequency solutions on the past and future static regions, defined with respect to the past and future timelike Killing vectors, and let $\mathcal{F}^{\rm in}$ and $\mathcal{F}^{\rm out}$ be the corresponding Fock spaces.

To describe the scattering of a free field, one must have a well-defined Cauchy problem describing the evolution of complex solutions to the classical wave equation. If initial data for a free field φ on $\Sigma_{\rm in}$ have a unique time evolution to a free field $U\varphi$ on $\Sigma_{\rm out}$, then the S matrix $\mathcal{S}$ is determined by the Bogoliubov operators α and β that give the positive- and negative-frequency parts of $U\varphi$, for $\varphi\in\mathcal{H}^{\rm in}$:

$$\alpha\varphi=(U\varphi)_+ \ , \tag{2}$$

$$\beta\varphi=(U\varphi)_- \ . \tag{3}$$

The S matrix exists if and only if $\mathrm{Tr}(\beta\beta^\dagger)<\infty$. As emphasized in [1], when $\mathcal{S}$ exists, its unitarity is implied by the conservation of the inner product on the one-particle Hilbert space, constructed from the symplectic product of solutions to the classical field equation. For any free field, conservation of the symplectic product is, in turn, implied by Gauss' law, which holds irrespective of the existence of a causal structure in the region between $\Sigma_{\rm in}$ and $\Sigma_{\rm out}$.

Conservation of the inner product is shown explicitly as follows. Denote by ϕ, ν^A, ψ, and F^{ab}, respectively, a scalar, two-component spinor (defined only on an orientable spacetime), four-component spinor, and electromagnetic field (antisymmetric tensor), satisfying the free-field equations

$$\begin{gathered}0=K\phi\equiv(-\Box+m^2+\xi R)\phi \ , \\ 0=\nabla_{AA'}\nu^A=\sigma^a{}_{AA'}\nabla_a\nu^A \ , \\ 0=(\gamma^a\nabla_a+m)\psi \ , \quad 0=\nabla_b F^{ab} \ .\end{gathered} \tag{4}$$

The corresponding products are

$$\begin{gathered}\langle\phi_2|\phi_1\rangle=\frac{1}{i}\int d\Sigma_a\bar{\phi}_2\overleftrightarrow{\nabla}^a\phi_1 \ , \\ \langle\nu_2|\nu_1\rangle=\int d\Sigma_{AA'}\bar{\nu}_2^{A'}\nu_1^A=\int d\Sigma_a\sigma^a{}_{AA'}\bar{\nu}_2^{A'}\nu_1^A \ , \\ \langle\psi_2|\psi_1\rangle=\int d\Sigma_a\bar{\psi}_2\gamma^a\psi_1 \ , \\ \langle F_2|F_1\rangle=\frac{1}{i}\int d\Sigma_a(\overline{A}_{2b}F_1^{ab}-\bar{F}_2^{ab}A_{1b}) \ .\end{gathered} \tag{5}$$

In the last equality, A_a is any vector potential for F_{ab}, i.e., any asymptotically regular vector field for which $F_{ab}=\nabla_a A_b-\nabla_b A_a$, and the product is gauge invariant. If N is the spacetime region bounded by Σ_1 and Σ_2, conservation of each of the products follows from Gauss' law:

$$\begin{aligned}
\langle U\phi_2|U\phi_1\rangle-\langle\phi_2|\phi_1\rangle &= \frac{1}{i}\int_{\partial N} d\Sigma_a \bar{\phi}_2 \overleftrightarrow{\nabla}^a \phi_1 = \frac{1}{i}\int_N d\tau \nabla_a(\bar{\phi}_2 \overleftrightarrow{\nabla}^a \phi_1)=0 \;, \\
\langle U\nu_2|U\nu_1\rangle-\langle\nu_2|\nu_1\rangle &= \int_{\partial N} d\Sigma_{AA'}\bar{\nu}_2^{A'}\nu_1^A = \int_N d\tau \nabla_{AA'}(\bar{\nu}_2^{A'}\nu_1^A)=0 \;, \\
\langle U\psi_2|U\psi_1\rangle-\langle\psi_2|\psi_1\rangle &= \int_{\partial N} d\Sigma_a \bar{\psi}_2\gamma^a\psi_1 = \int_N d\tau \nabla_a(\bar{\psi}_2\gamma^a\psi_1)=0 \;, \\
\langle UF_2|UF_1\rangle-\langle F_2|F_1\rangle &= \frac{1}{i}\int_{\partial_N}(\bar{A}_{2b}F_1^{ab}-\bar{F}_2^{ab}A_{1b}) = \frac{1}{i}\int_N d\tau \nabla_a(\bar{A}_{2b}F_1^{ab}-\bar{F}_2^{ab}A_{1b})=0 \;.
\end{aligned} \tag{6}$$

Here $d\tau$ is an element of four-volume and $d\Sigma_{AA'}=\sigma^a_{AA'}d\Sigma_a$.

To summarize, *If a free-field wave equation has a unique solution on N for arbitrary initial data on $\Sigma_{\rm in}$, and if the in-vacuum has an image with finite norm in $\mathcal{F}_{\rm out}$, then the S matrix exists and is unitary.*

Based on what is now known about the Cauchy problem on spacetimes with CTC's, one expects that for a broad class of such spacetimes the scattering of free fields is well defined and unitary. The work that has been done is summarized below. The simplest class of spacetimes with CTC's are static, and for these, most of the key questions are decided. For other spacetimes with CTC's, making precise the arguments for a well-defined Cauchy problem may be more difficult.

Morris and Thorne [2] (see also Morris,Throne, and Yurstever [3], Friedman *et al.* [4]) considered spacetimes in which the two mouths of a wormhole move toward one another, as seen by the external spacetime. Figure 1 shows three examples, (i)–(iii), of wormhole spacetimes, constructed by removing two solid cylinders (copies of $D^3\times\mathbb{R}$) from Minkowski space and identifying the boundaries (copies of $S^2\times\mathbb{R}$) in a way that produces CTC's. The metric inherited from Minkowski space is continuous, but not smooth, but one can choose a metric to make the spacetime smooth everywhere. We shall follow the usual terminology in calling the identified cylindrical boundary the history of the wormhole "throat" and calling the curved region surrounding each copy of the throat the wormhole "mouth." In (i), the left and right cylindrical boundaries are identified after a time translation τ, producing a static spacetime in which closed timelike curves are forever present when τ is greater than the spatial distance between the cylinders. In (ii), the cylinders are related by a boost and a translation, and CTC's are confined to a finite region between the past and future Cauchy horizons. The spacetime of (iii) illustrates a more general motion of the mouths, the "twin-paradox" example studied in Ref. [3].

For smooth, static spacetimes of the form (i), although there are no complete spacelike hypersurfaces, one can prove the existence of a massless scalar field for arbitrary initial data on $\mathcal{J}^-$ [7]. That is, for arbitrary, smooth initial data with finite energy on $\mathcal{J}^-$, there is a smooth, asymptotically regular solution to the massless Klein-Gordon equation:

$$-\Box\phi=0 \;. \tag{7}$$

The proof uses a generalized spectral decomposition that relies on the fact that the spacetime is static.

It appears to be easy to extend the proof to other massless free fields, although we have not yet explicitly done so, and an extension to fields with nonzero rest mass is also likely to be straightforward. The method of proof does not, however, allow one to treat more general spacetimes, and one's expectation that there is a well-defined Cauchy problem for the CTC spacetimes (ii) and (iii) rests on the argument given in Refs. [3,4].

The argument is that, because light traversing or reflecting off of a wormhole diverges as if reflected from a spherical mirror, one can construct a solution by a convergent multiple scattering series. An incoming wave from $\mathcal{J}^-$ may be thought of as initially scattering off of the wormhole mouths, producing at each mouth some purely outgoing scattered wave that is composed of a reflected part from the initial scattering at that mouth and a transmitted part that comes from the initial scattering at the other mouth. These first-scattered waves in turn scatter again, producing new reflected and transmitted waves, and so on. The convergence of the scattering series is slowest in the short-wavelength limit, and this limit can be treated by geometrical optics. In geometrical optics, the amplitude of a light ray that traverses a wormhole of radius a, with mouths separated by a distance d and moving toward one another with speed v, decreases by a factor $\gamma a/2d$. The frequency increases by $\sqrt{(1+v)/(1-v)}$, and one consequently expects $\partial^n\phi/\partial t^n$ to be finite at the Cauchy horizon, when

$$n<\ln(a/2d)/\ln(1-v) \;. \tag{8}$$

(This corrects Eq. (6) of Ref. [4], in which the focusing factor $\gamma=(1-v^2)^{-1/2}$ was omitted.)

For the static spacetime (i) Friedman and Morris [9] have a proof of convergence of the multiple scattering series for $\lambda>\Lambda$, any Λ, where λ is the wavelength and the separation between mouths is sufficiently large: $a/d<\epsilon$, for some ϵ. Thus, for small mouths, one can prove convergence for all $\lambda>\Lambda$ with $\Lambda\ll a$. For the remaining range, the wavelength is short enough that geometrical optics must be valid ($\lambda<\Lambda\ll a$), and convergence had already been shown in the geometrical optics limit. We therefore have little doubt that, at least for handles with small mouths, the multiple scattering series converges.

III. FAILURE OF UNITARITY FOR INTERACTING FIELDS

The initial steps in constructing a perturbative scattering theory for interacting fields formally go through

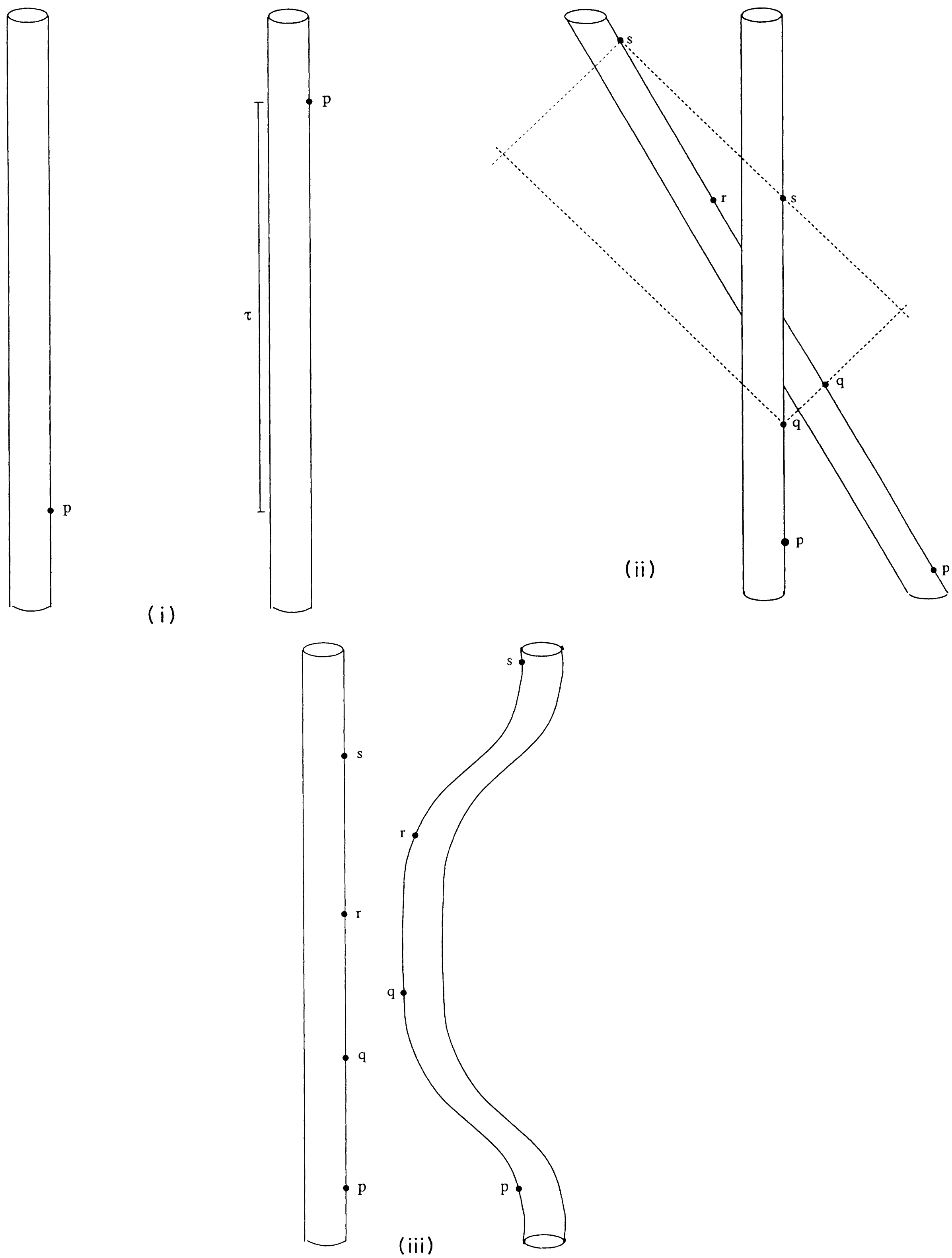


FIG. 1. Spacetimes with a Lorentzian wormhole are constructed by removing two solid cylinders (copies of $D^3\times R$) from Minkowski space and identifying the bounding cylinders. Three wormhole spacetimes with CTC's are depicted, with identified points labeled by the same letter. CTC's are the timelike curves joining identified points of the boundary. In spacetime (i), the cylinders are related by a constant timelike translation, and closed timelike curves are forever present. In spacetimes (ii) and (iii), the CTC's are confined to a finite region bounded by a past and future Cauchy horizons. The region is roughly outlined in (ii) by dotted null lines.

without change in a spacetime with CTC's: in Ref. [1], we carried through a formal path-integral reduction of S-matrix elements to products of Feynman propagators in a way that is independent of the causal structure. Let us follow the conventions of [1], denoting by $|i\cdots j\rangle_{\rm in}$ an n-particle state in $\mathcal{F}_{\rm in}$ and by $|i\cdots j\rangle_{\rm out}$ the image of $|i\cdots j\rangle_{\rm in}$ under the action of the free-field S matrix:

$$|i\cdots j\rangle_{\rm out}=\mathcal{S}^{(0)}|i\cdots j\rangle_{\rm in}\ . \tag{9}$$

As in [1] we restrict our detailed discussion to a $\lambda\phi^4$ theory.

As noted in the Introduction, perturbative unitarity relies on a series of relations, which are satisfied if the Feynman propagator Δ_F has the causal form

$$i\Delta_F(x,y)=\theta(x^0-y^0)D(x,y)+\theta(y^0-x^0)\bar{D}(x,y)\ , \tag{10}$$

where $D(x,y)$ is the Wightman function. We now examine two of the unitarity relations of paper I for the wormhole spacetimes of the previous section. We show first that the propagator does not satisfy the pointwise unitarity relation [Eq. (76) of paper I]:

$$-[i\Delta_F(x,y)]^2-[\overline{i\Delta_F(x,y)}]^2+[D(x,y)]^2+[\overline{D(x,y)}]^2=0\ . \tag{11}$$

As noted below, this is not quite sufficient to prove that unitarity fails, because the S-matrix amplitudes involve only a smearing of the relation with solutions to the Klein-Gordon equation, Eq. (25). We can, however, show that unitarity fails, by showing that the smeared relation, Eq. (85) of paper I,

$$-i\lambda\int d\tau\,\bar{f}_j(x)f_k(x)[i\Delta_F(x,x)-\overline{i\Delta_F(x,x)}]=0\ , \tag{12}$$

is not satisfied. The failure of the relations arises essentially from the fact that, when there is no causal structure, the propagator cannot have the causal form (10).

In spacetime (ii), the CTC's are confined to a bounded region and there are spacelike hypersurfaces $\Sigma_{\rm in}$ and $\Sigma_{\rm out}$. To treat (i), these must be replaced by $\mathcal{I}^-$ and $\mathcal{I}^+$. Let F_I be an orthonormal basis for the space $\mathcal{H}^{\rm in}$ of solutions to the Klein-Gordon equation that have positive frequency in the past, and let $D(x,y)$ be the distribution

$$D(x,y)=\sum\bar{F}_J(x)F_J(y)=\bar{D}(y,x)\ , \tag{13}$$

agreeing in the past with the Wightman function. To see that the propagator does not have the form (10) is not difficult. The spacetime can be depicted by a single chart, consisting of Minkowski space with two cylinders removed, if one identifies the cylindrical boundaries. For the static spacetime (i) the cylinders are identified after a time translation by τ: in terms of the Minkowski time t of an observer moving along the Killing vector (parallel to the cylinders), a point p_1 on the left cylinder is identified with a point p_2 on the right cylinder for which

$$t_2=t_1+\tau\ . \tag{14}$$

In the spacetime (ii), p_2 is related to p_1 by a spacetime translation together with a boost. For ease of visualization, Figs. 2–4 refer to spacetime (i), but the arguments apply equally well to spacetime (ii).

For a solution f to the scalar wave equation, the function and its normal derivative are continuous across the cylindrical boundary. Let $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ be the unit vectors at p_1 and p_2 that are outward normals to the left and right cylinders and are normal to the timelike Killing vectors that run along the cylinders (see Fig. 2). Because a vector pointing inward at p_1 is identified with a vector pointing out at p_2 (it is tangent to a path that goes in to p_1 and goes out from p_2), the boundary conditions have the form

$$f(p_1)=f(p_2)\ , \tag{15a}$$

$$\hat{\mathbf{n}}_1\cdot\nabla f(p_1)=-\hat{\mathbf{n}}_2\cdot\nabla f(p_2)\ . \tag{15b}$$

Then $D(x,y)$ satisfies the boundary conditions (15a) and (15b) in each of its arguments. The right-hand side of equation Eq. (10), with x^0 the Minkowski time, makes sense in the spacetime outside the cylinders, but it is not continuous across the cylindrical boundaries when there are CTC's—when the identified points are timelike separated with respect to causal structure of the spacetime outside the cylinders.

Let $t=x^0$ be the Minkowski time of an observer moving parallel to the right cylinder, so that t is a scalar defined outside the cylinders, but discontinuous from p_2 to p_1. When the radius a of the cylinders goes continu-

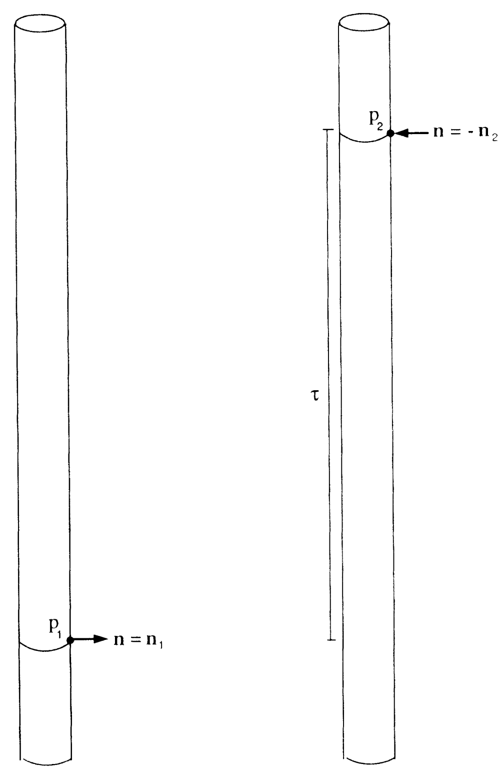


FIG. 2. Fields on the spacetime of Fig. 1(i) are continuous at identified points p_1 and p_2. Continuity of a field's derivative along a curve that enters at p_2 and leaves at p_1 implies that the derivative of the field along the outward normal, $\hat{n}_1$ at p_1 has opposite sign to its derivative along the outward normal, $\hat{n}_2$ at p_2.

ously to zero, $D(x,y)$ goes continuously to its value for the globally hyperbolic spacetime M_0 with $a=0$. On M_0, $D(x,y)$ and $D(y,x)$ are equal only if x and y are spacelike separated. Consequently, for at least a finite range of a, we have

$$D(x,y) \neq D(y,x) \tag{16}$$

for x to the past or to the future of y in M_0.

Define an error $E(x,y)$ measuring the departure of the propagator from the causal form (10), in the manner

$$i\Delta_F(x,y)=\theta(x^0-y^0)D(x,y) + \theta(y^0-x^0)\bar{D}(x,y)+E(x,y) \ . \tag{17}$$

First we want to show that $E(x,y)$ does not everywhere vanish. As above, let p_1 and p_2 be the corresponding points on the left and right cylinders and let y be a point to the past of p_2 and to the future of p_1 with respect to the causal structure on the spacetime outside the cylinders (Fig. 3). We have

$$i\Delta_F(p_2,y)=D(p_2,y)+E(p_2,y) = D(p_1,y)+E(p_2,y) \ , \tag{18}$$

by the boundary condition (15a) satisfied by D. But the propagator satisfies the same boundary conditions:

$$\Delta_F(p_1,y)=\Delta_F(p_2,y) \ . \tag{19}$$

From Eqs. (13) and (17) we have

$$D(y,p_1)+E(p_1,y)=D(p_1,y)+E(p_2,y) \ . \tag{20}$$

Thus, from Eq. (16), it follows that E cannot everywhere vanish.

We can also show that E is everywhere purely imaginary. With Δ_F of the form (17), because both the Wightman function and the imaginary part of the propagator satisfy the Klein-Gordon equation in both arguments, then so must the real part of the error function E. Since the initial data for the Klein-Gordon operator acting on E must vanish on $\mathcal{J}^-$ (the identified cylinders are timelike and do not affect the initial null data), we must also have

$$E+\bar{E}=0 \ , \tag{21}$$

for the entire spacetime.

Finally, we show that Eq. (11) is not everywhere satisfied. To lowest order in a, Eqs. (11) and (17) imply

$$0=-[D(x,y)+E(x,y)]^2-\overline{[D(x,y)+E(x,y)]}^2 + D(x,y)^2+\overline{D(x,y)}^2 \tag{22}$$

$$=-2(DE+\overline{DE}) \ , \tag{23}$$

for x to the future of y. Then, from Eqs. (21) and (23), we

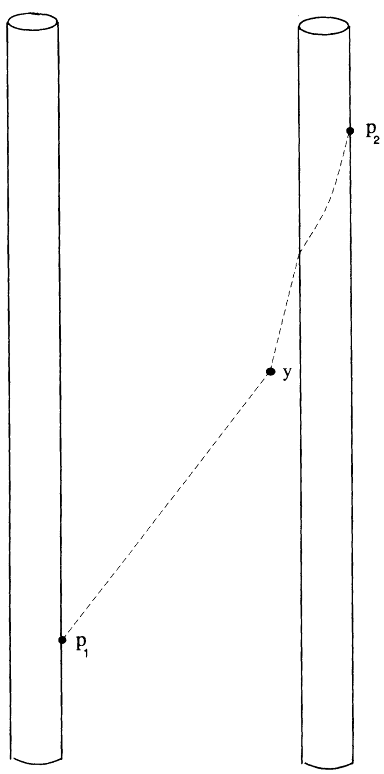


FIG. 3. A future-directed timelike curve joins the point y to the copy p_2 of the point p, and a past-directed timelike curve joins y to the copy p_1 of the same point. Because one cannot define a time ordering of points in a region of CTC's, the Feynman propagator does not have the causal form it takes in a globally hyperbolic spacetime.

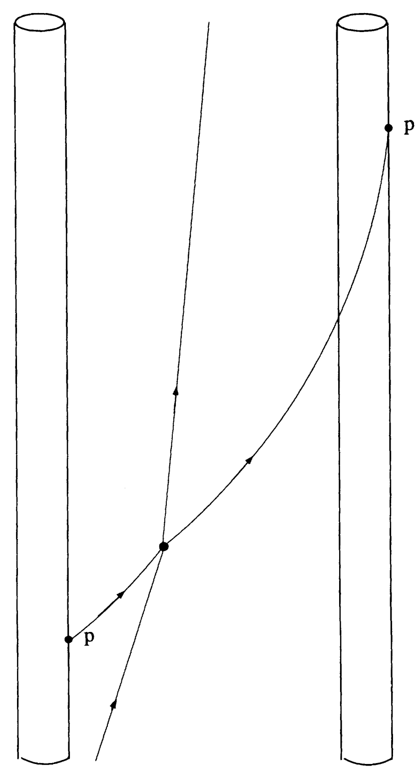


FIG. 4. A tadpole diagram contributes to the classical scattering of a one-particle state, when its loop is a CTC.

have

$$0 = DE + \overline{DE} = (D - \overline{D})E \ , \tag{24}$$

implying that $E(x,y)$ vanishes whenever x is to the future of y, contradicting Eq. (20). Consequently the pointwise unitarity relation (11) is not satisfied. Unitarity requires that the corresponding integrated relation

$$0 = \int dx\, dy\, f_i(x) f_{(j}(x) f_k(y) f_{l)}(y) \times \{ -[i\Delta_F(x,y)]^2 - [\overline{i\Delta_F(x,y)}]^2 + [D(x,y)]^2 + [\overline{D(x,y)}]^2 \} \ , \tag{25}$$

for all complex solutions f_i, f_j, f_k, f_l to the Klein-Gordon equation. Because of the symmetrization, this is a slightly weaker requirement than the pointwise unitarity relation, but, given the failure of the pointwise relation, we think it highly likely that (25) is violated for the $\lambda\phi^4$ theory on spacetimes of the form (i) and (ii).

We can show that unitarity is violated to order λ for the one-particle to one-particle transition or for a transition related to this one by crossing symmetry. For unitarity to hold, Eq. (12) must be satisfied. If this relation holds for each value of the radius a of the wormhole throat, its derivative with respect to a must vanish:

$$0 = \frac{d}{da} \int d^4x\, \bar{f}_j(x) \mathrm{Im}[i\Delta_F(x,x)] f_k(x) \Big|_{a=0} = \int d^4x\, \bar{f}_j^{(0)}(x) f_k^{(0)}(x) \mathrm{Im} \frac{d}{da}[i\Delta_F(x,x)] \Big|_{a=0} \ , \tag{26}$$

where $f_j^{(0)}$ is a solution to the Klein-Gordon equation on Minkowski space.

Because any function of the form $e^{ik\cdot x}$ can be written as the product of two plane waves $f_j^{(0)}$ and $f_k^{(0)}$, Eq. (26) implies that every Fourier component of $\mathrm{Im}(d/da)[i\Delta_F(x,x)]$ vanishes. Thus (almost everywhere),

$$\mathrm{Im} \frac{d}{da}[i\Delta_F(x,x)] \Big|_{a=0} = 0 \ . \tag{27}$$

The propagator for spacetime (i) is discussed in Appendix A. If a is the radius of the wormhole throat, $\mathrm{Im}[i\Delta_F(x,x)]$ is given by

$$\mathrm{Im}[i\Delta_F(x,x)] = 2\,\mathrm{Im}[\hat{D}_{01}(x,x) + \hat{D}_{-10}(x,x)] + o(a) \ . \tag{28}$$

The quantities $\hat{D}_{01}$ and $\hat{D}_{-10}$ are corrections to the propagator corresponding to one traversal of the wormhole. For a static wormhole spacetime with points on the first and second copies of the throat identified after a time translation τ:

$$\hat{D}_{01} = \int d\omega \sum_{lm} \frac{k}{\pi} \bar{t}_{l\omega} j_l(kr_1) Y_{lm}(\hat{\mathbf{r}}_1) \bar{h}_l^{(1)}(kr_2) \bar{Y}_{lm}(\hat{\mathbf{r}}_2) e^{-i\omega\tau} \ , \tag{29}$$

$$\hat{D}_{-10} = \int d\omega \sum_{lm} \frac{k}{\pi} t_{l\omega} h_l^{(1)}(kr_1) Y_{lm}(\hat{\mathbf{r}}_1) j_l(kr_2) \bar{Y}_{lm}(\hat{\mathbf{r}}_2) e^{-i\omega\tau} \ . \tag{30}$$

In these formulas, $\mathbf{r}_1$ ($\mathbf{r}_2$) is the position vector of the point x with respect to an origin centered in the first (second) wormhole throat, and k is the magnitude of the momentum of a particle with energy ω. Finally $t_{l\omega}$ is the transmission coefficient through the wormhole's throat:

$$t_{l\omega} = \frac{i}{(ka)^2} [2h_l^{(1)}(ka) h_l^{(1)\prime}(ka)]^{-1} \ . \tag{31}$$

The propagator itself is singular, and its singularity should be canceled by the mass-renormalization counterterm. The counterterm, however, is real and does not contribute to $\mathrm{Im}[i\Delta_F(x,x)]$.

From the form of h_l for small argument we have

$$\frac{d}{da} t_{l\omega} \Big|_{a=0} = \frac{ik}{2} \delta_{0l} \ , \quad t_{l\omega}|_{a=0} = 0 \ . \tag{32}$$

Then

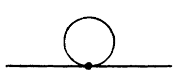
$$\frac{d}{da} \mathrm{Im}[i\Delta_F(x,x)] \Big|_{a=0} = \mathrm{Im} \int d\omega \frac{ik^2}{4\pi^2} [-j_0(kr_1) \bar{h}_0^{(1)}(kr_2) + j_0(kr_2) h_0^{(1)}(kr_1)] e^{-i\omega\tau}$$
$$= (8\pi^2 r_1 r_2)^{-1} \int d\omega [-\sin(kr_1)\cos(kr_2 + \omega\tau) + \sin(kr_2)\cos(kr_1 - \omega\tau)] \ . \tag{33}$$

Because the integral is a nonvanishing distribution, the unitarity relation (12) is violated. Boulware reached the same conclusion for the tadpole diagram in the case of Gott spacetimes, and the argument presented above was stimulated by conversation with him.

The loss of unitarity for interacting fields is likely to be related to a loss of unitarity that Klinkhammer and Thorne [18] have found for a nonrelativistic billiard ball that can interact with itself when it traverses a closed timelike curve. Using a WKB approximation they construct a path integral to propagate the wave function from Σ_{in} to Σ_{out}. Their results are based on the analysis of a classical billiard ball, in which they and Echeverria had shown that, at least for a wide class of initial data, "glancing blows" allow solutions to the Cauchy problem [8]. The solutions, however are not unique: for some initial data, a billiard ball can loop several times through the wormhole before hitting itself, and different solutions correspond to the ball's looping different numbers of times. The path integral is dominated by these different classical alternatives, weighting equally all classical paths whose length is short enough that the WKB approximation remains valid. For longer paths—many loops—the spread of the wave packet quickly diminishes the amplitude for additional looping through the wormhole.

Loss of unitarity for the billiard ball system follows from the fact that the final amplitude depends on how many classical paths there are. If all initial data had the same number M of classical solutions, one could regain unitarity by multiplying the final amplitude by $M^{-1/2}$ to obtain a normalized wave function. Because M depends on the initial data, the evolution is not unitary. As in the case of interacting fields, the difficulty is unrelated to a billiard ball being trapped forever. The set of trapped classical solutions has measure zero in the space of timelike geodesics, and the amplitude for a trapped billiard ball vanishes.

The billiard ball system should be similar to the evolution of a one-particle state in the $\lambda\phi^4$ theory, if the Compton wavelength $1/\mu$ corresponding to the physical mass of the scalar field, is small compared to the wormhole radius a and the state is chosen to have small velocity in the frame of the wormhole mouths. The hard-sphere potential of the billiard balls is replaced by the attractive $\lambda\phi^4$ interaction, but one would expect similar near collisions of a particle with itself to correspond to classical solutions; and these would dominate the path integral for states that approximate the non-relativistic WKB wave function. The one-particle–one-particle scattering to lowest order in λ corresponds to a tadpole diagram,

 ,

in which, as shown in Fig. 4, the loop of the diagram is dominated by the classical solution for a particle that traverses a CTC.

For a $\lambda\phi^4$ field with small λ, however, there is likely to be only one classical solution for given initial data. If this is the case, the closest connection to the Klinkhammer-Thorne and Politzer work is that diagrams that would contribute to their one-particle–one-particle scattering analysis fail to be unitary for any value of λ. In the weak-field case one can see the loss of unitarity directly in a perturbative approach to quantum field theory. When the interaction is strong, one can infer a loss of unitarity from the multiplicity of classical solutions; but unitarity fails whether or not the classical solutions are unique.

Recent work by Deutsch [19] also considers quantum mechanics on spacetimes with closed timelike curves, with inputs and outputs from a black box modeling "chronology-violating networks." Deutsch assumes that the classical evolution of fields is beset by paradoxes, some of which (such as the grandfather paradox) prevent consistent classical evolutions. As outlined in the Introduction, we do not share this view. On the basis of the examples that have been studied, we think it more likely that (to state a rough conjecture) on smooth, stable spacetimes whose CTC's are isolated—confined to a compact region—one can find classical solutions for generic initial data. Deutsch avoids the paradox by a departure from standard quantum mechanics in which particles traveling around CTC's are described by an arbitrary density matrix. A resulting nonuniqueness is resolved by a minimization of entropy.

IV. DISCUSSION AND CONCLUSIONS

What are the consequences of a lack of unitarity in the evolution that maps Fock space at a time prior to any CTC's to that at a later time? Does a consistent probability interpretation of quantum mechanics survive, or is it impossible to make sense of quantum field theory on spacetimes with CTC's? We begin with a discussion in a Copenhagen framework, showing that the loss of unitarity leads to inconsistent alternatives for computing probabilities for outcomes of the same experiment. (Similar analyses are given by Sorkin [33] and Jacobson [34].) We then argue that a version of the sum-over-histories interpretation of quantum mechanics can still make sense, although one pays a price for the loss of unitarity. Even in a region of spacetime to the past of any CTC's, the probabilities assigned to the outcomes of measurements can be affected by the fact that CTC's form in the future. One can only assign probabilities to paths that begin in the distant past, prior to any region of CTC's, and that end in the distant future, after all such regions. Probabilities can be assigned to decohering paths that include the histories of local measuring instruments. We expect them to agree with standard probabilities for the outcomes of experiments that involve no interaction (in past or future) with regions containing CTC's.

The key difficulty that arises from a loss of unitarity is an ambiguity in the assignment of probabilities for events occurring *before* the region of CTC's. To understand the ambiguity, consider a quantum system consisting of a microscopic subsystem interacting with a macroscopic, but quantum mechanical, measuring instrument. The combined system is described, as usual, by a tensor product of states $|a\rangle$ of the microscopic subsystem and states $|I\rangle$ of the measuring instrument. In the Copenhagen interpretation, a macroscopically large measuring instrument can be included in or excluded from the description of a quantum system at the discretation of the interpreter. When the measuring instrument is included in the quantum system, its macroscopic nature enforces a rapid decoherence that allows the probabilities of experimental outcomes to agree with probabilities assigned to final states of the subsystem by itself, with the instrument regarded as classical.

Suppose that the microscopic subsystem is initially in state $|a\rangle$ just prior to the spacelike hypersurface $\Sigma_{\rm in}$, and that the microscopic system and the instrument interact on $\Sigma_{\rm in}$. As a result of the interaction, the combined system will be in a state

$$\sum_c |c\rangle|I_c\rangle\langle c|a\rangle \ . \tag{34}$$

In this expression, $|I_c\rangle$ is a state of the instrument associated with the state $|c\rangle$ of the microscopic system. The states $|c\rangle$ are orthonormal, as are the states $|I_c\rangle$.

After their interaction on $\Sigma_{\rm in}$, the instrument and the microscopic system decouple. We assume that, between $\Sigma_{\rm in}$ and $\Sigma_{\rm out}$, an instrument in the state $|I_c\rangle$ remains in that state, so that on $\Sigma_{\rm out}$ the state $|I_c\rangle$ has evolved to $e^{i\phi_c}|I_c\rangle$. (One could imagine the instrument's avoiding the CTC region or passing through it with negligible interaction.) The evolution of the microscopic system between $\Sigma_{\rm in}$ and $\Sigma_{\rm out}$ is governed by an operator $\mathcal{V}$ which will not in general be unitary. Then the state $|\Psi\rangle$ of the combined system on $\Sigma_{\rm out}$ has the form

$$|\Psi\rangle=\sum_c \mathcal{V}|c\rangle e^{i\phi_c}|I_c\rangle\langle c|a\rangle \ . \tag{35}$$

If we observe the instrument on $\Sigma_{\rm out}$, the Copenhagen probability $P^C(I_b,\Sigma_{\rm out})$ that it will be found in the state $|I_b\rangle$, associated with state $|b\rangle$ of the microscopic subsystem is

$$\begin{aligned} P^C(I_b,\Sigma_{\rm out}) &= \sum_d \left|\sum_c \langle d|\mathcal{V}|c\rangle\langle I_b|I_c\rangle\langle c|a\rangle\right|^2 \Big/ \langle\Psi|\Psi\rangle \\ &= \sum_d |\langle d|\mathcal{V}|b\rangle\langle b|a\rangle|^2/\langle\Psi|\Psi\rangle \\ &= \langle b|\mathcal{V}^\dagger\mathcal{V}|b\rangle|\langle b|a\rangle|^2/\langle\Psi|\Psi\rangle \ . \end{aligned} \tag{36}$$

If $\mathcal{V}$ is unitary, this agrees with the conventional result

$$P^C(I_b,\Sigma_{\rm out})=P^C(I_b,\Sigma_{\rm in})=|\langle b|a\rangle|^2 \ . \tag{37}$$

If the failure of unitarity were simply due to an overall normalization of the time evolution operator $\mathcal{V}$ (e.g., for a system with a Hamiltonian with constant imaginary part, corresponding to particles decaying with a given half-life), we would again have the usual result (37), because of the normalization factor $\langle\Psi|\Psi\rangle^{-1}$ in Eq. (36). The difficulty we face is more severe, however. The expectation value of $\mathcal{V}^\dagger\mathcal{V}$ does depend on the state, and one finds inconsistent probabilities by observing the instrument at different times, $\Sigma_{\rm in}$ and $\Sigma_{\rm out}$.

Even though it decouples from the microscopic system before unitarity fails (before the Cauchy horizon bounding the region of CTC's), the instrument gives different results for the measurement depending on whether the instrument is checked before or after the epoch of CTC's: $P^C(I_b,\Sigma_{\rm out})\neq P^C(I_b,\Sigma_{\rm in})$. This is quite disturbing. If the measuring instrument has a memory, according to the Copenhagen interpretation, the memory will change from before the epoch of CTC's to after (even if it does not interact with the region of CTC's). A more attractive alternative is to abandon Copenhagen in favor of a path-integral interpretation.

Within the framework of the path-integral interpretation, one can apparently regain a consistent assignment of probabilities for fields on a spacetime in which the CTC's are confined to a compact region. A path integral has the additional advantage that it apparently allows one to define probabilities for measurements performed in a region containing CTC's. Features of the interpretation essential to our discussion are presented in the following two paragraphs. We consider only experiments that can be described by restrictions on the paths in configuration space of the field ϕ, and outline in the Appendix a more complete version of the interpretation. (A detailed review and list of references is given in Ref. [20].) The remainder of this section is intended to be largely self-contained.

As before, let $\Sigma_{\rm in}$ and $\Sigma_{\rm out}$, respectively, be spacelike hypersurfaces to the past and future of all CTC's. For a scalar field ϕ, a configuration-space path integral governs the time evolution of state vectors in the field representation. State vectors in the in- and out-Fock spaces are identified with functionals $\Psi(\phi|_{\Sigma_{\rm in}})$ and $\Psi(\phi|_{\Sigma_{\rm out}})$ (our notation conforms to that of paper I). Suppose that one can devise a measuring instrument that makes a class $\mathcal{C}$ of histories negligibly interfere with the complementary class of all other histories. The probability $P(\mathcal{C})$ that the history of the system belongs to $\mathcal{C}$ is then [21–23]

$$P(\mathcal{C})=\frac{\int_{\phi|\Sigma_{\rm out}}\left|\int_{\phi\in\mathcal{C}} e^{iS(\phi)}\Psi(\phi|_{\Sigma_{\rm in}})\right|^2}{\int_{\phi|\Sigma_{\rm out}}\left|\int_{\{\phi\}} e^{iS(\phi)}\Psi(\phi|_{\Sigma_{\rm in}})\right|^2} \ , \tag{38}$$

where $\{\phi\}$ is the class of all fields between $\Sigma_{\rm in}$ and $\Sigma_{\rm out}$ and the inner integrals include fields on $\Sigma_{\rm in}$ but not on $\Sigma_{\rm out}$.

To each outcome of a measurement corresponds a restriction on the possible fields in the vicinity of the measuring instrument. The probability for a particular outcome is given by (38), with $\mathcal{C}$ the class of all paths obeying the restriction. This prescription assigns an unambiguous probability for measurements made at any time, and the probability is independent of the choice of $\Sigma_{\rm in}$ and $\Sigma_{\rm out}$, as long as they are, respectively, to the past and future of any CTC's.

For the system we considered at the beginning of this section, we suppose that the state $|b\rangle$ of the microscopic subsystem corresponds to a restriction on $\phi|_{\Sigma_{\rm in}}$ and to a corresponding class $\mathcal{B}$ of fields. When one includes a measuring instrument, the joint state $|b\rangle|I_b\rangle$ similarly corresponds to a class $\tilde{\mathcal{B}}$ of fields + instrument paths. For the class $\tilde{\mathcal{B}}$, the measuring instrument's path at $\Sigma_{\rm in}$ is in the region of configuration space associated with a reading I_b of the instrument at $\Sigma_{\rm in}$. By our prescription, the probability for this is proportional to the sum over all paths in $\tilde{\mathcal{B}}$ that start on $\Sigma_{\rm in}$ and terminate on $\Sigma_{\rm out}$:

$$P(b,\Sigma_{\rm in})=\langle b|\mathcal{V}^\dagger\mathcal{V}|b\rangle\langle a|b\rangle\langle b|a\rangle/\langle\Psi|\Psi\rangle=P(b,\Sigma_{\rm out}) \ . \tag{39}$$

If the instrument remains intact and in an eigenstate $|B\rangle$ until reaching $\Sigma_{\rm out}$, this sum agrees with what one would find in the Copenhagen framework if one resolved the ambiguity of a Copenhagen interpretation by choosing to measure $|B\rangle$ on $\Sigma_{\rm out}$. Here, however, the measuring instrument need not remain intact; no observer need look at it on $\Sigma_{\rm out}$, and the prescription is unambiguous.

The prescription, however, means that probabilities of experiments performed to the past of any CTC's can be affected by whether or not CTC's will form. Even to give a probability interpretation to an initial wave function on $\Sigma_{\rm in}$, one must compute a path integral to $\Sigma_{\rm out}$. That is, the probability density for finding the field ϕ on $\Sigma_{\rm in}$ is a sum of the form (38), where $\mathcal{C}$ is the set of all fields ϕ that start at $\phi|_{\Sigma_{\rm in}}$.

The fact that one must, in principle, compute a path integral in order to find probabilities in the present is already present in a milder form, even without CTC's, if one adopts the prescription that a class of histories must decohere in order to have a well-defined probability. One cannot, in principle, assign probabilities to alternatives on $\Sigma_{\rm in}$ without computing a path integral of the form (38), extending arbitrarily far to the future, in order to decide whether the alternatives decohere. In practice, however, decohering alternatives are obvious, and no such integrals need be evaluated. We expect that, at least for microscopic CTC's, one can similarly dispense with path integrals extending over times long compared to the experiment, and that one will not ordinarily encounter violations of causality on scales large compared to the size of the CTC's. A plausible conjecture is that on spacetimes whose CTC's are confined to a compact region, ordinary quantum mechanics is valid unless one designs an experiment to probe that region. Properly designed experiments, however, can violate causality (exhibit probabilities of outcomes different from those of ordinary quantum mechanics) in regions to the past of any CTC's.

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APPENDIX A: PROPAGATOR ON TIME-TUNNEL SPACETIMES

Let M, g_{ab} be a time-tunnel spacetime, an asymptotically flat spacetime all of whose CTC's thread a wormhole. That is, M,g has CTC's, but its universal covering space does not. We shall assume that there are asymptotically flat spacelike hypersurfaces $\Sigma_{\rm in}$ and $\Sigma_{\rm out}$ to the past and future of all CTC's, and that these hypersurfaces are generalized Cauchy surfaces—surfaces for which smooth, finite-energy data for the scalar wave equation has a unique solution ϕ on M with ϕ and $\nabla\phi$ in L_2. For a static time tunnel, $\Sigma_{\rm in}$ and $\Sigma_{\rm out}$ can be replaced by $\mathcal{I}^-$ and $\mathcal{I}^+$.

We formally construct the propagator on M by means of a multiple scattering series [9,4], a variant of which is given in Eq. (A17) below in terms of a family of covering spaces. Let $\tilde{M}_N, \tilde{g}$ be the N-cover of M, shown in Fig. 5. The projection π: $\tilde{M}_N \to M$, is locally an isometry that covers each point $x \in M$ by N points in $\tilde{M}_N$. A global isometry T: $\tilde{M}_N \to \tilde{M}_N$ maps each point $X \in \tilde{M}$ over x to the next point TX over x: On $\tilde{M}_3$ (Fig. 5), for example,

$$TX_{-1} = X_0 , \quad TX_0 = X_1 , \quad TX_1 = X_{-1} . \tag{A1}$$

If ϕ satisfies the Klein-Gordon equation on M,g, then $\tilde{\phi} = \phi \circ \pi$ satisfies the Klein-Gordon equation on $\tilde{M}_N$.

A tilde will be used to denote pullbacks of functions, covariant tensors, and operators from M to $\tilde{M}_N$. In particular the pullbacks to $\tilde{M}_N$ of the Wightman function and Feynman propagator are given by

$$\tilde{D} = D \circ (\pi \times \pi) , \quad \tilde{\Delta}_F = \Delta_F \circ (\pi \times \pi) . \tag{A2}$$

Then $\tilde{\Delta}_F$ satisfies

$$\tilde{K}_X \tilde{\Delta}_F = \tilde{K}_Y \tilde{\Delta}_F = -\tilde{\delta}(X,Y) , \tag{A3}$$

where

$$\tilde{\delta}(X,Y) = \delta(\pi(X), \pi(Y)) = \sum_m \delta(X, T^m(Y)) . \tag{A4}$$

If one regards $\tilde{M}_N, \tilde{g}$ as a spacetime, ignoring its role as a covering space, one can define a Wightman function $\hat{D}(X,Y)$ and Feynman propagator $\hat{\Delta}_F(X,Y)$ by requiring that they satisfy

$$\tilde{K}_X \hat{D} = \tilde{K}_Y \hat{D} = 0 , \quad \tilde{K}_X \hat{\Delta}_F = \tilde{K}_Y \hat{\Delta}_F = -\delta(X,Y) , \tag{A5}$$

and have in the (static) past the form

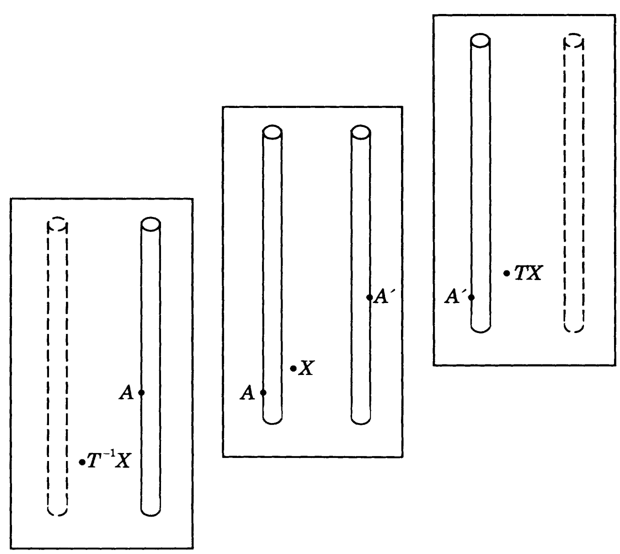

FIG. 5. The spacetime M_3 is constructed by identifying corresponding points on the cylindrical boundaries represented by solid lines. To construct the covering space $\tilde{M}_3$, one identifies points on the dashed cylinders as well. The points $T^{-1}X$, X, and TX correspond to the same point $x \in M$ but are not identified in the covering space.

$$\hat{D}(X,Y)={}_{\rm in}\langle 0|\phi(X)\phi(Y)|0\rangle_{\rm in}\ , \tag{A6}$$

$$\hat{\Delta}_F(X,Y)=\theta(X^0-Y^0)\hat{D}(X,Y)+\theta(Y^0-X^0)\hat{D}(Y,X)\ . \tag{A7}$$

Then

$$\tilde{\Delta}_F=\frac{1}{N}\sum_{m,n}\hat{\Delta}_F\circ(T_m\times T_n)\ ,\qquad \tilde{D}=\frac{1}{N}\sum_{m,n}\hat{D}\circ(T_m\times T_n)\ . \tag{A8}$$

To check Eq. (A8), let $\{f_k\}$ be an orthonormal basis for $\mathcal{H}_{\rm in}$, the Hilbert space constructed from solutions that have positive frequency with respect to the past timelike Killing vector on M. Then $\{\hat{f}_k\}=\{N^{-1/2}\tilde{f}_k\}$ is orthnormal on $\tilde{M}_N$ and can be completed to an orthonormal basis $\{\hat{f}_q\}$ by adjoining an additional set of orthonormal functions for which $\Sigma_m \hat{f}_q\circ T^m=0$. We have

$$\begin{aligned}\sum_{m,n}\hat{D}\circ(T_m\times T_n)&=\sum_{m,n}\bar{\hat{f}}_q\hat{f}_q\circ(T_m\times T_n)\\&=\sum_{m,n}(\bar{\tilde{f}}_k\circ T_m)(\tilde{f}_k\circ T_n)\frac{1}{N}\\&=N^2\bar{\tilde{f}}_k\tilde{f}_k\frac{1}{N}\\&=N\tilde{D}\ ,\end{aligned} \tag{A9}$$

in agreement with the second equality in (A8).

Because $\tilde{\Delta}_F$ and $N^{-1}\Sigma_{m,n}\hat{\Delta}_F\circ(T_m\times T_n)$ have the same form in the static past we need only verify Eq. (A3) to show that the corresponding relation for Δ_F holds: Because T_M is an isometry, $\tilde{K}_X(T_m\times T_n)=T_m\times T_n\tilde{K}_X$, and we have

$$\begin{aligned}\tilde{K}_X\left[\frac{1}{N}\sum\hat{\Delta}_F\circ(T_m\times T_n)\right]&=\frac{1}{N}\sum(\tilde{K}_X\hat{\Delta}_F)\circ(T_m\times T_n)\\&=\frac{1}{N}\sum(-\delta)\circ T_m\times T_n\\&=-\sum_m\delta\circ(1\times T_m)\\&=-\tilde{\delta}\ .\end{aligned} \tag{A10}$$

Equation (A8) implies

$$\begin{aligned}\tilde{D}(X,Y)&=\frac{1}{N}\sum_{pq}\hat{D}(T_pX,T_qY)\\&=\frac{1}{N}\sum_{pn}\hat{D}(T_pX,T_{p-n}Y)\\&=\sum_n\hat{D}(X,T_{-n}Y)\ .\end{aligned} \tag{A11}$$

The final equality here follows from the symmetry $\hat{D}(X,Y)=\hat{D}(TX,TY)$ and the fact that the sum over n in the second line is thus independent of the value of p. Similarly,

$$\begin{aligned}i\tilde{\Delta}_F=\frac{1}{N}\sum_{pq}i\hat{\Delta}_F(T_pX,T_qY)&=\frac{1}{N}\sum_{pq}[\theta(T_pX,T_qY)\hat{D}(T_pX,T_qY)+\theta(T_pY,T_qX)\hat{D}(T_pY,T_qX)]\\&=\frac{1}{N}\sum_{pn}[\theta(T_pX,T_{p-n}Y)\hat{D}(T_pX,T_{p-n}Y)+(X\leftrightarrow Y)]\\&=\sum_n[\theta(X,T_{-n}Y)\hat{D}(X,T_{-n}Y)+(X\leftrightarrow Y)]\ ,\end{aligned} \tag{A12}$$

where we have used the relation

$$\theta(TX,TY)=\theta(X,Y)\ ,\quad \hat{D}(TX,TY)=\hat{D}(X,Y)\ . \tag{A13}$$

Note that N does not appear in Eqs. (A11) and (A12); if the series converge, the equations are also correct as relations on the (globally hyperbolic) universal covering space.

Let us now approximate $\tilde{M}_N,\tilde{g}$ by leaving out the outermost wormhole. More precisely, suppose the spacetime M,g is obtained from $\mathbb{R}^4$ by removing two solid cylinders C_1 and C_2 and identifying each point $x\in\partial C_1$ with the point $\alpha(x)\in\partial C_2$. One can construct $\tilde{M}_N$ from N copies of $\mathbb{R}^4-C_1\cup C_2$ by identifying $\alpha(x)$ on the kth copy of ∂C_2 with x on the $(k+1)$st copy of ∂C_1, ending with the identification of the last copy of $\alpha(x)\in\partial C_2$ with the first copy of $x\in\partial C_1$. The approximating space M_N lacks this final identification: from the first and last copies of $\mathbb{R}^4$, only one cylinder (C_2 and C_1, respectively) are removed.

Then M_N is globally hyperbolic, and we can write

$$i\hat{\Delta}_N=\theta(X,Y)\hat{D}_N(X,Y)+\theta(Y,X)\hat{D}_N(Y,X)\ . \tag{A14}$$

The corresponding approximations to $\tilde{D}_F$ and $\tilde{\Delta}_F$ are given by Eqs. (A11) and (A12), with $\hat{D}$ and $\hat{\Delta}_F$ replaced by $\hat{D}_N$ and $\hat{\Delta}_N$. Thus,

$$i\tilde{\Delta}_N=\sum_{-N/2}^{N/2}[\theta(X,T_{-n}Y)\hat{D}_N(X,T_{-n}Y)+(X\leftrightarrow Y)]\ . \tag{A15}$$

This version of the multiple scattering series is correct to order $N/2$ when X and Y are on the middle (zeroth) sheet. If the series converges, then

$$\tilde{D}(X,Y)=\lim_{N\to\infty}\sum_{-N/2}^{N/2}\hat{D}_N(X,T_{-n}Y) \tag{A16}$$

and

$$i\tilde{\Delta}_F=\lim_{N\to\infty} i\tilde{\Delta}_N \ . \tag{A17}$$

We can compute Δ_F explicitly to first order in the multiple scattering series in terms of the first-order corrections to the eigenfunctions, for the spacetime with boosted wormhole mouths or a static spacetime [(i) and (ii) of Fig. 1, respectively], with a flat metric outside the removed cylinders. The calculation is simplest for zero-rest-mass fields, because initial data can be specified on $\mathcal{J}^-$, and $\mathcal{J}^-(M_N)$ consists of N disjoint copies of $\mathcal{J}^-(M)$. We shall choose on M_N an orthonormal basis $\hat{f}_{(m)j}(x)$ by setting as initial data on $\mathcal{J}^-$:

$$\hat{f}_{(m)j}(X)=\begin{cases} f_j(x) \ , & X\in m\text{th sheet} \ , \\ 0 \ , & X\notin m\text{th sheet} \ . \end{cases} \tag{A18}$$

Then

$$\langle \hat{f}_{(m)j}|\hat{f}_{(n)k}\rangle=\delta_{mn}\delta_{jk} \ . \tag{A19}$$

We have

$$\hat{D}(X,Y)=\sum_{mj}\hat{f}_{(m)j}(X)\bar{\hat{f}}_{(m)j}(Y)=\sum_m \hat{D}_{mm} \ , \tag{A20}$$

where

$$\hat{D}_{mn}=\sum_j \hat{f}_{(m)j}(X)\bar{\hat{f}}_{(n)j}(Y) \ . \tag{A21}$$

From the symmetry

$$\hat{f}_{(m)j}(T_pX)=\hat{f}_{(m-p)j}(X) \ , \tag{A22}$$

we have

$$\hat{D}_{mn}(T_pX,T_qY)=\hat{D}_{m-pn-q}(X,Y) \ . \tag{A23}$$

Then Eqs. (A11) and (A20) give

$$\tilde{D}(X,Y)=\sum_{mn}\hat{D}_{mm+n}(X,Y) \ . \tag{A24}$$

An analogous relation for the propagator follows from Eqs. (A20) and (A12):

$$i\tilde{\Delta}_F(X,Y)=\sum_{mn}[\theta(X,T_{-n}Y)\hat{D}_{mm+n}(X,Y) + \theta(Y,T_{-n}X)\hat{D}_{mm+n}(Y,X)] \tag{A25}$$

For X,Y in the pth and qth sheets, respectively, $\hat{D}_{mn}(X,Y)\sim(a/d)^{|m-p|+|n-q|}$, because $f_{(m)j}(X)$ requires $|m-p|$ transmissions. We expect that (A24) and (A25) converge to distributions when the multiple scattering series converges to solutions f_j of the Klein-Gordon equation.

We can now calculate the propagator to first order in the multiple scattering series. From Eqs. (A15) and (A17), with X and Y on the zeroth sheet, of M_3, we have

$$\hat{f}_{(m)j}(T_mX)=O(a^0) \ , \quad \hat{f}_{(m)j}(T_{m\pm1}X)=O(a) \ ,$$
$$\hat{D}_{mn}(X,Y)=O(a^{|m|+|n|}) \ .$$

Equation (A25) implies

$$\begin{aligned} i\tilde{\Delta}_F(X,Y)=&\,\theta(X,Y)\hat{D}_{00}+\theta(Y,X)\bar{\hat{D}}_{00}+\theta(X,T^{-1}Y)(\hat{D}_{01}+\hat{D}_{-10})(X,Y)+\theta(X,TY)(\hat{D}_{0-1}+\hat{D}_{10})(X,Y)\\ &+\theta(Y,T^{-1}X)(\hat{D}_{01}+\hat{D}_{-10})(Y,X)+\theta(Y,TX)(\hat{D}_{0-1}+\hat{D}_{10})(Y,X)\\ =&\,\theta(X,Y)\hat{D}_{00}+\theta(Y,X)\hat{D}_{00}+\theta(TX,Y)(\hat{D}_{01}+\hat{D}_{-10})(X,Y)+\theta(X,TY)(\hat{D}_{0-1}+\hat{D}_{10})(X,Y)\\ &+\theta(TY,X)(\hat{D}_{01}+\hat{D}_{-10})(Y,X)+\theta(Y,TX)(\hat{D}_{0-1}+\hat{D}_{10})(Y,X) \ . \end{aligned} \tag{A26}$$

Note that Eq. (A26) is the $O(a)$ approximation to $\tilde{\Delta}_F$ of Eq. (A25).

Finally, we need the first-order correction to the eigenfunctions $f_{(m)j}$. As eigenfunctions on Minkowski space, we use the normalized spherical Bessel functions, $\sqrt{(k/\pi)}j_l(kr)Y_{lm}e^{-i\omega t}$. Let us begin by considering the simplest case, in which a single wormhole joins two identical copies of Minkowski space. The exact solution corresponding to an incoming spherical wave is a sum of a reflected wave outgoing from the first mouth and a transmitted wave outgoing from the second mouth. Denote by $r_{l\omega}$ and $t_{l\omega}$ the reflection and transmission coefficients for a wave $j_l(kr)Y_{lm}(\hat{\mathbf{r}}_1)e^{-i\omega t}$ centered about the first mouth, and let $\mathbf{r}_1(X)$ and $\mathbf{r}_2(TX)$ be vectors from the centers of the two mouths. Exact eigenfunctions on this simplest wormhole spacetime are then of the form

$$\begin{aligned} f_{lm\omega}(x)=e^{-i\omega t}\sqrt{k/\pi}[&j_l(kr_1)Y_{lm}(\hat{r}_1)\\ &+r_{l\omega}h_l^{(1)}(kr_1)Y_{lm}(\hat{\mathbf{r}}_1)\\ &+e^{i\omega\tau}t_{l\omega}h_l^{(1)}(kr_2)Y_{lm}(\hat{\mathbf{r}}_2)] \ . \end{aligned} \tag{A27}$$

The boundary conditions, Eqs. (15a) and (15b), become

$$j_l(ka)+r_{l\omega}h_l^{(1)}(ka)=t_{l\omega}h_l^{(1)}(ka) \tag{A28}$$

and

$$kj_l'(ka)+r_{l\omega}kh_l^{(1)'}(ka)=-t_{l\omega}kh_l^{(1)'}(ka)\ , \tag{A29}$$

determining the transmission and reflection coefficients:

$$r_{l\omega}=\frac{h_l^{(1)}(ka)j_l'(ka)+h_l^{(1)'}(ka)j_l(ka)}{2h_l^{(1)}(ka)h_l^{(1)'}(ka)}\ , \tag{A30}$$

$$t_{l\omega}=\frac{h_l^{(1)}(ka)j_l'(ka)-h_l^{(1)'}(ka)j_l(ka)}{2h_l^{(1)}(ka)h_l^{(1)'}(ka)} =\frac{i}{(ka)^2 2h_l^{(1)}(ka)h_l^{(1)'}(ka)}\ . \tag{A31}$$

Finally, again to $O(a)$, we have

$$\hat{D}_{01}=\int d\omega \sum_{lm}\frac{k}{\pi}\bar{t}_{l\omega}j_l(kr_1)Y_{lm}(\hat{\mathbf{r}}_1)\bar{h}_l^{(1)}(kr_2)\bar{Y}_{lm}(\hat{\mathbf{r}}_2)e^{-i\omega\tau}\ , \tag{A32}$$

$$\hat{D}_{-10}=\int d\omega \sum_{lm}\frac{k}{\pi}t_{l\omega}h_l^{(1)}(kr_1)Y_{lm}(\hat{\mathbf{r}}_1)j_l(kr_2)\bar{Y}_{lm}(\hat{\mathbf{r}}_2)e^{-i\omega\tau}\ . \tag{A33}$$

To obtain the last equality, one can either use the translation formula of spherical Bessel functions [24] or use the fact that the zeroth-order form of $D(x,y)$ is translation invariant to write it as a sum of spherical Bessel functions centered about the left mouth.

APPENDIX B: A PATH-INTEGRAL INTERPRETATION

We briefly review the path-integral interpretation of quantum mechanics used in Sec. IV. In standard quantum mechanics, ideal measurements are associated with projection operators on a Hilbert space. When CTC's are present, however, there are no global spacelike hypersurfaces, and one can construct a Hilbert space of states only on in and out regions. Thus, within the standard framework, one cannot speak of measurements made in a region of CTC's; one can at best describe only measurements made by instruments that maintain stable records of experimental outcomes until there are no longer any CTC's. In addition, as we have seen, a Copenhagen interpretation gives inconsistent probabilities for measurements made before and after a region of CTC's. A path-integral interpretation, on the other hand, allows one to define measurements in a region with CTC's in the same way that they are defined on a globally hyperbolic spacetime, and one can apparently maintain a consistent set of probabilities despite the loss of unitarity that characterizes the quantum evolution.

The view we adopt is similar to that presented in Sec. IV.3 of Hartle [20] and in Sinha and Sorkin [23] (although Sorkin does not regard decoherence as a prerequisite for the assignment of probability). It shares with the path-integral interpretation given in Feynman's [21] initial paper an assignment of probabilities to classes of paths in configuration space (see also Caves [22], Stachel [25], Mensky [26], Aharonov and Albert [27], Griffiths [28], Omnes [29], and Gell-Mann and Hartle [30]). It shares with the Everett interpretation a refusal to distinguish *in principle* macroscopic and microscopic systems—there is no separate "classical" domain. The entire Universe is assumed to be quantum mechanical, and the nearly classical behavior of macroscopic systems is in part the result of what Bohm calls "destruction of interference" by the random phases of a complex system [31,32]: in path-integral terminology, classes of paths in configuration space rapidly decohere, allowing one to identify the history of a physical system with any member of a class of macroscopically indistinguishable paths.

We begin by recapitulating a version of the path-integral interpretation, relating the formalism to a Schrödinger picture. In a wave-function terminology, two classes of fields $\mathcal{C}$ and $\mathcal{C}'$ are said to decohere if the wave functions at Σ_{out} obtained by summing over the class $\mathcal{C}$ has negligible overlap with that obtained from the class $\mathcal{C}'$. Let $\mathcal{C}\Psi_{\text{in}}$ be the state vector in $\mathcal{F}_{\text{out}}$ obtained from Ψ_{in} by summing over the class $\mathcal{C}$. That is, to obtain the value of $\mathcal{C}\Psi_{\text{in}}$ at $\phi|_{\Sigma_{\text{out}}}$ one sums over all fields ϕ in $\mathcal{C}$ that end at $\phi|_{\Sigma_{\text{out}}}$:

$$(\mathcal{C}\Psi_{\text{in}})(\phi|_{\Sigma_{\text{out}}})=\int_{\mathcal{C}}D\phi\, e^{iS}\Psi_{\text{in}}\ . \tag{B1}$$

Then the overlap of $\mathcal{C}\Psi_{\text{in}}$ and $\mathcal{C}'\Psi_{\text{in}}$ is called the decoherence functional $D(\mathcal{C},\mathcal{C}')$:

$$D(\mathcal{C},\mathcal{C}')=\langle \mathcal{C}'\Psi_{\text{in}}|\mathcal{C}\Psi_{\text{in}}\rangle =\int d\phi|_{\Sigma_{\text{out}}}\overline{(\mathcal{C}'\Psi_{\text{in}})}(\mathcal{C}\Psi_{\text{in}})\ , \tag{B2}$$

or

$$D(\mathcal{C},\mathcal{C}')=\int_{\mathcal{C}'}D\phi'\int_{\mathcal{C}}D\phi\,\delta(\phi'|_{\Sigma_{\text{out}}}-\phi|_{\Sigma_{\text{out}}})e^{-iS(\phi')+iS(\phi)}\overline{\Psi_{\text{in}}(\phi')}\Psi_{\text{in}}(\phi)\ . \tag{B3}$$

One can regard this last form as a definition of $D(\mathcal{C},\mathcal{C}')$ in language that refers to the classes of fields—not to a final wave function.

If $\{\phi\}$ is the set of all fields, one can assign a probability

$$P(\mathcal{C})=D(\mathcal{C},\mathcal{C})/D(\{\phi\},\{\phi\}) \tag{B4}$$

to each class of fields $\mathcal{C},\mathcal{C}',\ldots,$ if (a) the classes are exclusive and exhaustive ($\mathcal{C}\cap\mathcal{C}'=\emptyset, \mathcal{C}\cup\mathcal{C}'\cdots=\{\phi\}$) and (b) any pair of classes decohere: $D(\mathcal{C},\mathcal{C}')=0$.

Equation (B4) is equivalent to Eq. (41) of the text.

By including measuring instruments in the fundamental description of a system, one has substituted classes of paths for the projection operators of the Copenhagen interpretation. *In practice,* it is obviously helpful to be able to speak only about probabilities associated with a microscopic system without explicitly introducing a measuring instrument. If one is allowed only to look at classes of microscopic fields on spacetime, however, one cannot reproduce most states—an energy eigenstate of an atom, for example, cannot be described as a class of field paths. The formalism of most of the recent "sum-over-histories" work allows one to describe arbitrary states by generalizing the meaning of "history." Instead of considering only classes of paths in configuration space (equivalently, sequences of projection operators corresponding to sequences of regions in configuration space) one works in the canonical framework and defines a history as a sequence of arbitrary projection operators. This freedom, of course, is not available on spacetimes with CTC's.

Instead, we rely on the fact that an ordinary state of a macroscopic object can be approximated to any reasonable degree of accuracy by a description of the position of the object in its configuration space, i.e., in the large configuration space of the quantum fields that comprise it, or in a smaller configuration space needed for a less fundamental description. The state of a microscopic system can be specified in the way one learns about it or prepares it in practice, by specifying a history of the macroscopic objects, together with the Lagrangian describing their interaction with a microscopically described system. (A specification that incorporates the inaccuracy of one's knowledge of a macroscopic system is given, for example, by Caves [22].)

If CTC's pervade spacetime on a small scale, there will be no hypersurfaces to whose future (or past) the spacetime has a causal structure. Nevertheless, we expect that if one probes Planck-size distances only for a finite time, a sum over local geometries in the past and future will reproduce local Lorentz invariance and allow our description, with final and initial spacelike surfaces, to be an accurate approximation.

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