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Black Hole Evaporation and Higher-Derivative Gravity[1]

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We examine the role which higher-derivative gravity interactions may play in black hole evaporation. The thermodynamic properties of black holes in Lovelock gravity are described. In certain cases, the specific heat of a black hole becomes positive at a small mass. This results in an infinite lifetime for the black hole (and also allows it to achieve stable equilibrium with a thermal environment). Thus no conflict with unitary time evolution would arise in such theories.

One of the most celebrated predictions of quantum field theory in curved space is the Hawking effect [1], which results in the quantum instability of black holes. An external observer detects thermal radiation arising from a black hole with a temperature proportional to the surface gravity, κ,

$$k_B T = \frac{\hbar\kappa}{2\pi} = \frac{\hbar c^3}{8\pi G M}$$

The second equality holds for an uncharged spherically symmetric black hole of mass M in four dimensions. The specific heat of such a black hole is negative, so that the temperature rises as it loses energy. If this process continues indefinitely, the black hole would radiate away its entire mass in a finite amount of time. Such an event would be a disaster for quantum mechanics resulting in incoherence of the wave function, as follows. Beginning with an initial pure quantum state, it remains a pure quantum state in

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the future in the presence of the black hole. The observables describing the future state, though, are divided into two sets, those inside the horizon and those outside. An external observer makes measurements only of the latter, and so, observes a mixed state as thermal radiation must be. His state is described by the density matrix obtained by integrating over all internal observables. In principle the black hole acts as a reservoir of information which allows for the reconstruction of the full quantum state. If the black hole evaporates completely, though, this information is irrevocably lost. Hence a pure quantum state has evolved into a mixed state, which is a clear violation of one of the basic tenets of quantum theory, namely, unitary time evolution.

Exactly what happens in the final moments of evaporation is still a very open question, however. The relation of the temperature and the mass given above is derived for an explicit solution of Einstein's equations. These equations are an effective low-energy theory valid for small curvatures. One sees that the gravitational action will be modified by higher-derivative interactions in attempts to quantize gravity perturbatively as a field theory [2] or in the low-energy limit of string theory [3]. The latter prompted a recent spate of interest in Lovelock gravities. A Lovelock lagrangian is a sum of dimensionally extended Euler densities [4],

$$\mathscr{L} = \sum_{m=0}^{k} c_m \mathscr{L}_m$$

where c_m are arbitrary constants, and $\mathscr{L}_m$ is the Euler density of a $2m$-dimensional manifold

$$\mathscr{L}_m = 2^{-m} \delta^{a_1 b_1 \cdots a_m b_m}_{c_1 d_1 \cdots c_m d_m} R^{c_1 d_1}{}_{a_1 b_1} \cdots R^{c_m d_m}{}_{a_m b_m}$$

The generalized delta function $\delta^{a_1 \cdots b_m}_{c_1 \cdots d_m}$ is totally antisymmetric in both sets of indices. If only c_1 is nonvanishing, one recovers Einstein's theory. When perturbing about flat space, the Lovelock theories are ghost-free [5]. Exact solutions describing black holes have been found for these theories [6, 7]. We use the thermodynamics of these black holes as a model for the possible effects of higher-derivative interactions in the strong gravitational fields present in the final stages of black hole evaporation. Because the higher-derivative interactions are all exact divergences in four dimensions, we must consider space-times with $D \geqslant 5$. Since most candidates for the quantum theory of gravity involve more than four dimensions, this is not a drawback.

In many cases, the new solutions are qualitatively similar to the Schwarzschild black hole in higher dimensions [8]. However, certain instances are sufficiently different to warrant special attention. In particular, there are classes of solutions which give rise to thermodynamically stable black holes, that is, their specific heat is positive. Furthermore, the rate of evaporation slows to give these black holes an infinite lifetime. Since the black holes never vanish, none of the incoming quantum mechanical information is completely lost. Thus the conflict with unitary time evolution is very neatly avoided in these theories.

The solutions with positive specific heat may be divided into two categories [9]. The first arises in $2m+1$ dimensions for a theory including up to $\mathscr{L}_m$. In this case the temperature goes smoothly to zero as the size of the horizon shrinks to zero, and the mass approaches some finite value. There is a curvature singularity at $r=0$, and so at the minimum mass, the solution would correspond to a naked singularity. This limit is not reached in a finite time, however, since the temperature approaches zero too rapidly. The second category yields a remnant black hole similar to the familiar cases of extreme charged or spinning black holes. This behavior can occur only for theories including at least six derivative interactions (i.e., including $\mathscr{L}_m$ with $m \geqslant 3$). In these theories the solutions may have more than one horizon, and for certain critical values of the mass two horizons coalesce to form a degenerate zero-temperature horizon. We explicitly investigate the six-derivative theory and find examples of both kinds of behavior.

Wheeler found spherically symmetric solutions for Lovelock gravity of the form [7]

$$ds^2 = -f^2\,dt^2 + f^{-2}\,dr^2 + r^2\,d\Omega_{D-2}^2$$

where $d\Omega_{D-2}^2$ is a line element on the unit $(D-2)$-sphere, and

$$f^2 = 1 - r^2 F(r)$$

F is determined by solving for the real roots of a polynomial equation. For the six-derivative theory,

$$P(F) = F + aF^2 + bF^3 = \frac{\omega}{r^{D-1}}$$

where $a = (c_2/c_1)(D-3)(D-4)$, $b = (c_3/c_1)(D-3)(D-4)(D-5)(D-6)$, and ω is proportional to the mass of the black hole. For simplicity we have set $c_0 = 0$, which yields asymptotically flat solutions. The factors of $(D-n)$ express the fact that $\mathscr{L}_m$ makes no contribution for $D \leqslant 2m$. For $D=4$, $a=b=0$, and $f^2 = 1-(\omega/r)$, recovering the standard Schwarzschild metric with $\omega = 2GM/c^2$.

For $D \geqslant 7$, the cubic equation may be solved for F in closed form. For $a<0$ (which is the case of interest)

$$f^2 = 1 - \frac{r^2}{|a|}\left\{[A+(A^2+B)^{1/2}]^{1/3} + [A-(A^2+B)^{1/2}]^{1/3} + \frac{C}{3}\right\}$$

where

$$A = -\frac{C}{54}\left(2C^2 - 9C + 27\frac{\omega\,|a|}{r^{D-1}}\right)$$

$$B = \frac{C^3}{729}(3-C)^3$$

and $C = a^2/b$. When (A^2+B) becomes negative the metric is complex, indicating the presence of a singularity at $(A^2+B)=0$. In order to avoid such singularities at finite radius, we consider theories with

$$0 < \frac{a^2}{b} < 3$$

These singularities need not be naked, but a careful analysis reveals that degenerate horizons will not occur in those theories. The positions of the horizons are given by $f^2 = 0$. This is most simply expressed by

$$r_h^{D-3} + a r_h^{D-5} + b r_h^{D-7} = \omega$$

The restriction on a^2/b clearly requires $b>0$, and so the above equation will have more than one solution only if $a<0$.

The temperature of the horizon of a black hole is determined from the periodicity of the metric in imaginary time. To define quantum field propagators in these black hole backgrounds, one rotates to Euclidean time $t \to i\tau$. In order to produce a smooth Euclidean manifold, the Euclidean time must be identified with a certain periodicity β. This periodicity in imaginary time then appears in the propagators so defined and may be interpreted as indicating that the fields are in equilibrium with a heat bath of temperature $T = \hbar/(k_B \beta)$ [10, 11]. One finds that the periodicity is

$$\beta = \frac{2\pi}{\kappa}$$

where κ is the surface gravity of the horizon.

For the given metrics, the temperature is

$$T = \frac{\hbar}{4\pi k_B} \frac{\partial f^2}{\partial r}\bigg|_{r=r_h}$$
$$= \frac{\hbar}{4\pi k_B} \frac{1}{r_h} \frac{(D-3)\, r_h^4 + a(D-5)\, r_h^2 + b(D-7)}{r_h^4 + 2ar_h^2 + 3b}$$

Finding instances of zero-temperature horizons then reduces to solving the quadratic polynomial appearing in the numerator. In $D=7$, we find $T=0$ for $r_h=0$ and $r_h=(-a/2)^{1/2}$. The only other zero-temperature solutions occur in eight dimensions for negative a and

$$\frac{20}{9} < \frac{a^2}{b} < 3$$

A degenerate horizon then occurs at

$$r_h^2 = \frac{3\,|a|}{10}\left[1 \pm \left(1 - \frac{20}{9}\frac{b}{a^2}\right)^{1/2}\right]$$

The lower sign above is actually not of interest since it arises when the inner two of three horizons coalesce.

Seven dimensions and positive a provide an example of the first type of stability. There is only a single horizon which shrinks to zero as $\omega \to b$. The rate of mass loss is given by the luminosity $L \sim \mathscr{A} T^D$, where $\mathscr{A}$ is the area of the horizon. [Note that $\mathscr{A}$ has dimensions $(\text{length})^{D-2}$.] For a small horizon radius, $T \sim r_h \sim (\omega - b)^{1/2}$ and so

$$\frac{d\omega}{dt} \sim -r_h^5 T^7 \sim -(\omega - b)^6$$

This may be integrated to yield

$$\Delta t \sim (\omega - b)^{-5}\big|_{\omega_0}^{b} \to \infty$$

Thus the black hole requires an infinite amount of time to evaporate down to $\omega = b$, beginning with any mass where the temperature is finite.

An instance of the second stable category is found for $D=7$ when $a<0$. There are two horizons which coalesce for $\omega = \omega_c \equiv b - (a^2/4)$. As ω approaches ω_c, the horizon area remains finite, while $T \sim (\omega - \omega_c)^{1/2}$. The rate of energy loss is now

$$\frac{d\omega}{dt} \sim -(\omega - \omega_c)^{7/2}$$

which again results in an infinite time to reach the minimum mass. The eight-dimensional zero-temperature cases also fall in this class and produce similar results.

In conclusion, we have explicitly demonstrated that black holes are (essentially) stable in certain higher-derivative theories of gravity. These black holes act as reservoirs of quantum mechanical information for all time, and hence no conflict with unitary time evolution arises. We suggest, then, that the problems, which quantum gravity faces due to black hole evaporation, may be solved within the context of higher-derivative interactions.

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