Signal Analysis Primer and Applications

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Simons Foundation
12 December 2014

Research Background

- MEG-based Auditory Neuroscience
  - Cocktail-Party Auditory Processing
  - Role of Attention
  - Neural Representations of Speech
  - Fundamentally Temporally Neural Representations
- More at <http://www.isr.umd.edu/Labs/CSSL/simonlab/>

Outline

- Fourier Transform: *Why It's Useful*, and *What it Can/Cannot Do For You*
- Filters: *What They Do*, and *How They Do It*
- Filters: *Why So Many Different Kinds? Which Should I Use and When*?
- Grab Bag:
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The Fourier Transform

- **Every** Time-Domain Signal can be Re-expressed as a Sum of Sinusoids/Oscillations
- # of time points = # of frequencies
- Reciprocal relationship: *time resolution* (Δt) & *frequency span* (f_s)
- Reciprocal relationship: *frequency resolution* (Δf) & *time span* (T)

$$x[t] = \frac{1}{N}\sum_{k=0}^{N-1} X[f_k]\mathrm{e}^{i2\pi f_k t} \quad \text{where:}$$

$$t = \underbrace{0, \Delta t, 2\Delta t, \ \ldots \ , T - \Delta t}_{N}$$

$$f_k = \underbrace{0, \Delta f, 2\Delta f, \ \ldots \ , f_s - \Delta f}_{N}$$

$$f_s = sampling\ frequency = \frac{1}{\Delta t}$$

$$T = signal\ duration = \frac{1}{\Delta f}$$

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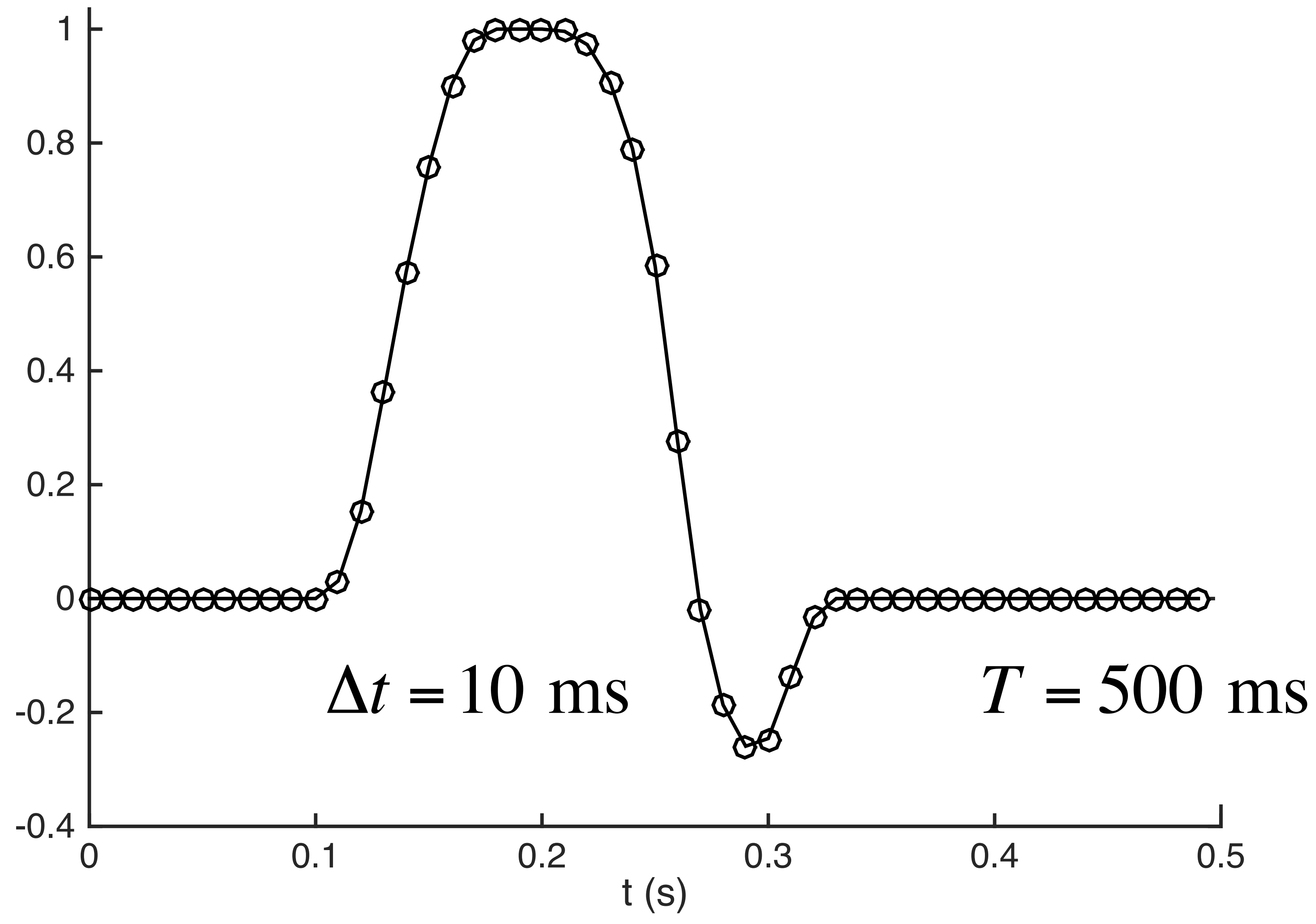

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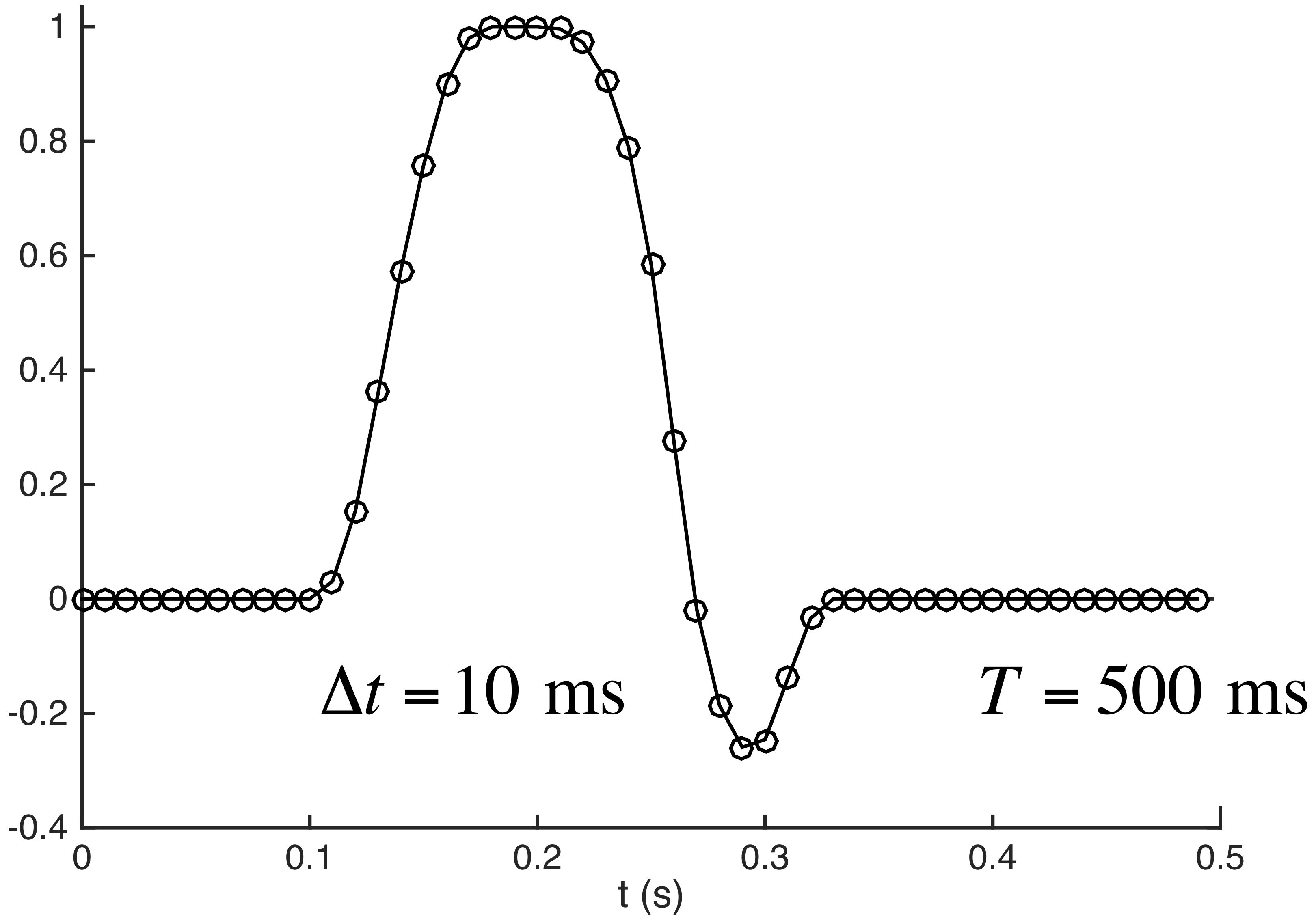

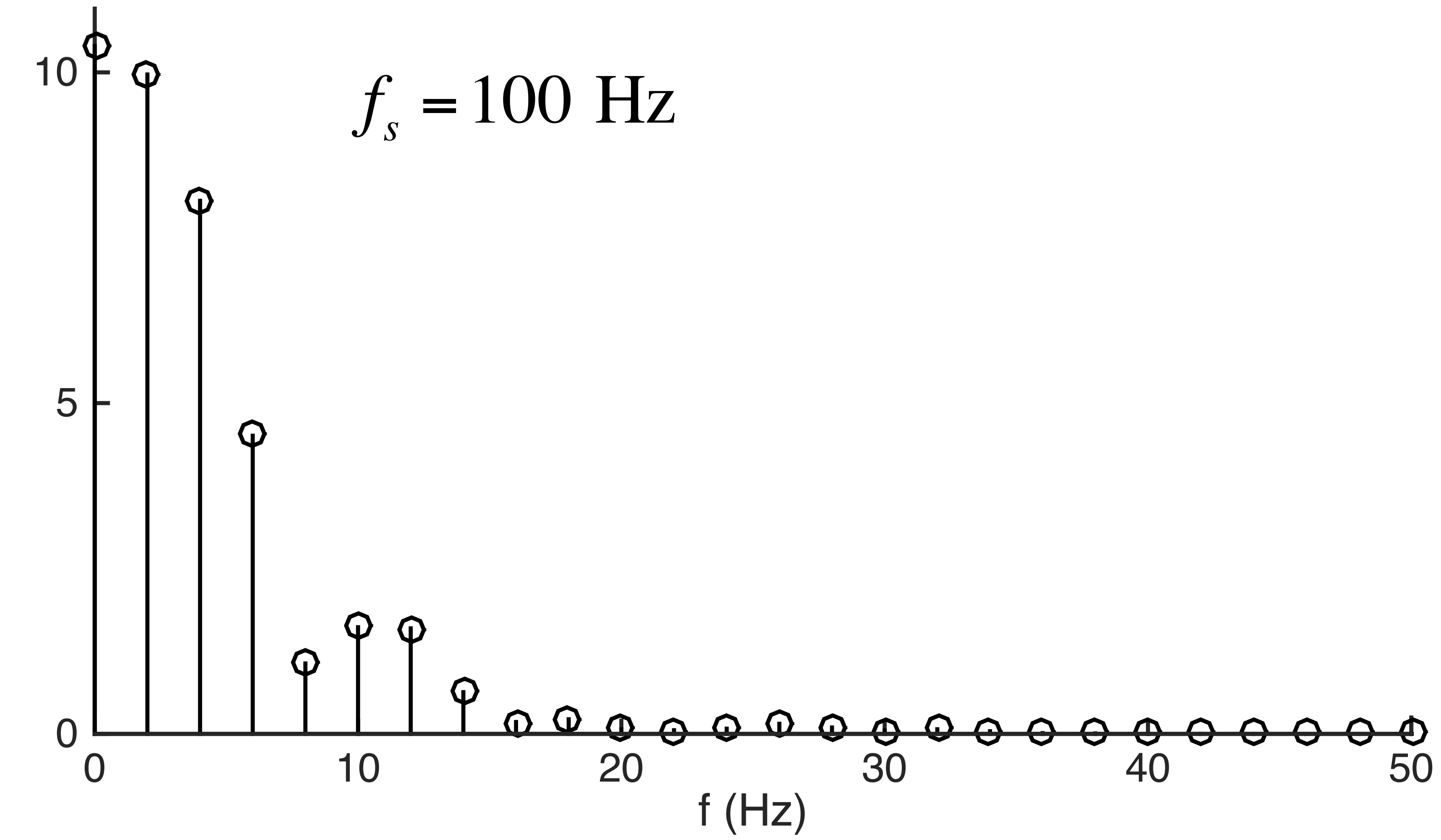

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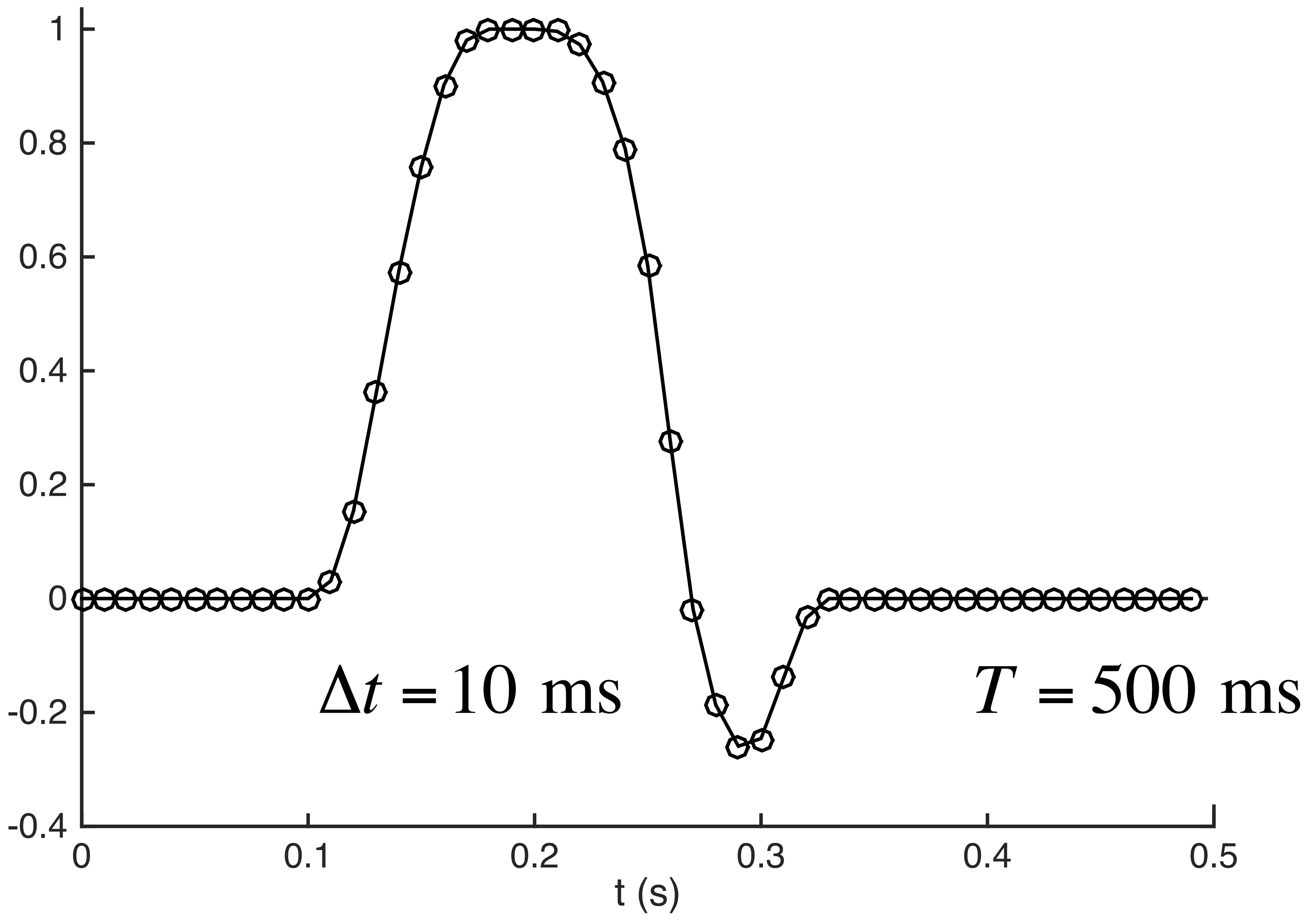

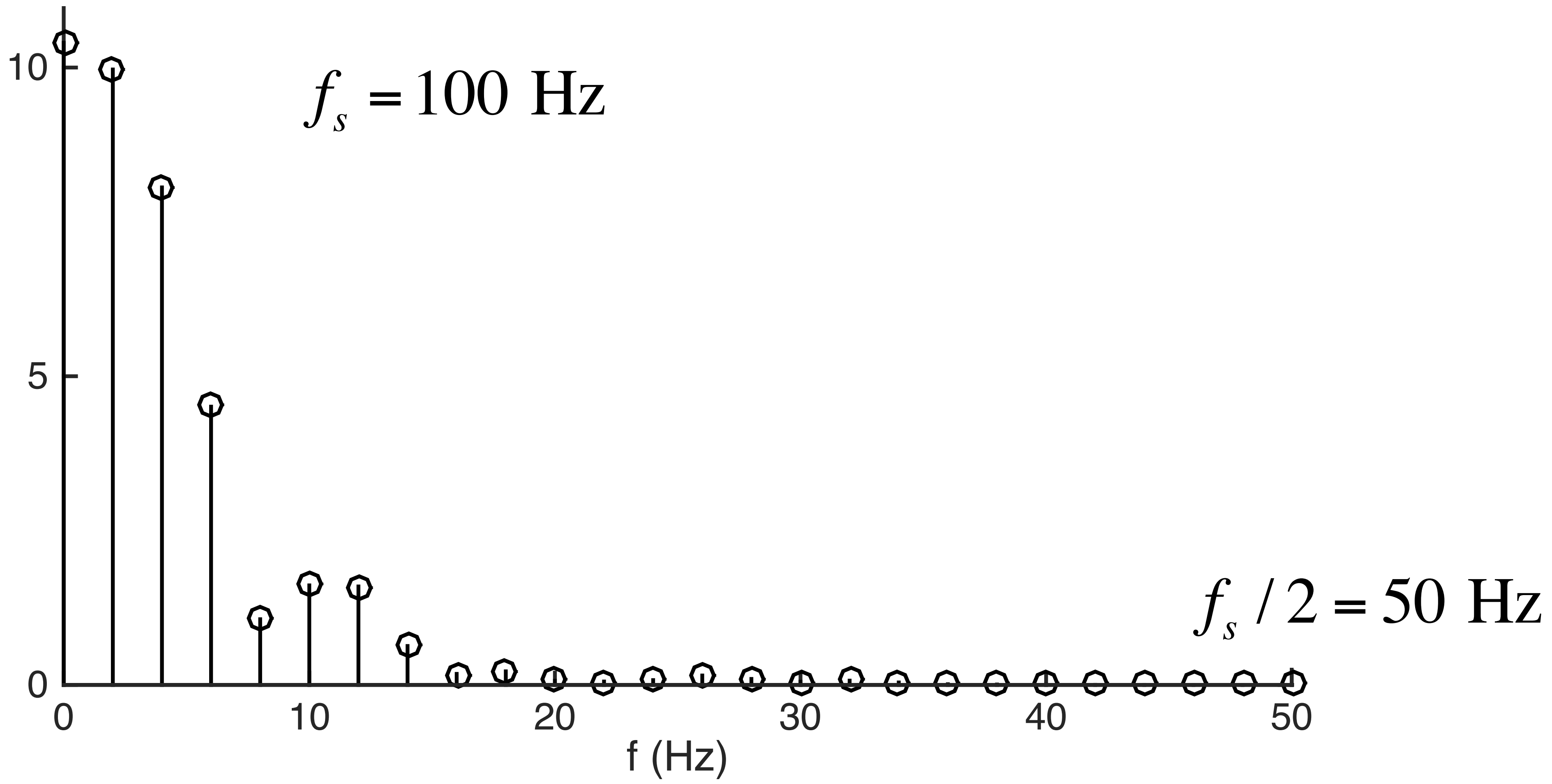

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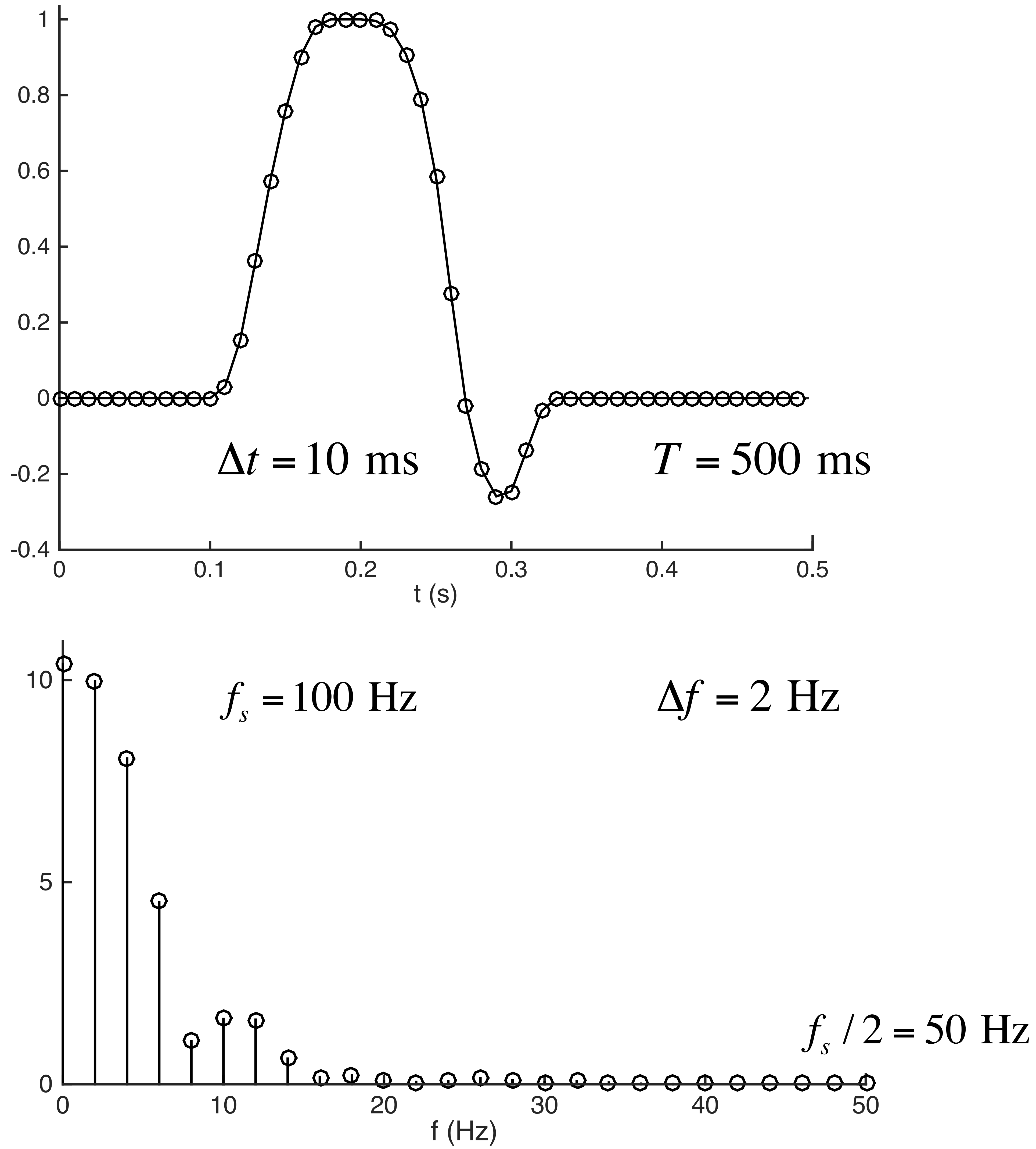

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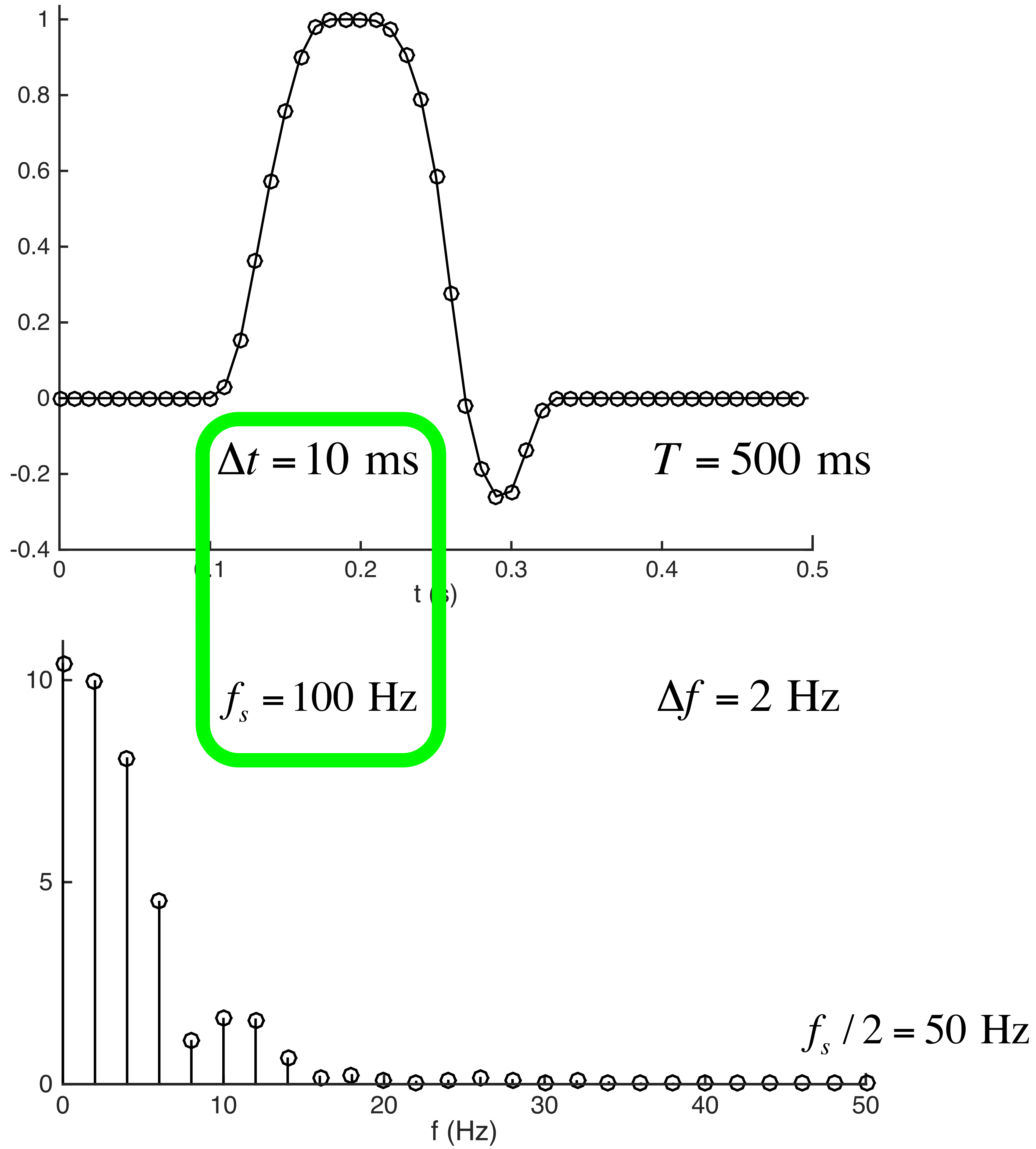

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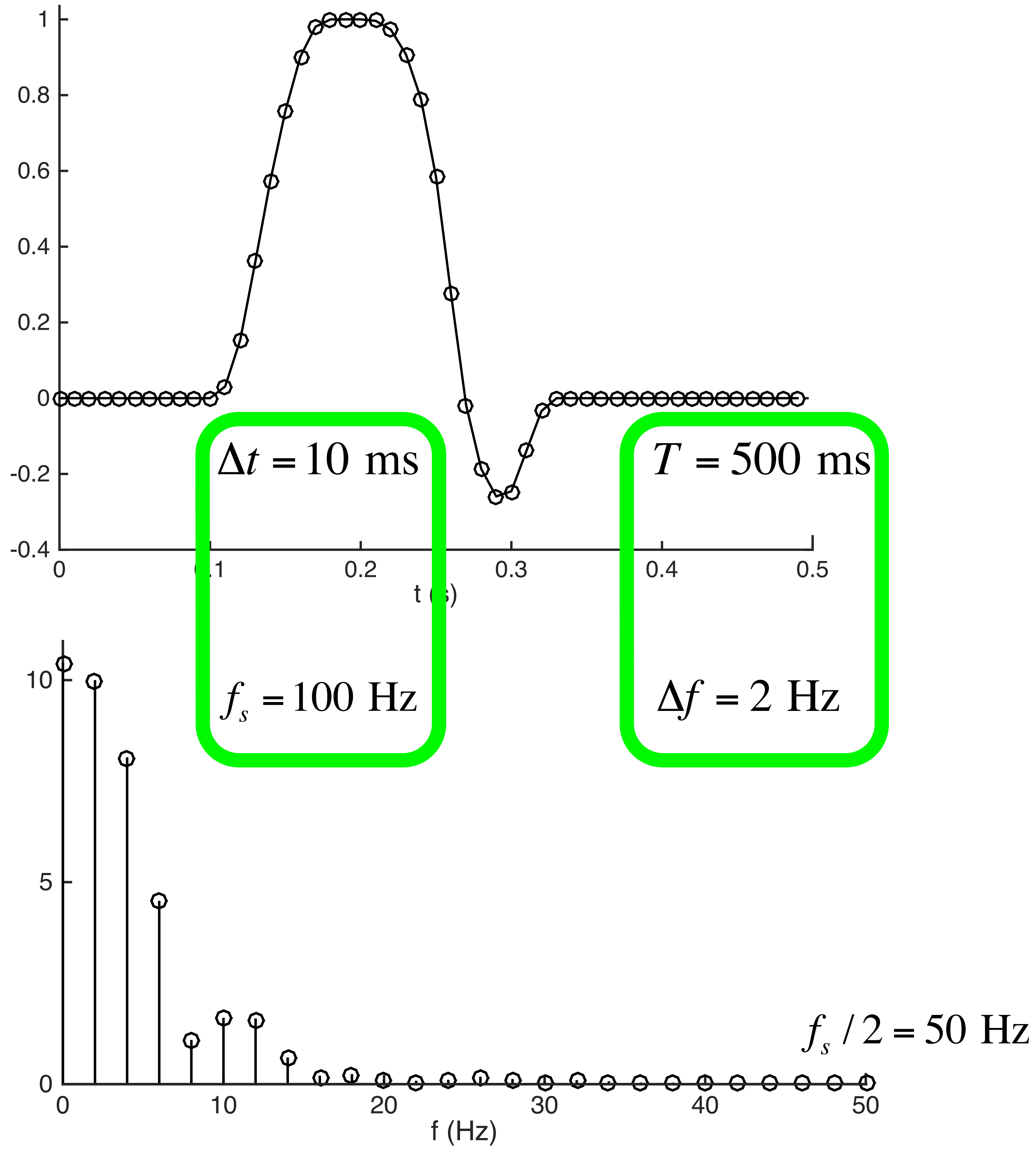

Fourier Transform: Time-Frequency Tradeoff

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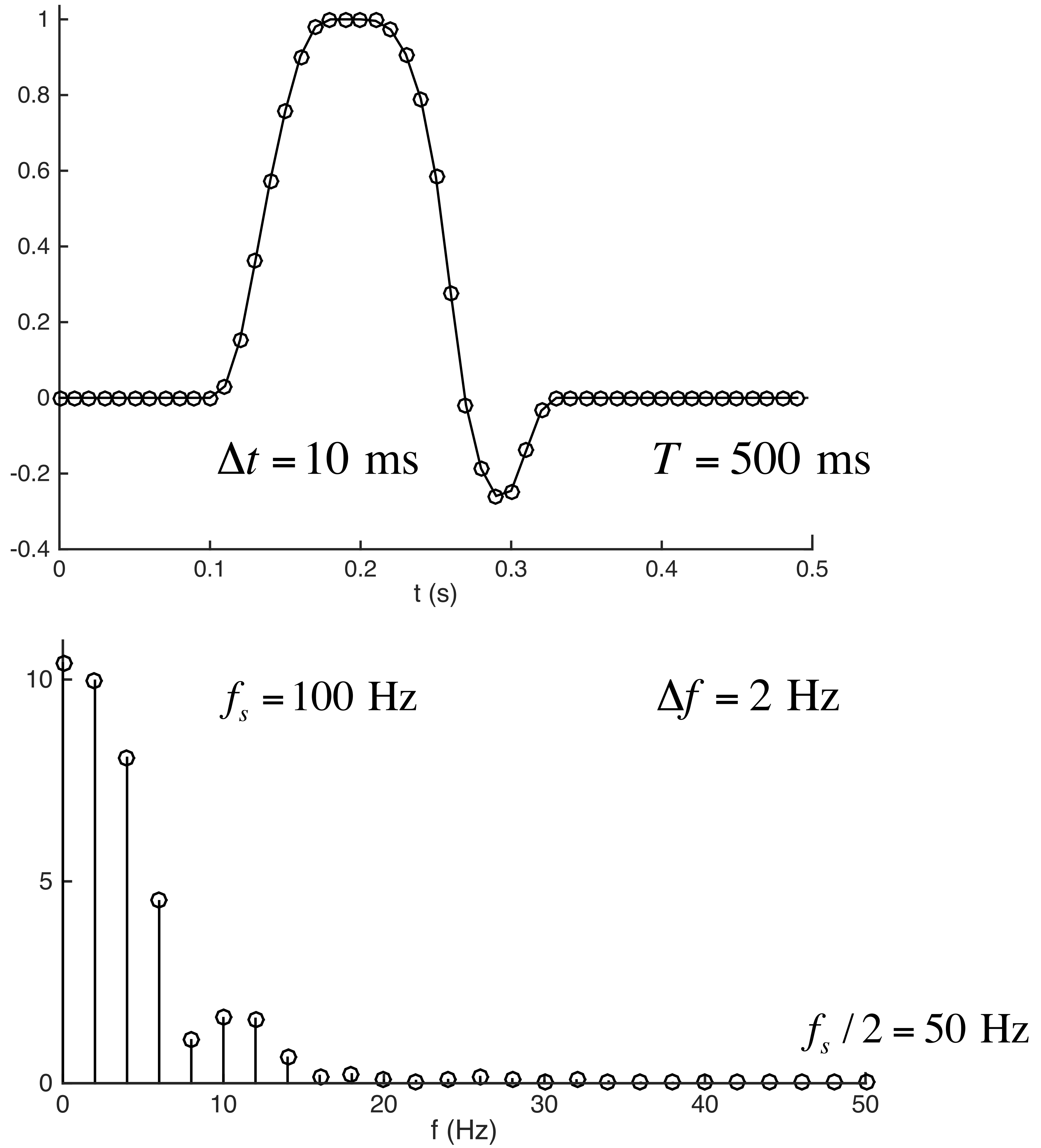

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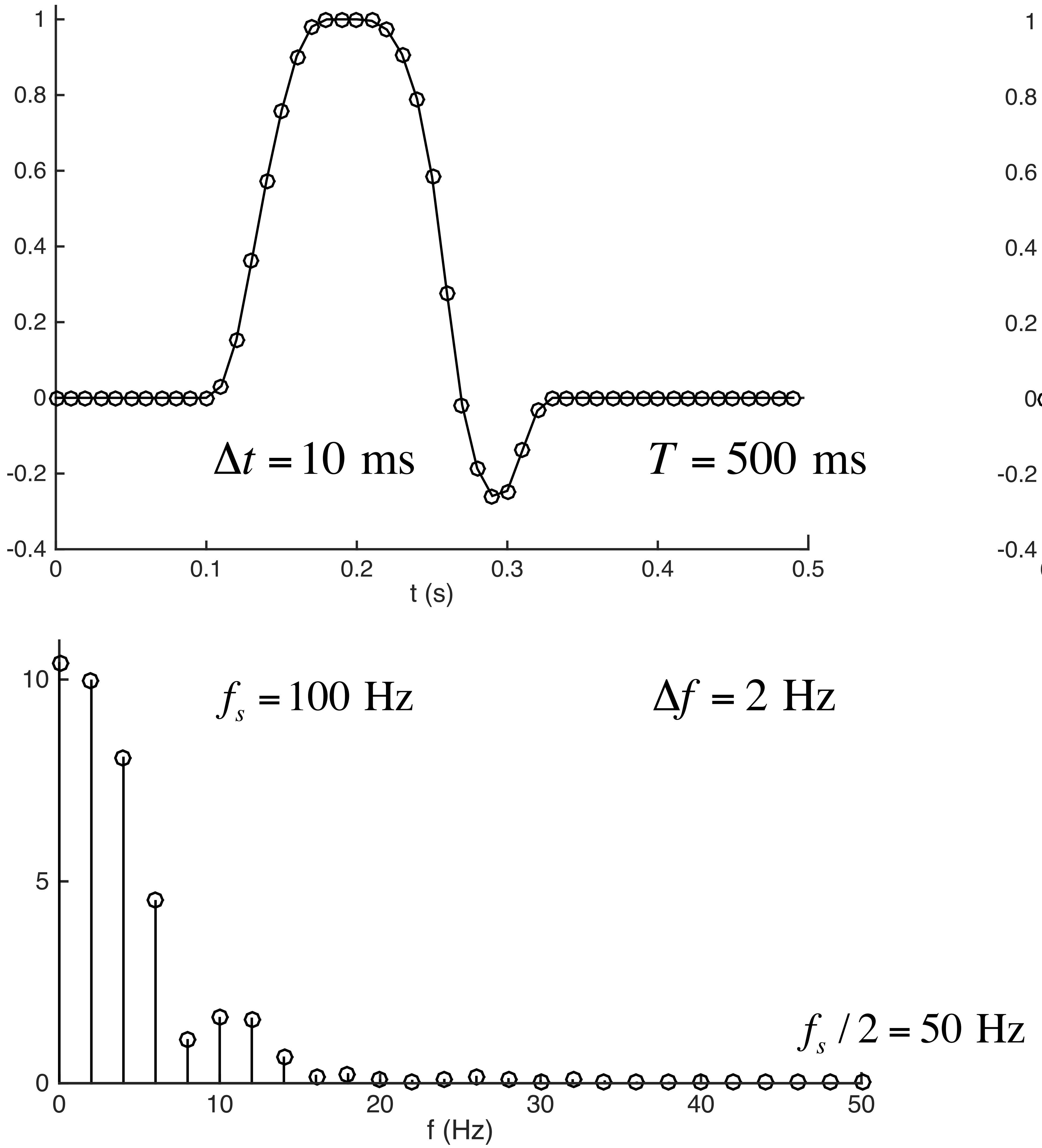

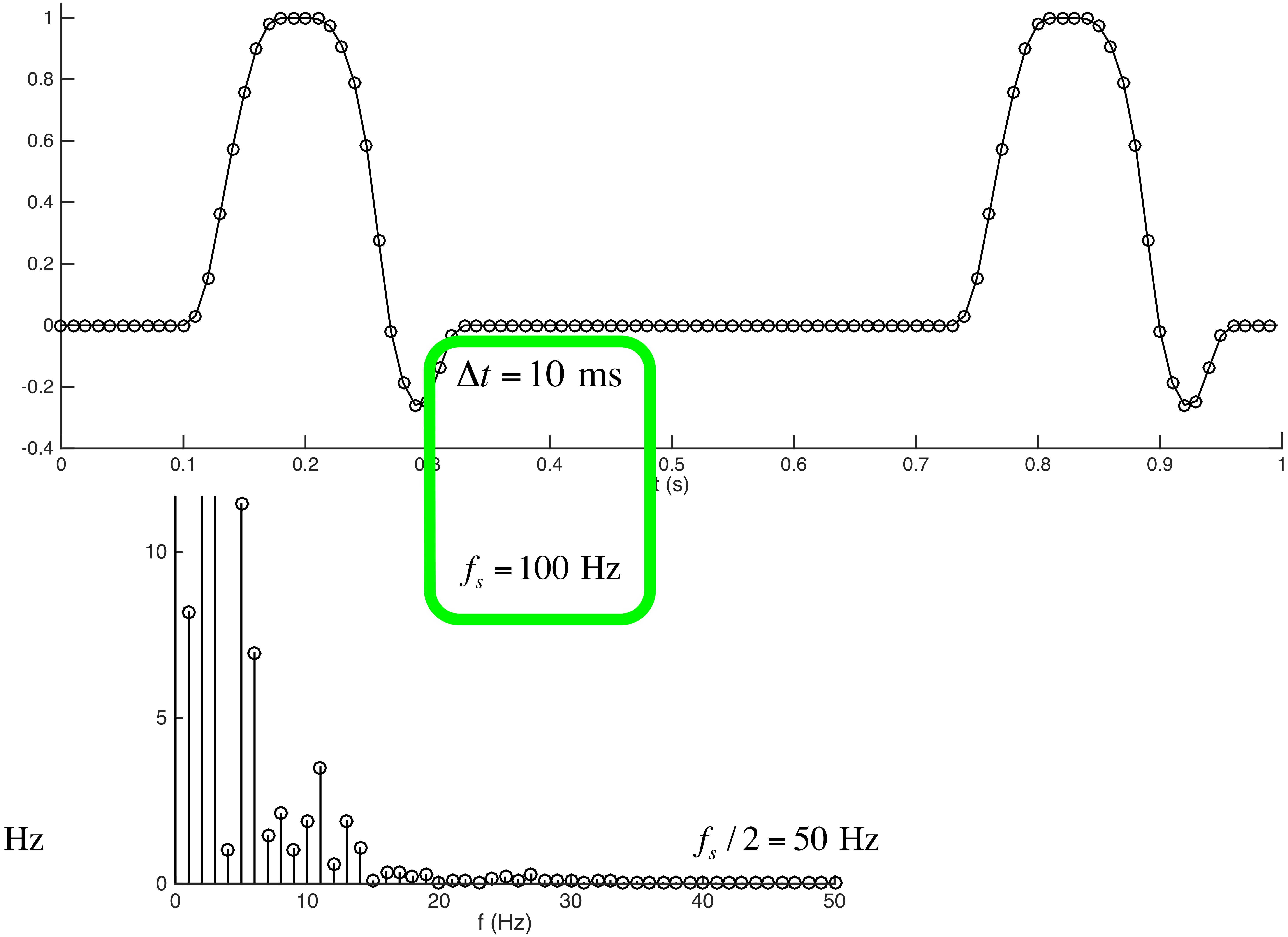

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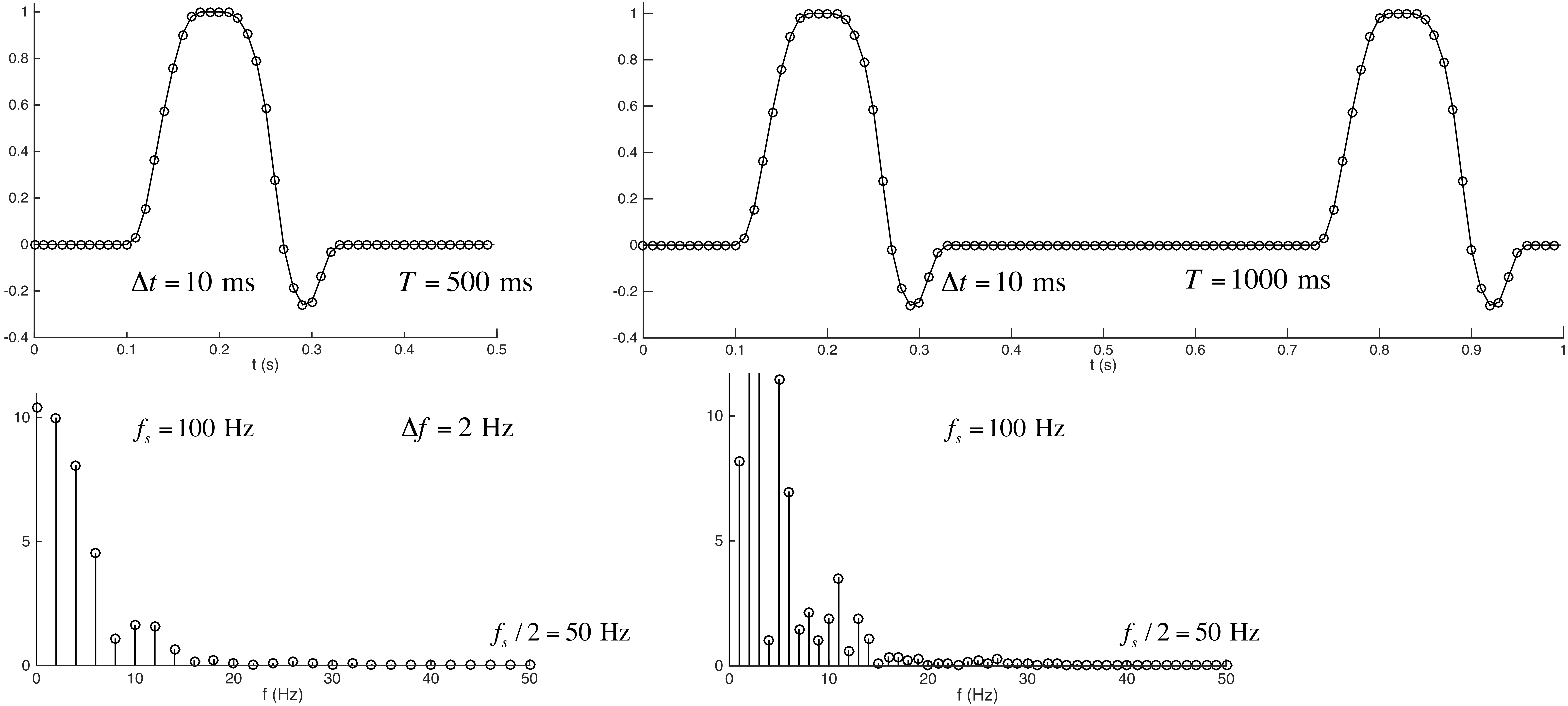

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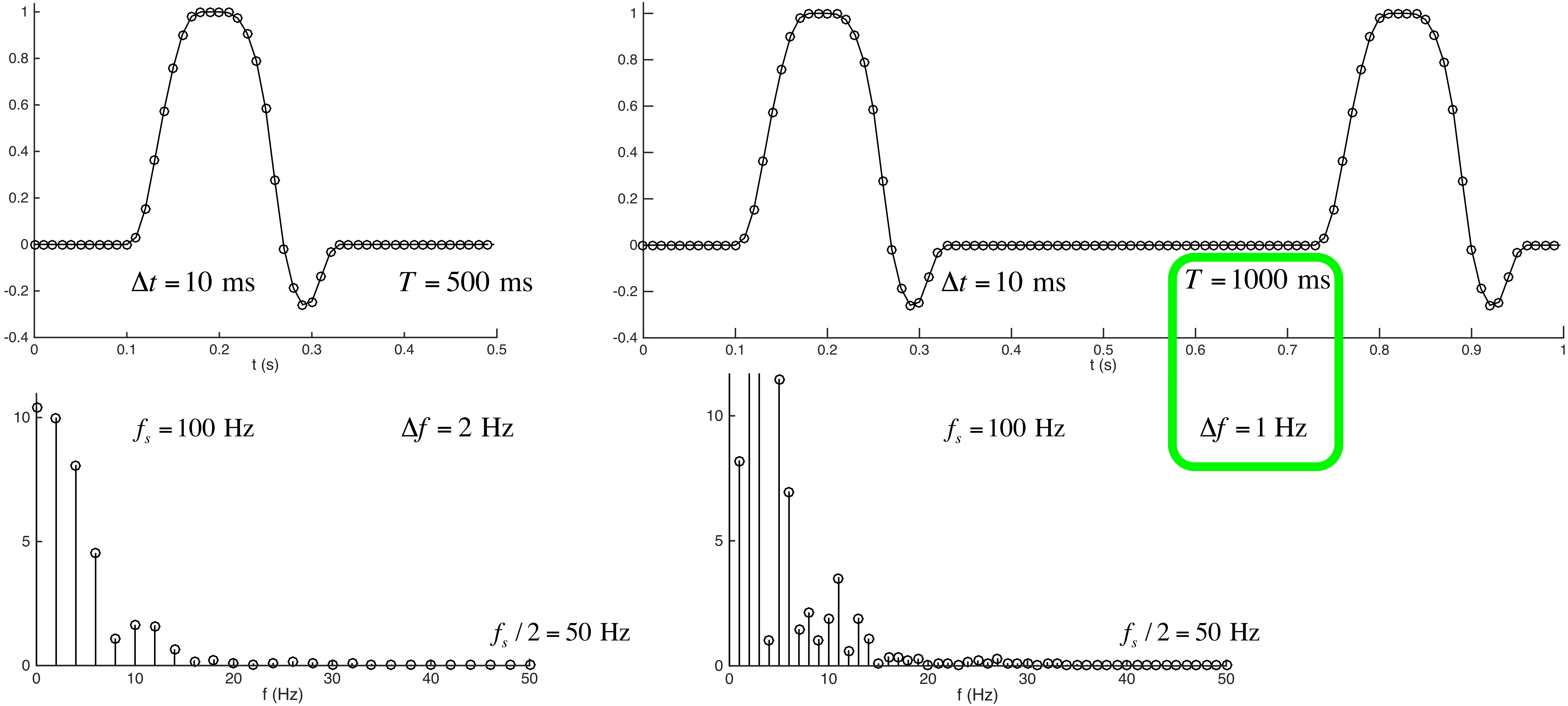
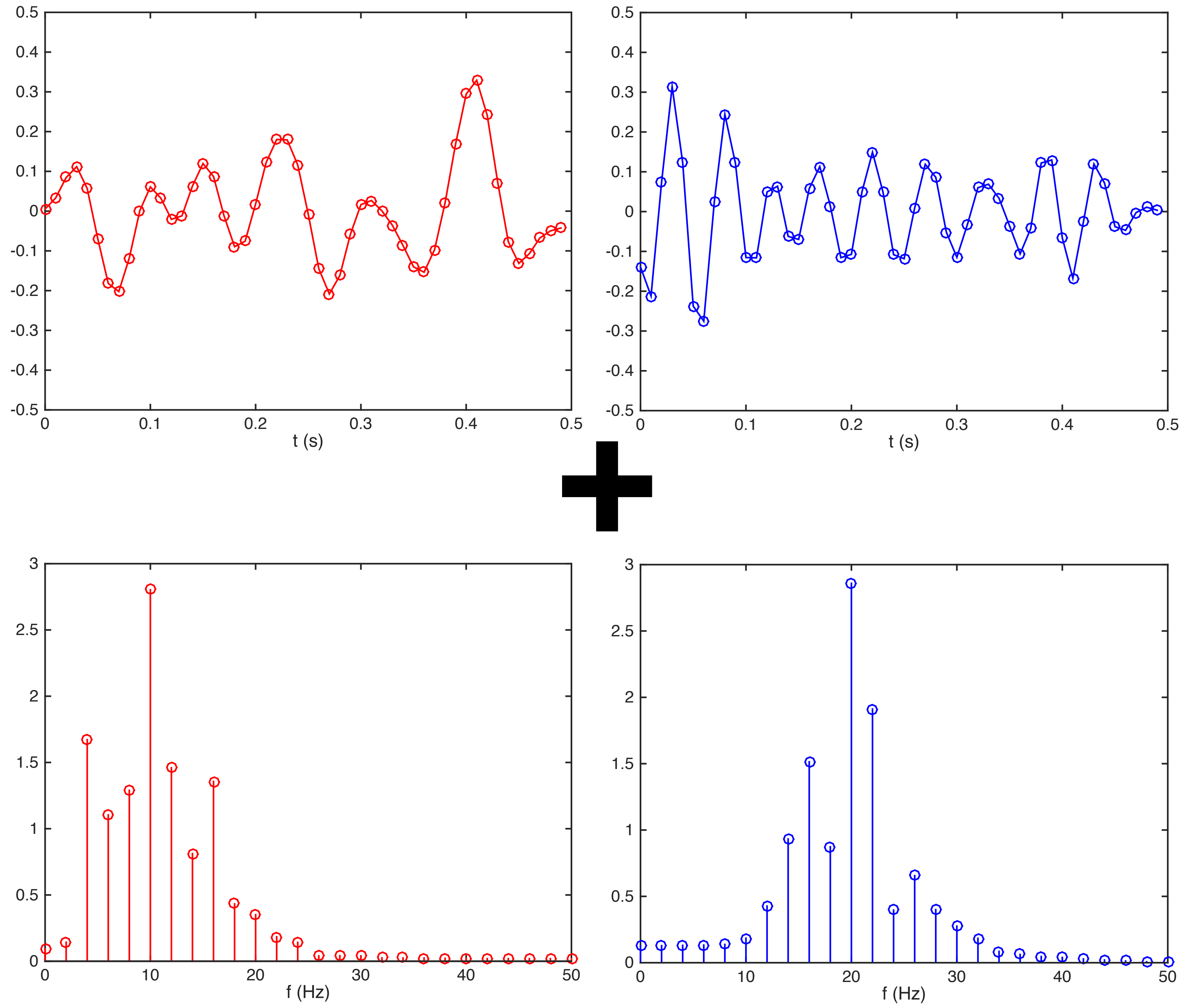


Fourier Transform: Practical Uses

- Measured Signals made up of several (many?) sources
- All overlap in time
- But overlap in frequency may be much less



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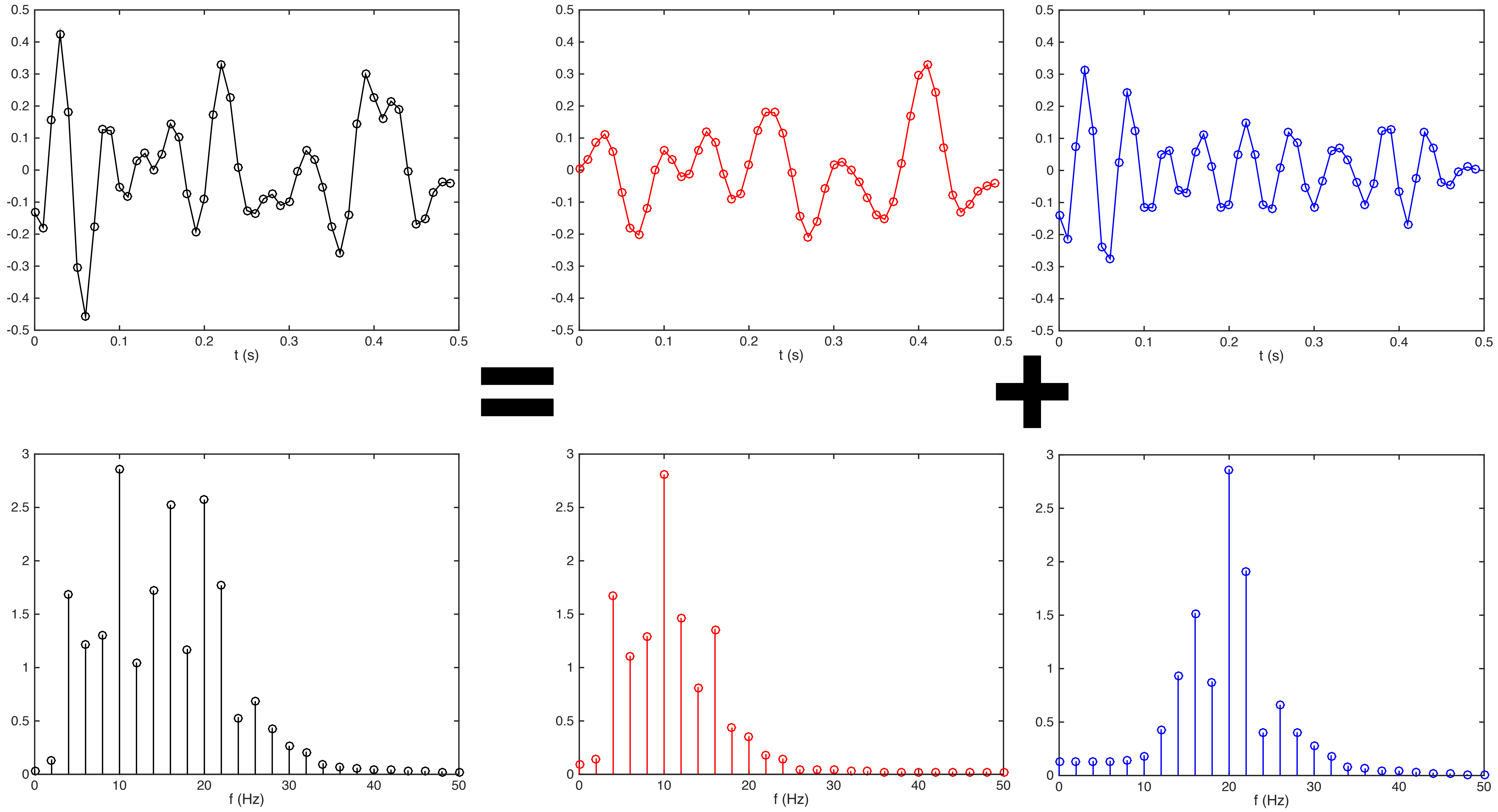

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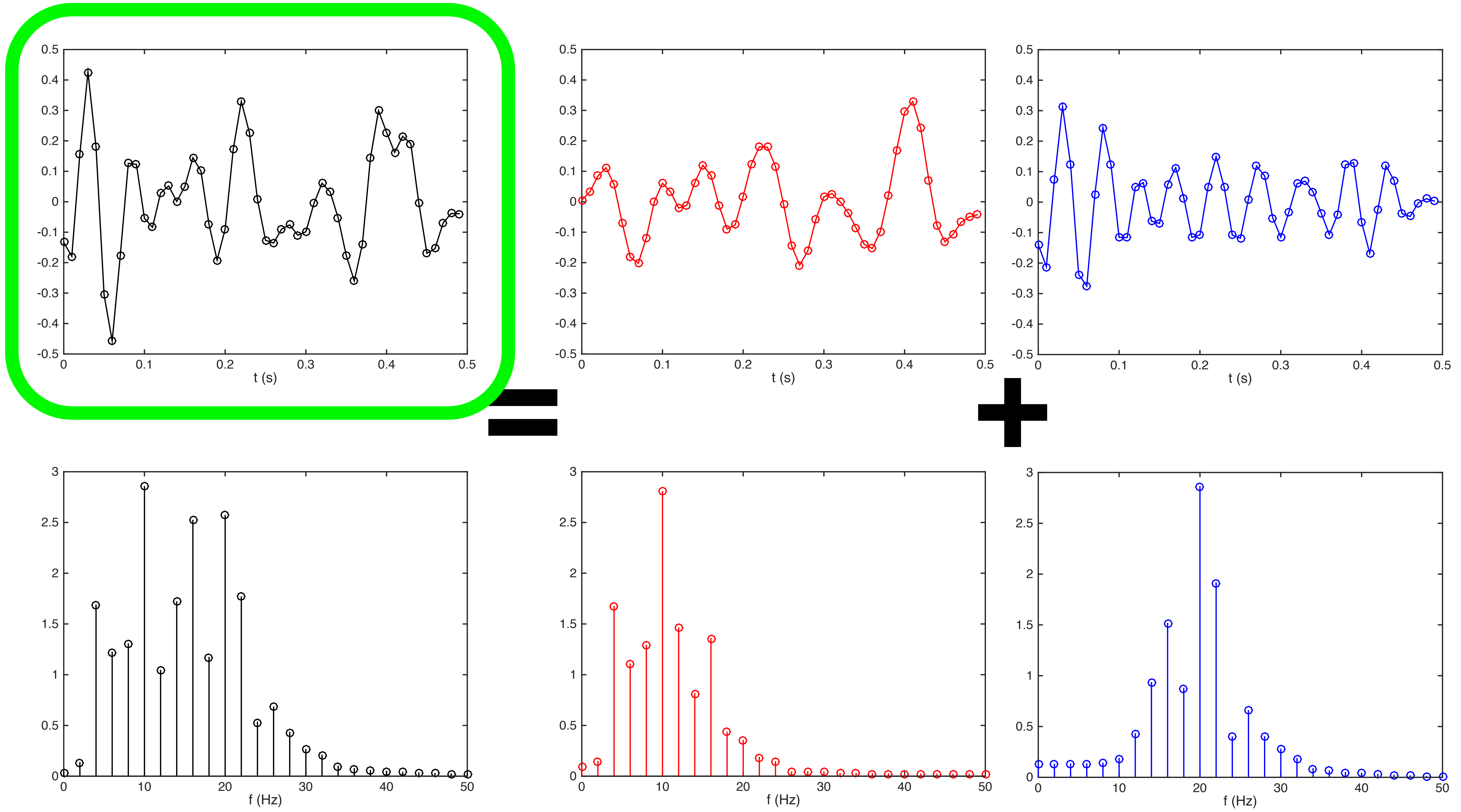

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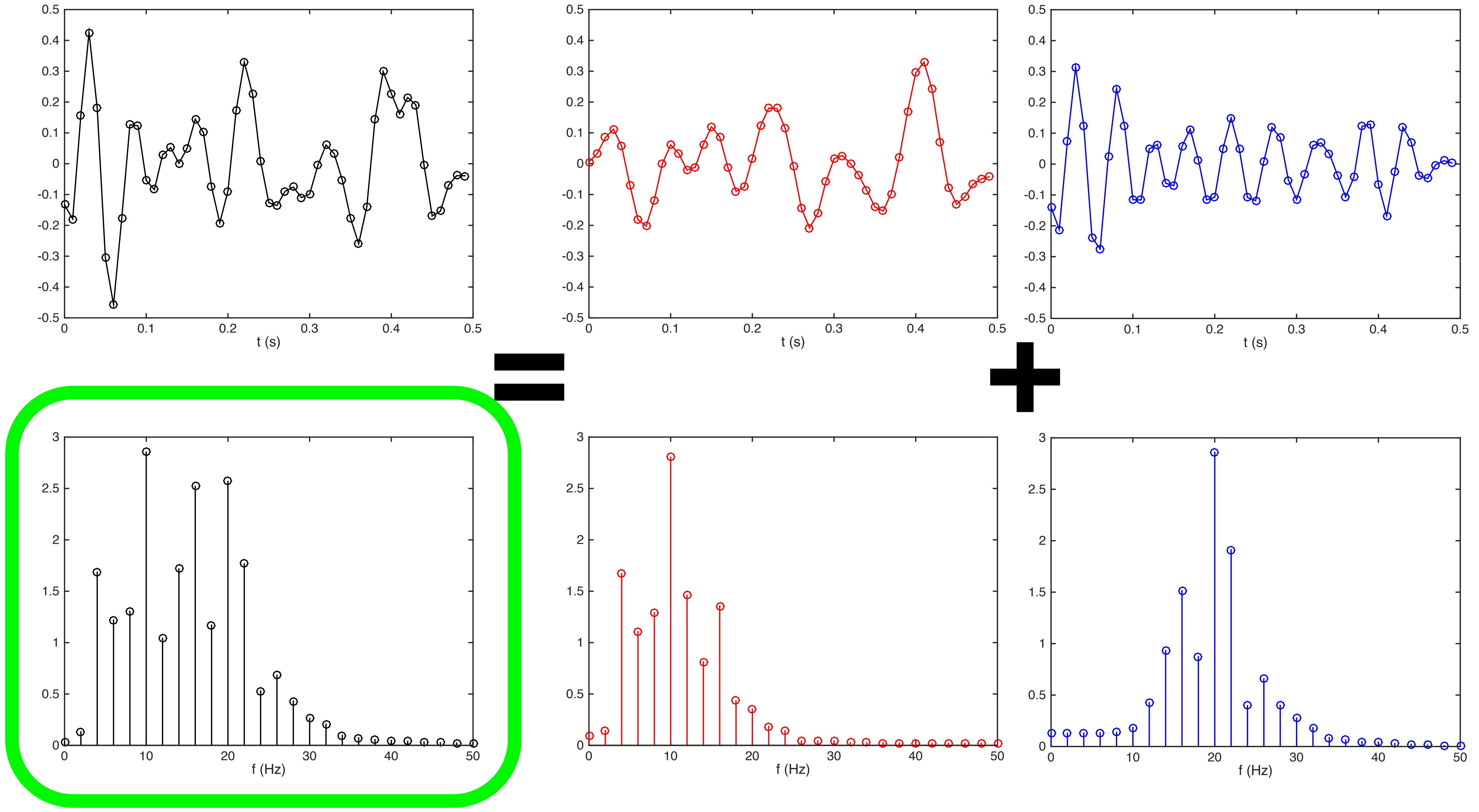

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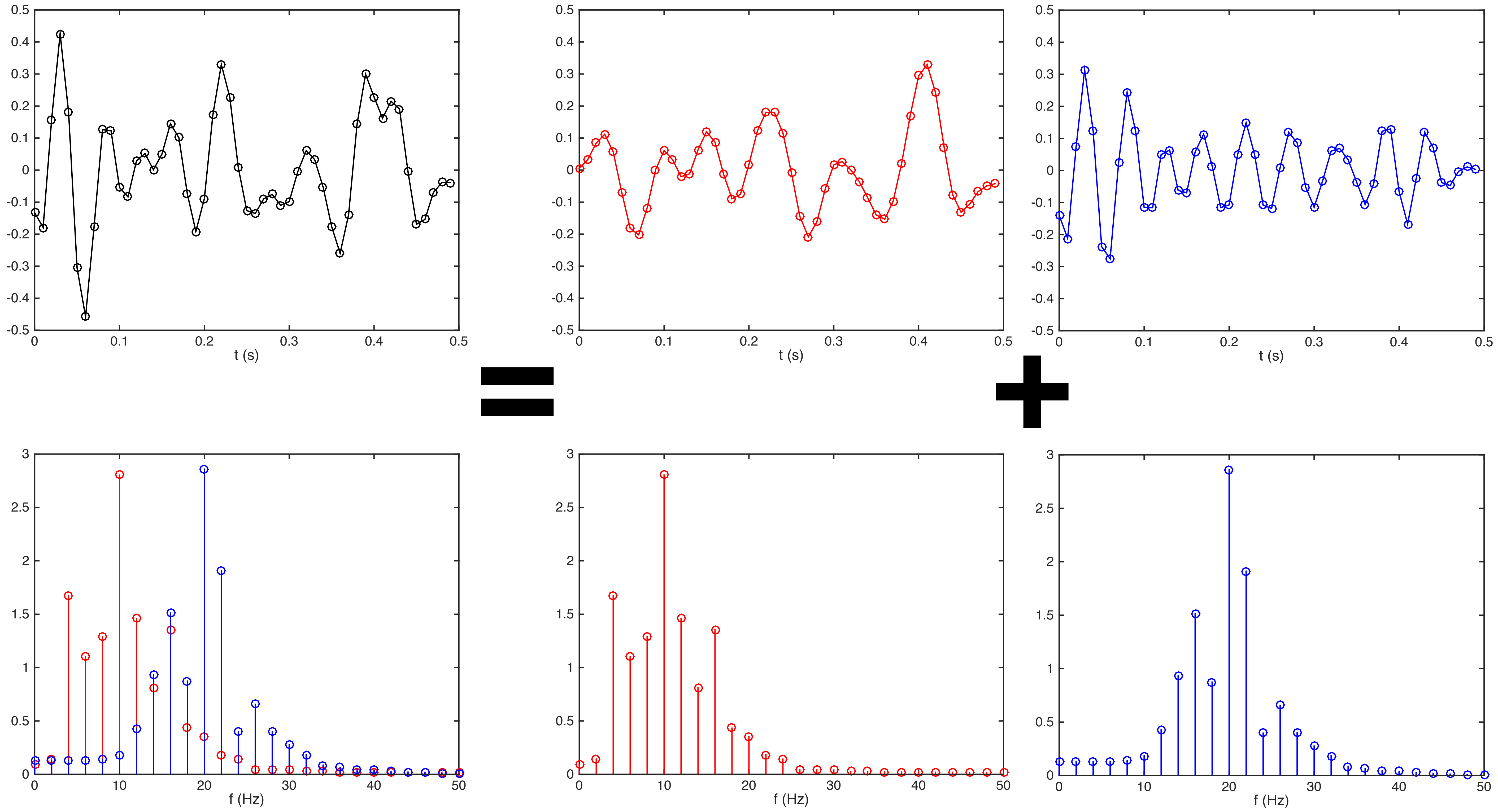

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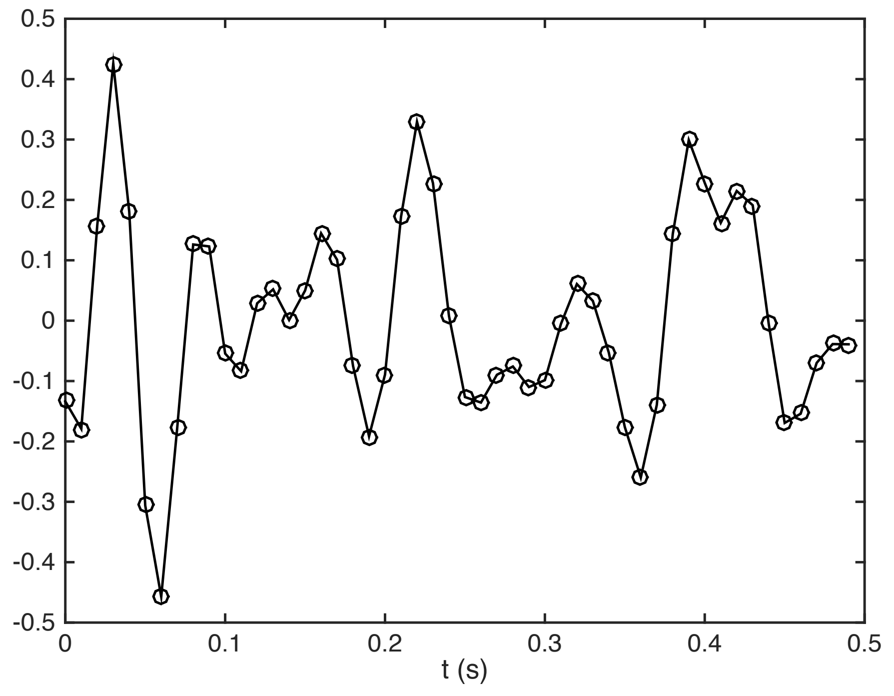

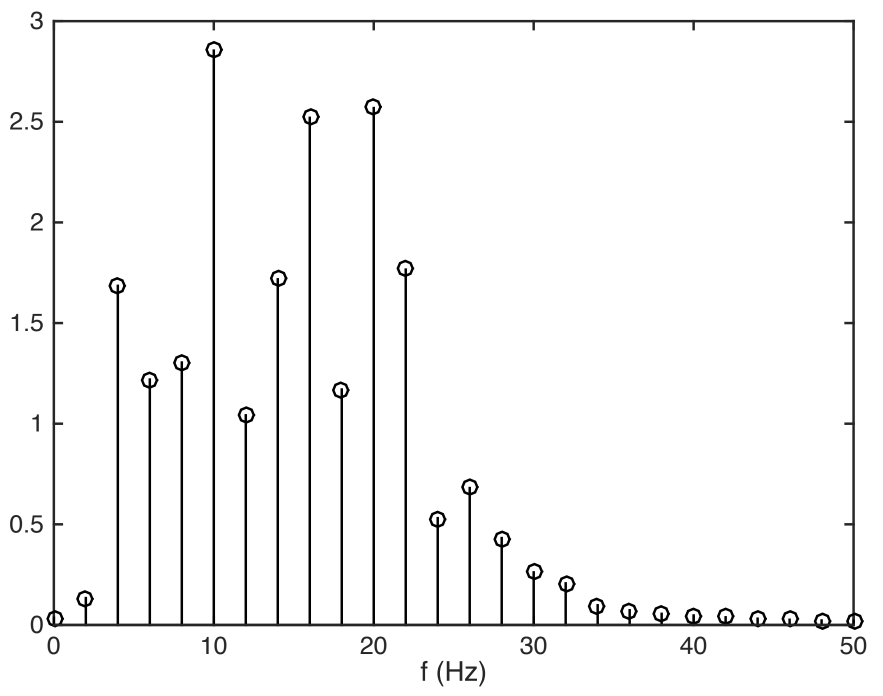

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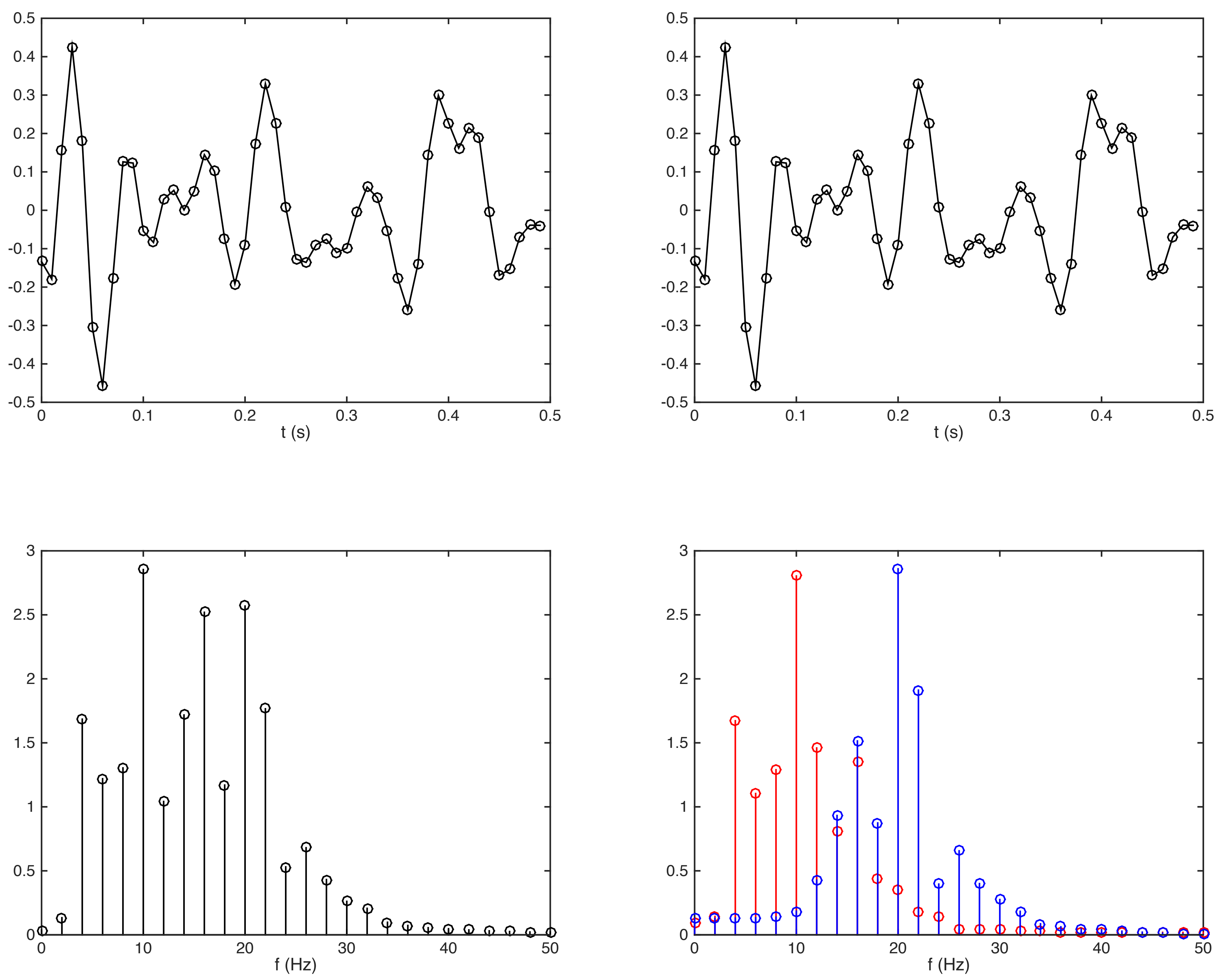

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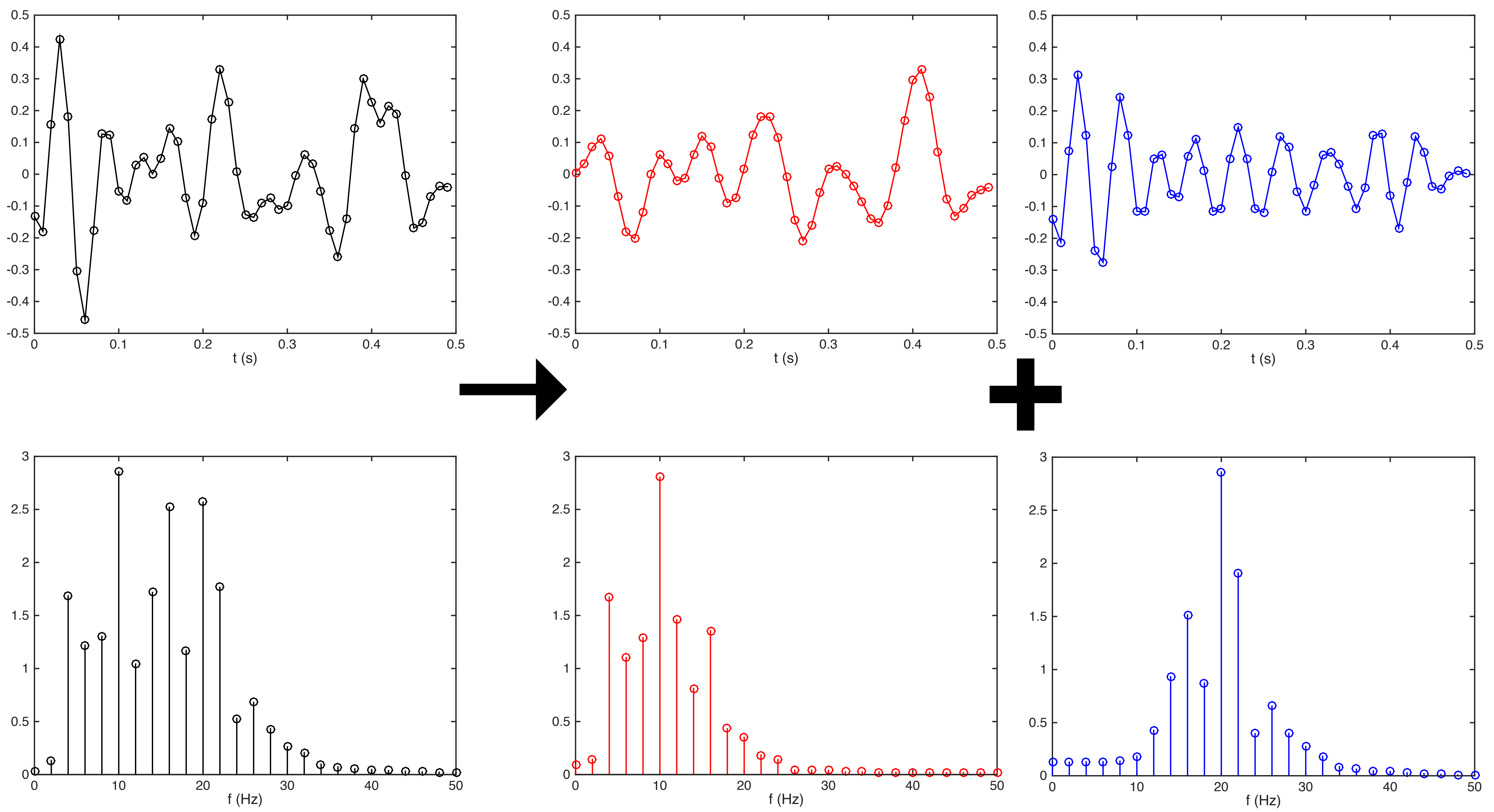

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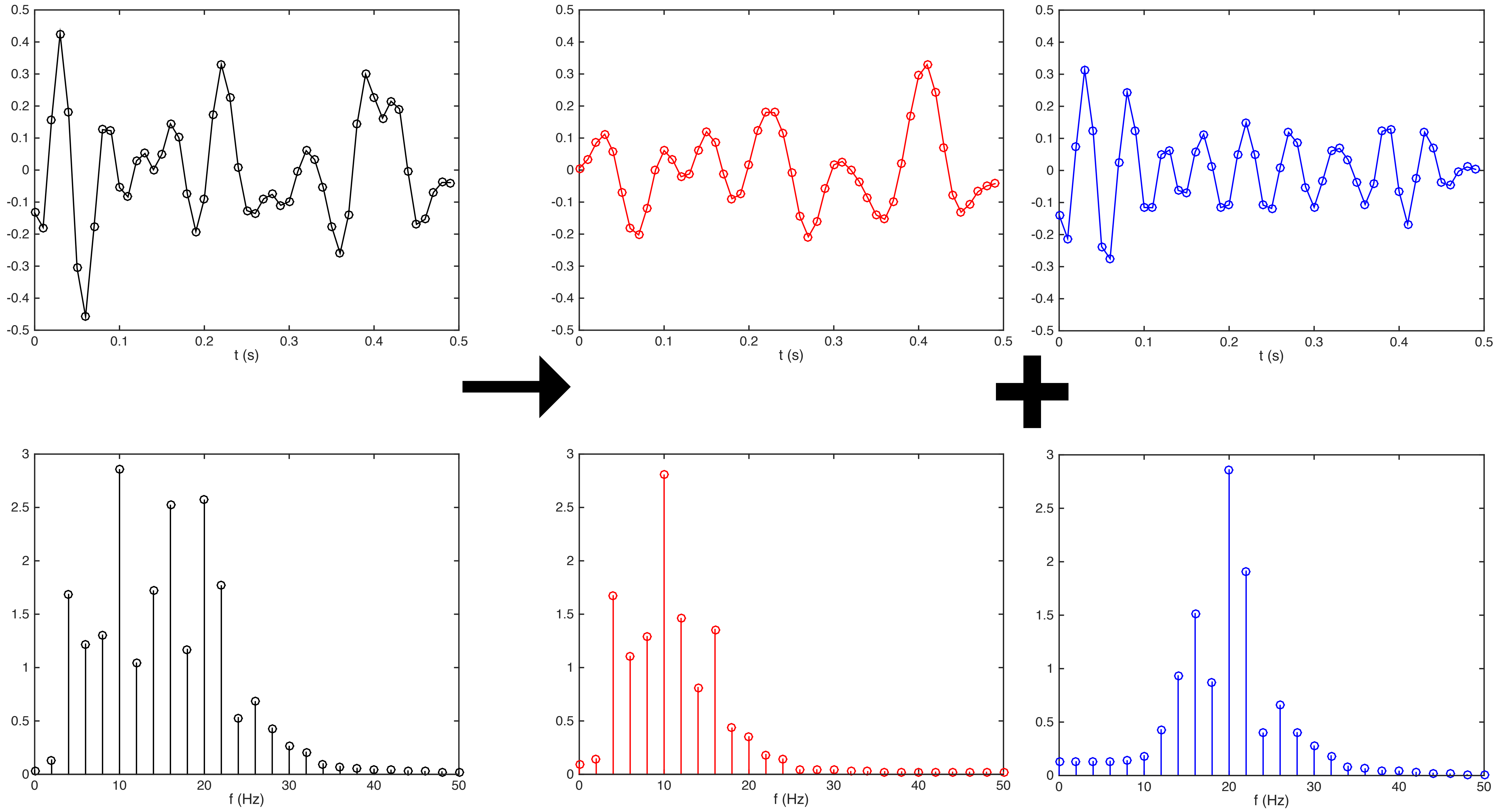

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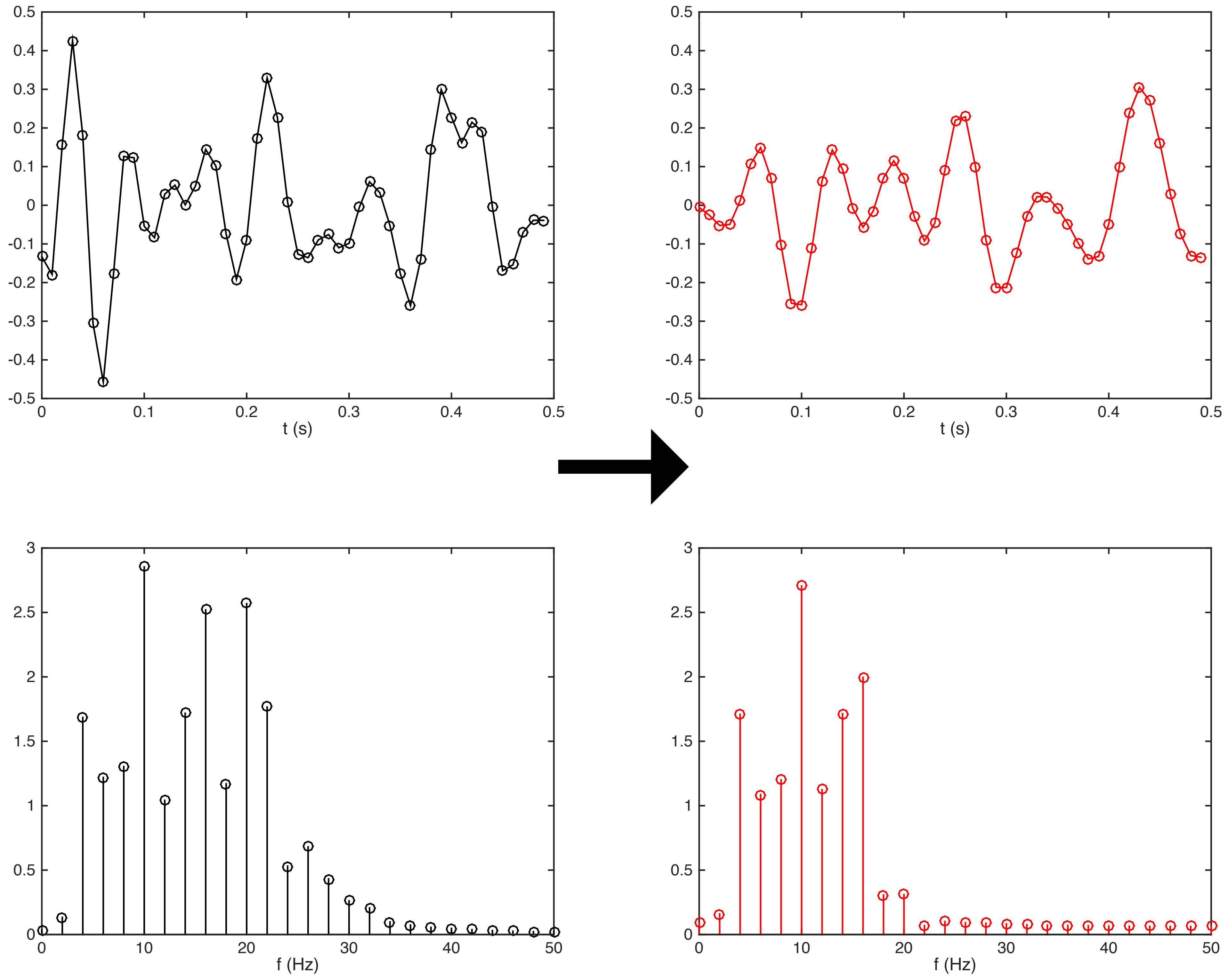
via filter



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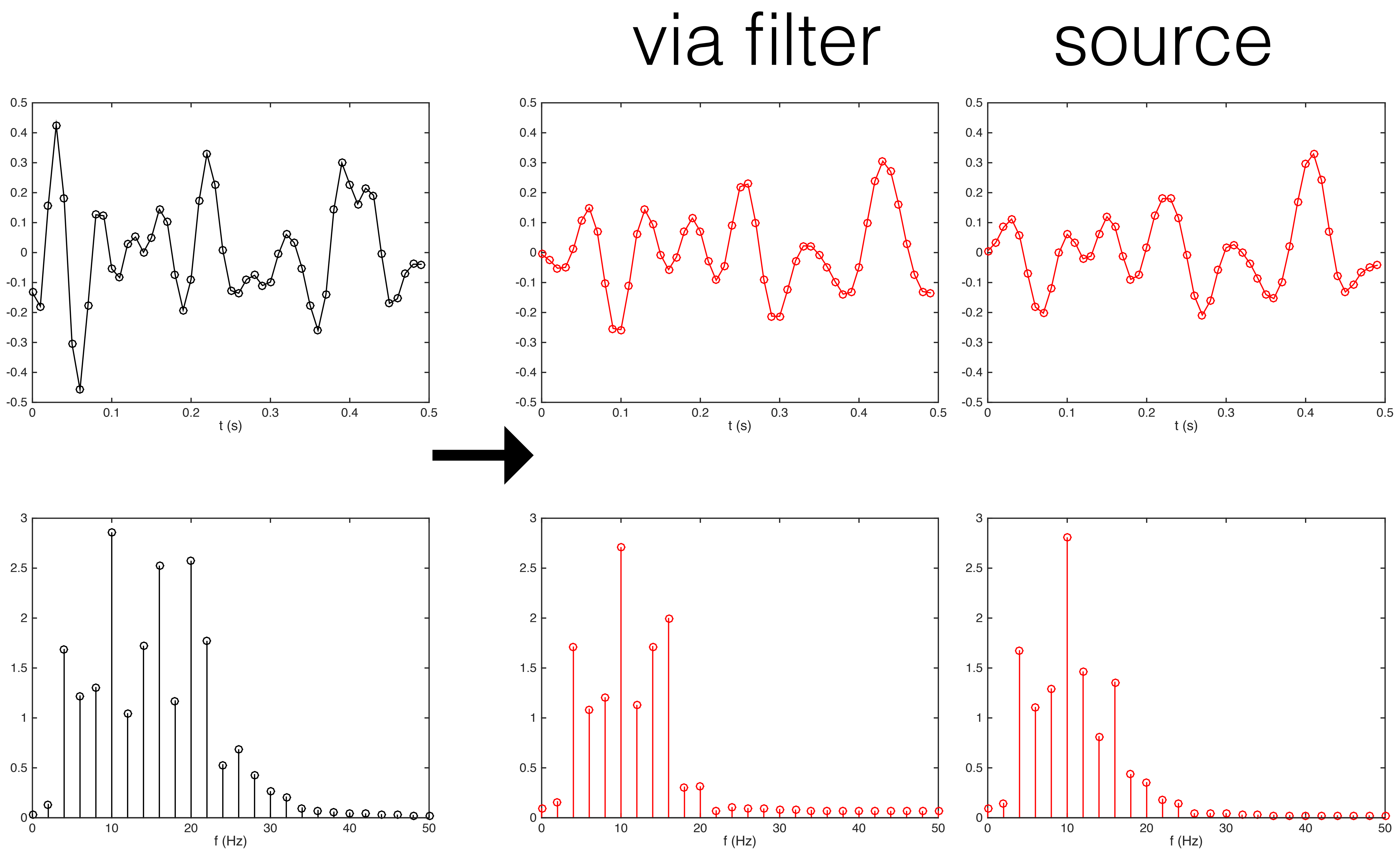

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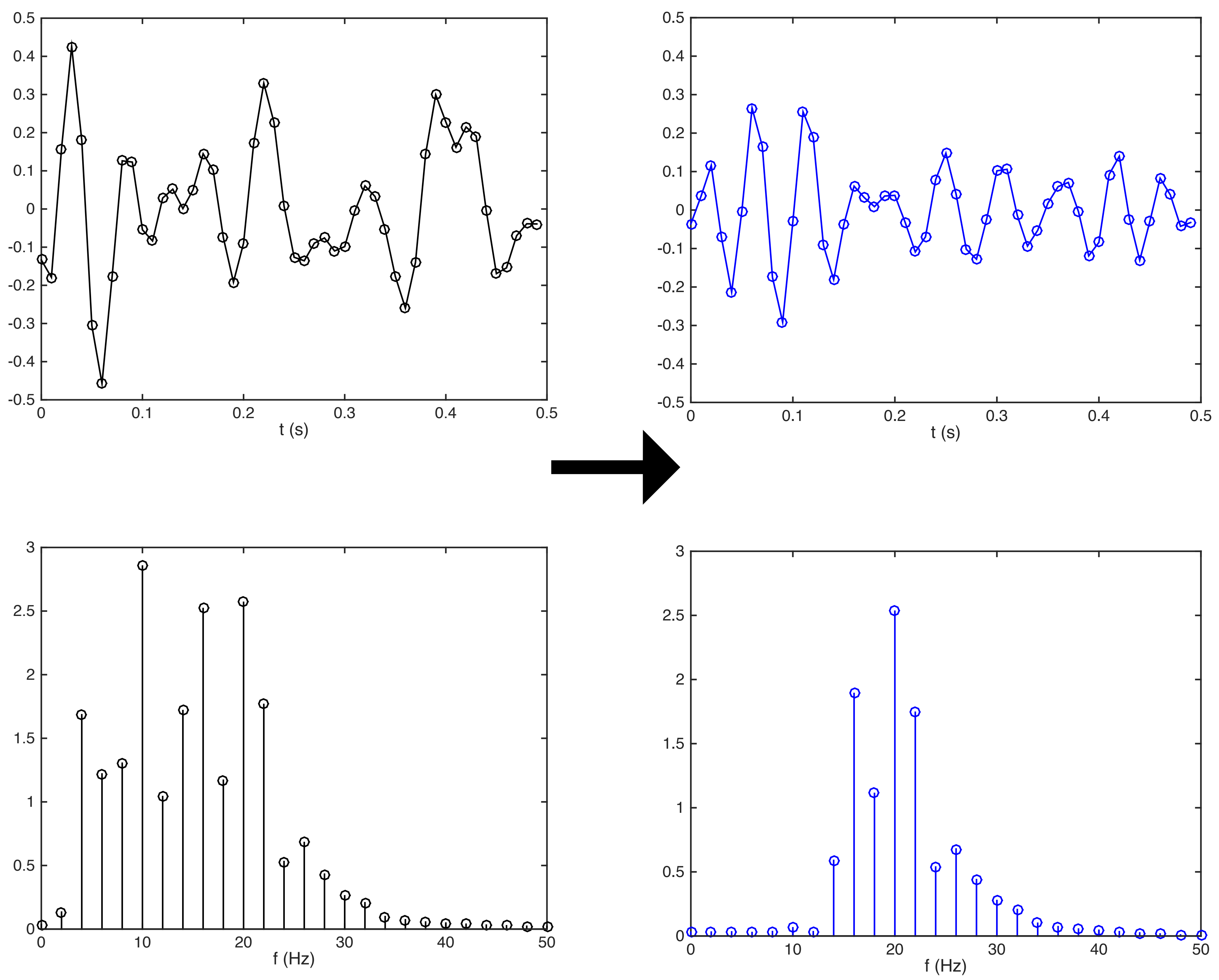

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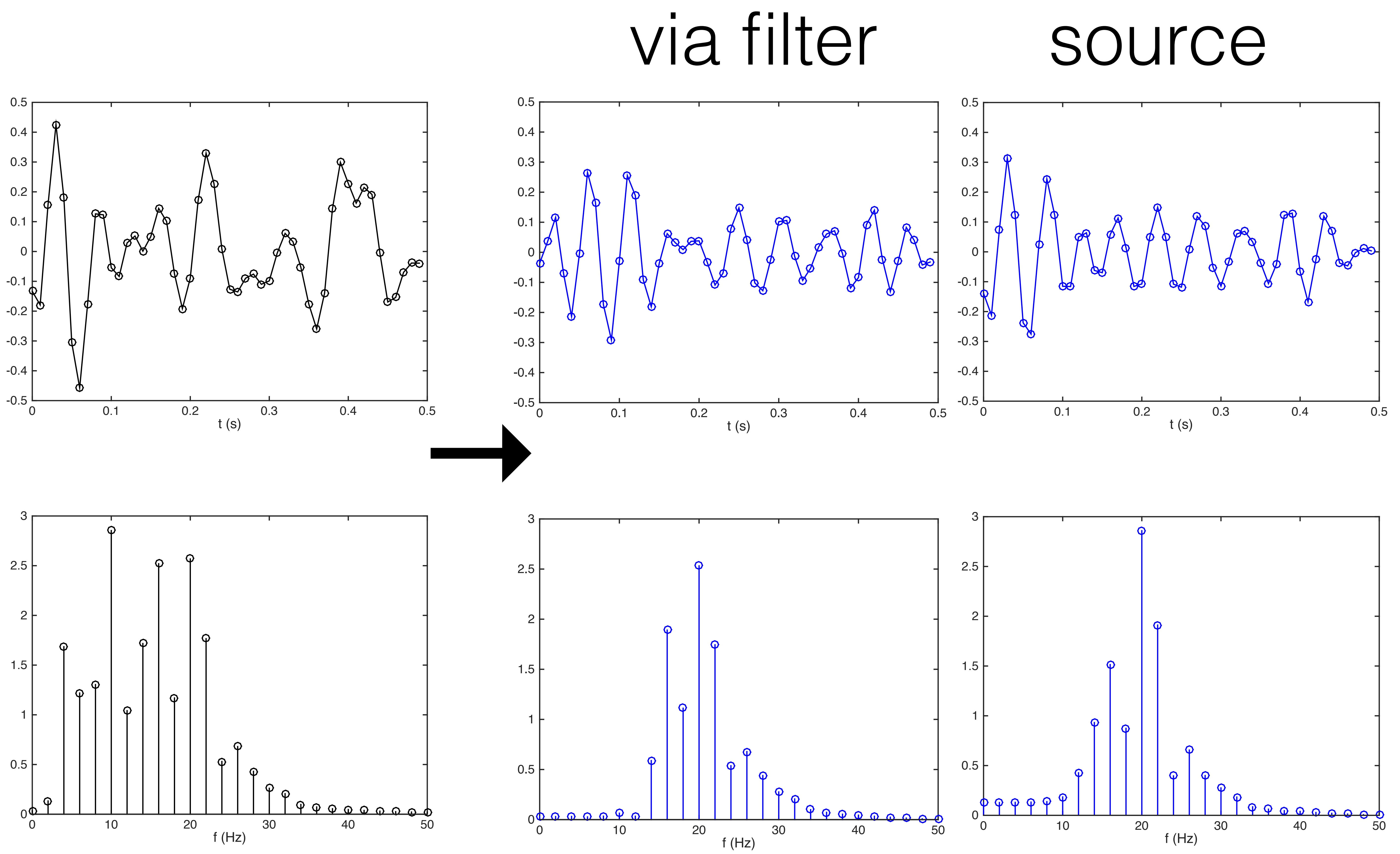

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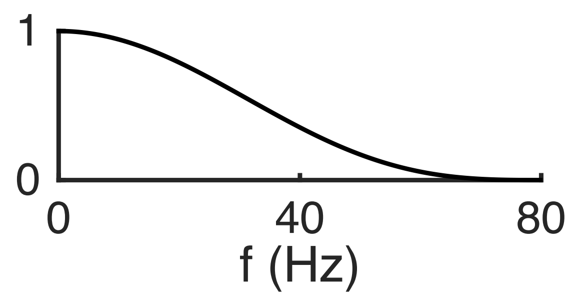
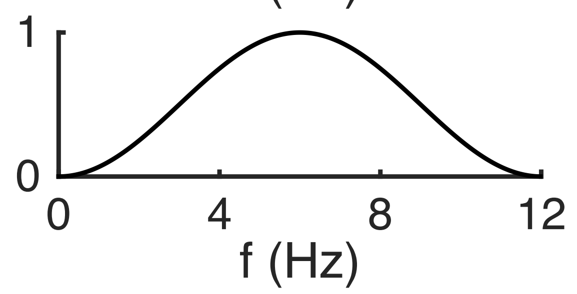
Filters: Frequency Selectivity

- *Frequency Selective* Filters

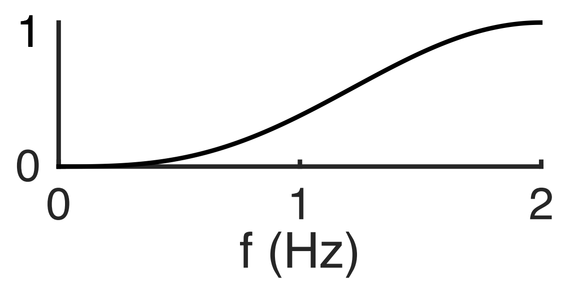
Low Pass



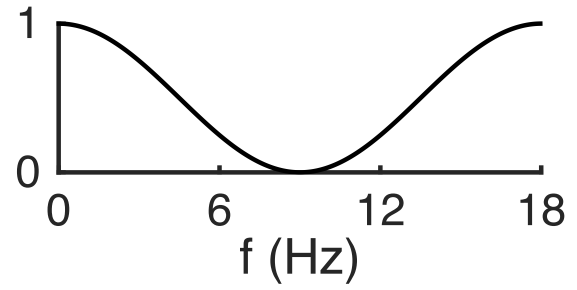
Band Pass



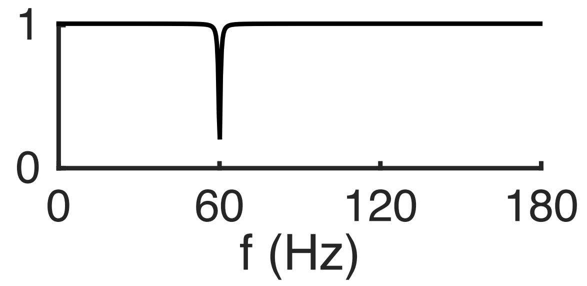
High Pass



Band Stop



Notch



and more…

Filters: Frequency Selectivity

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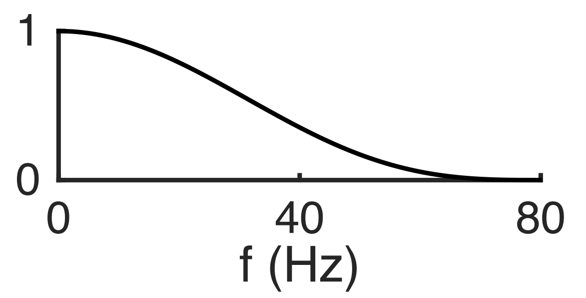

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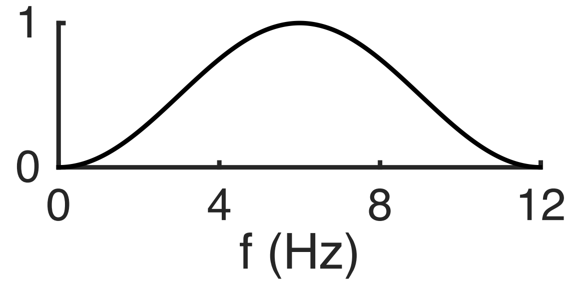

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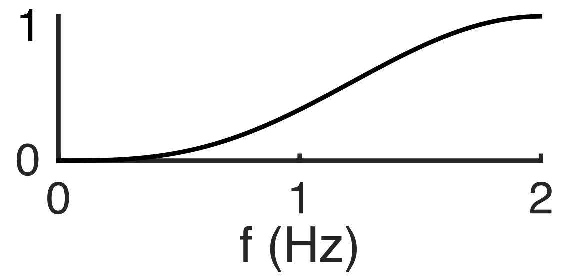

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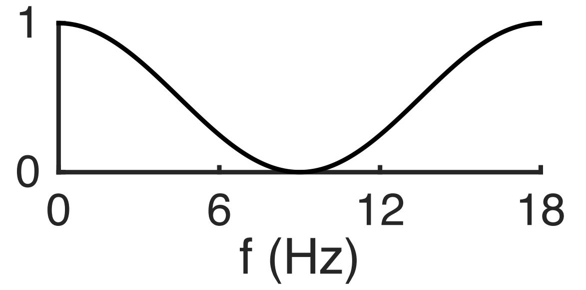

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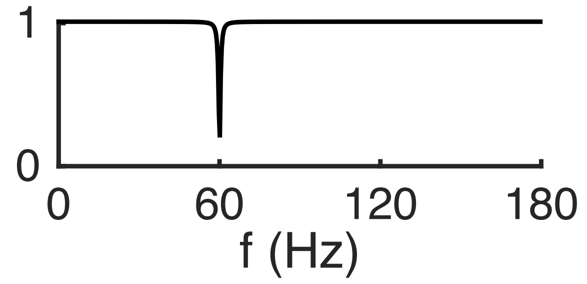

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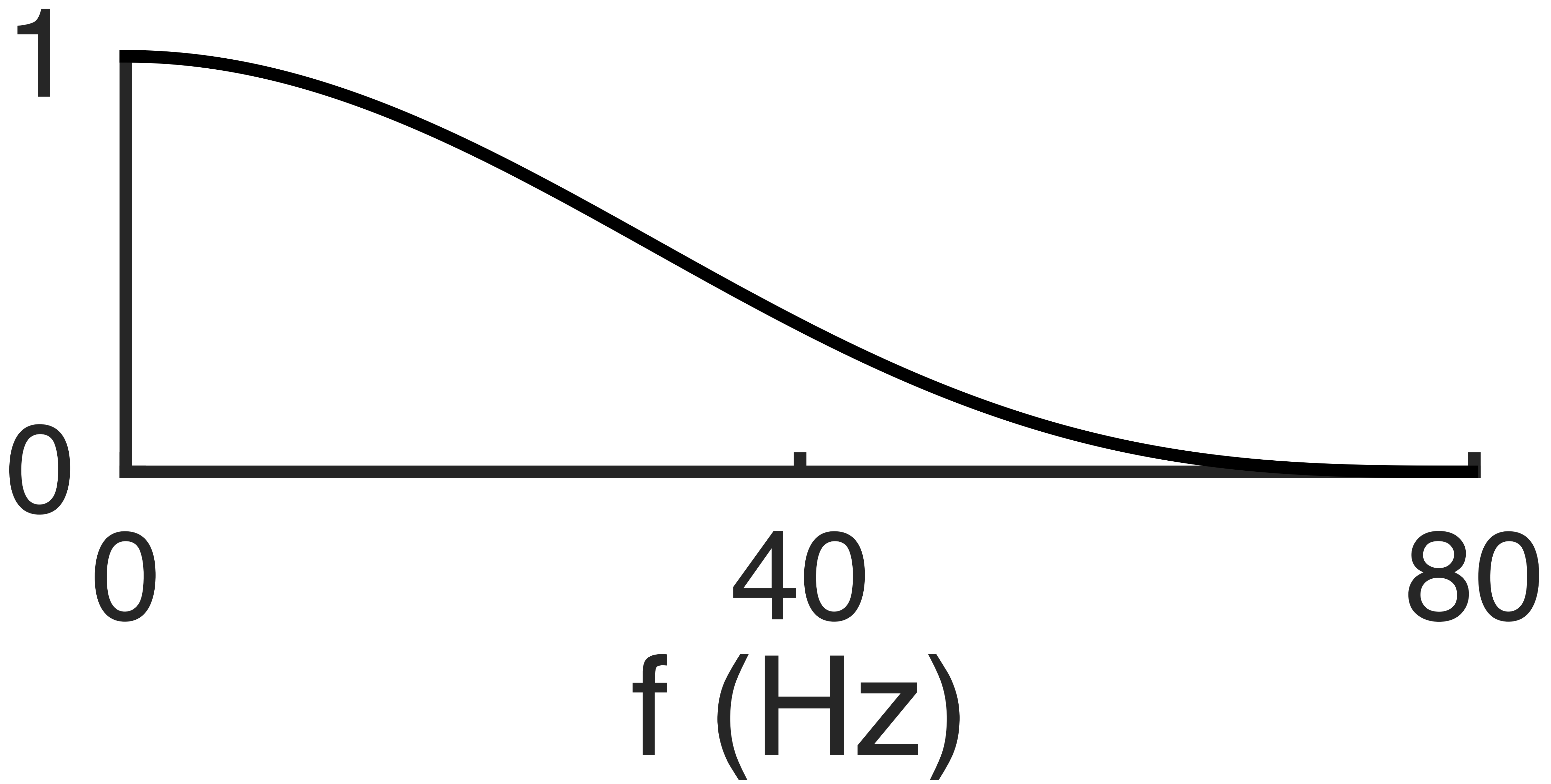

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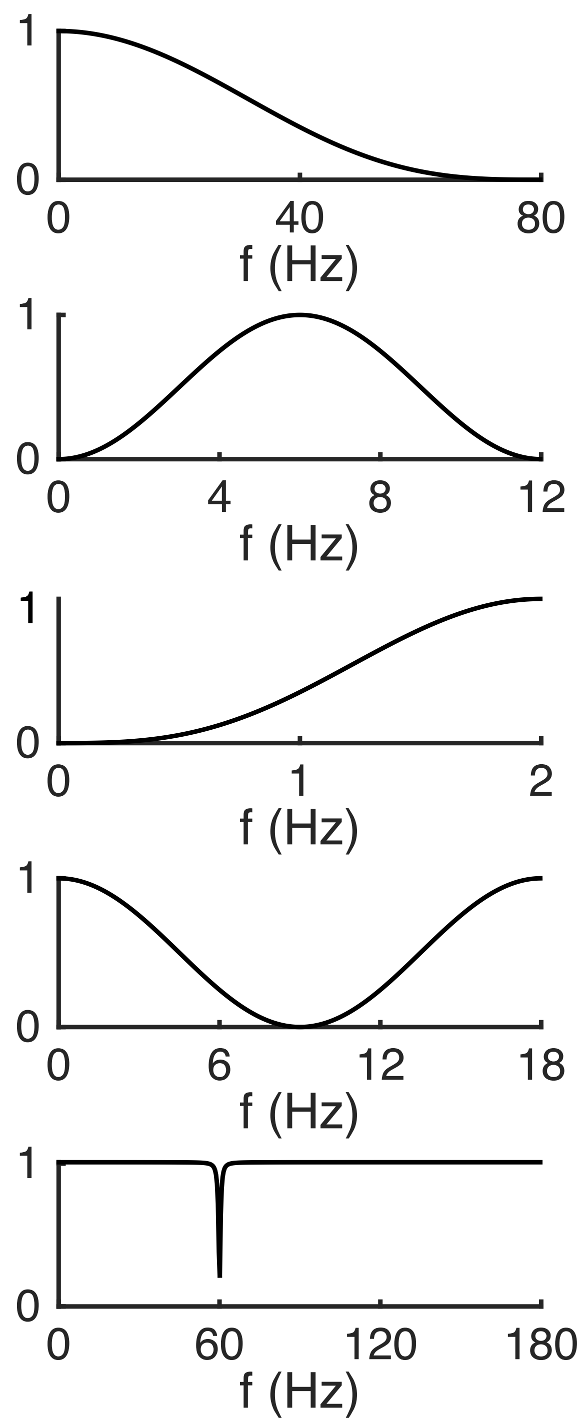

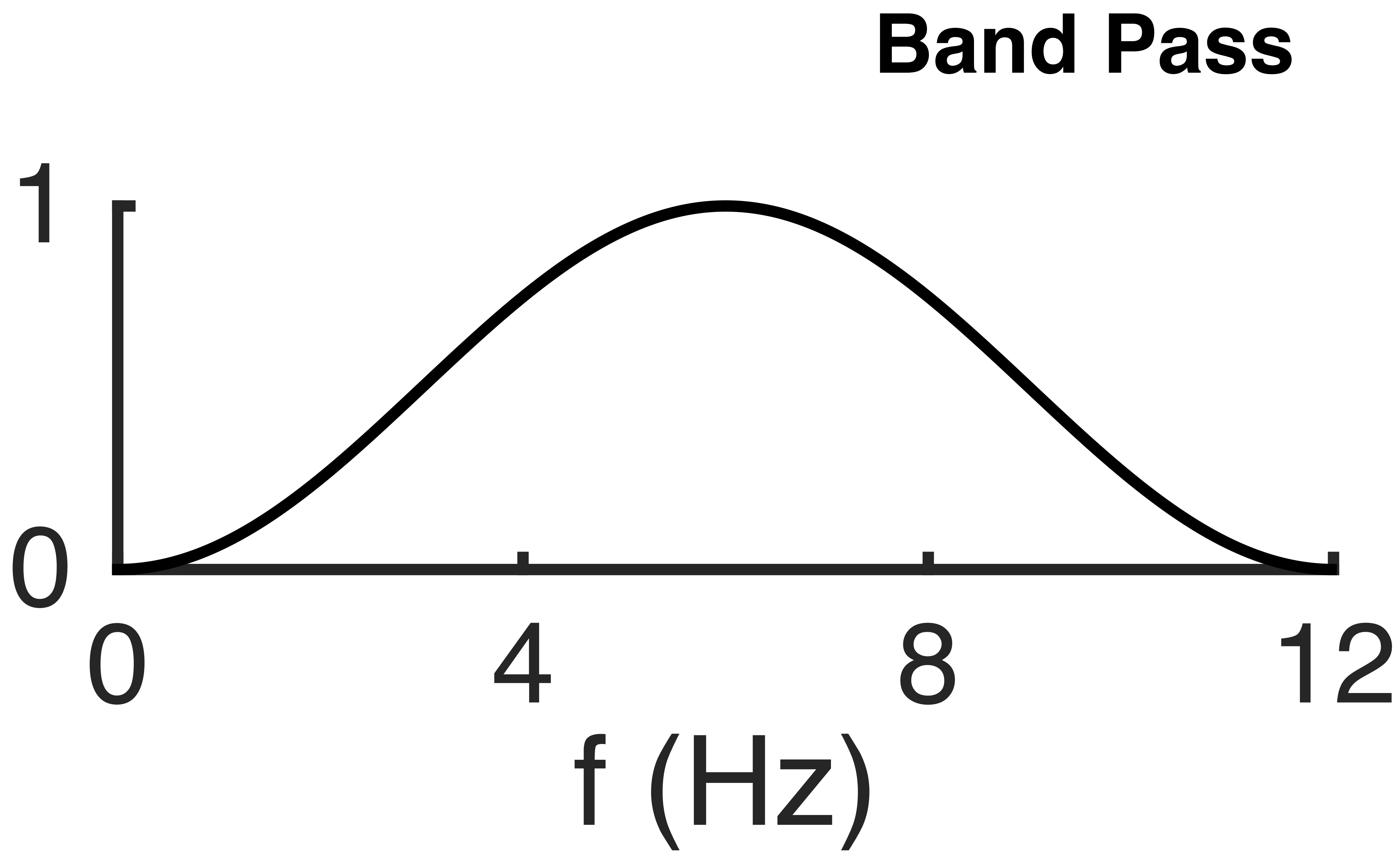

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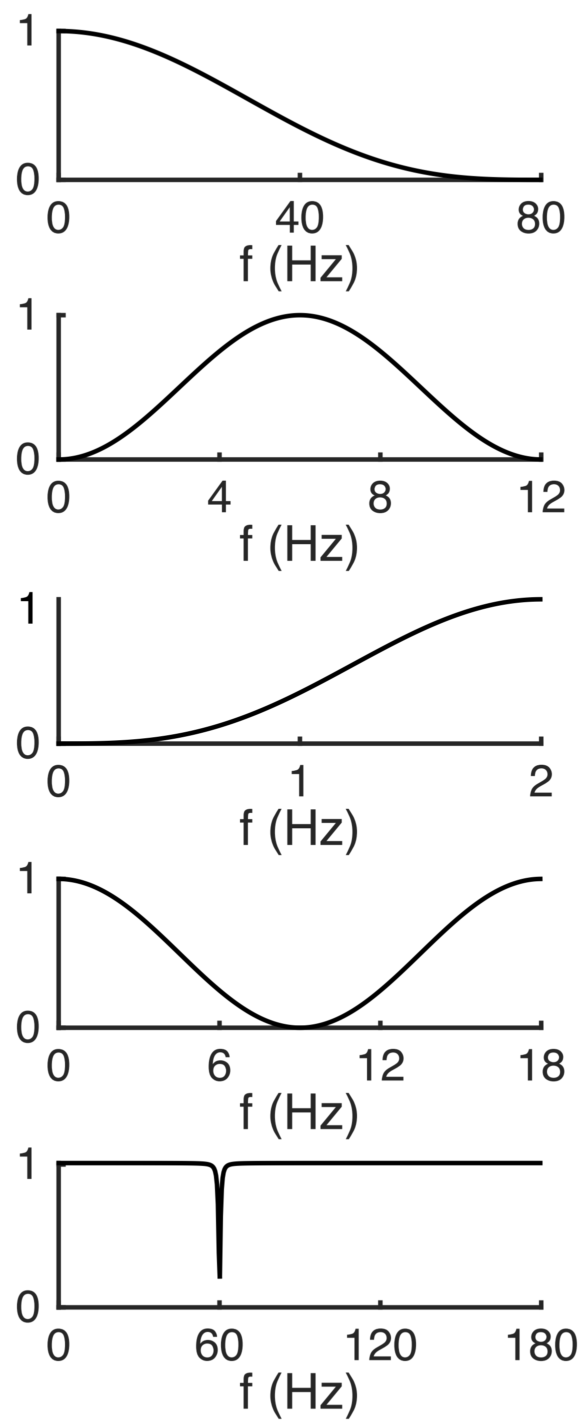

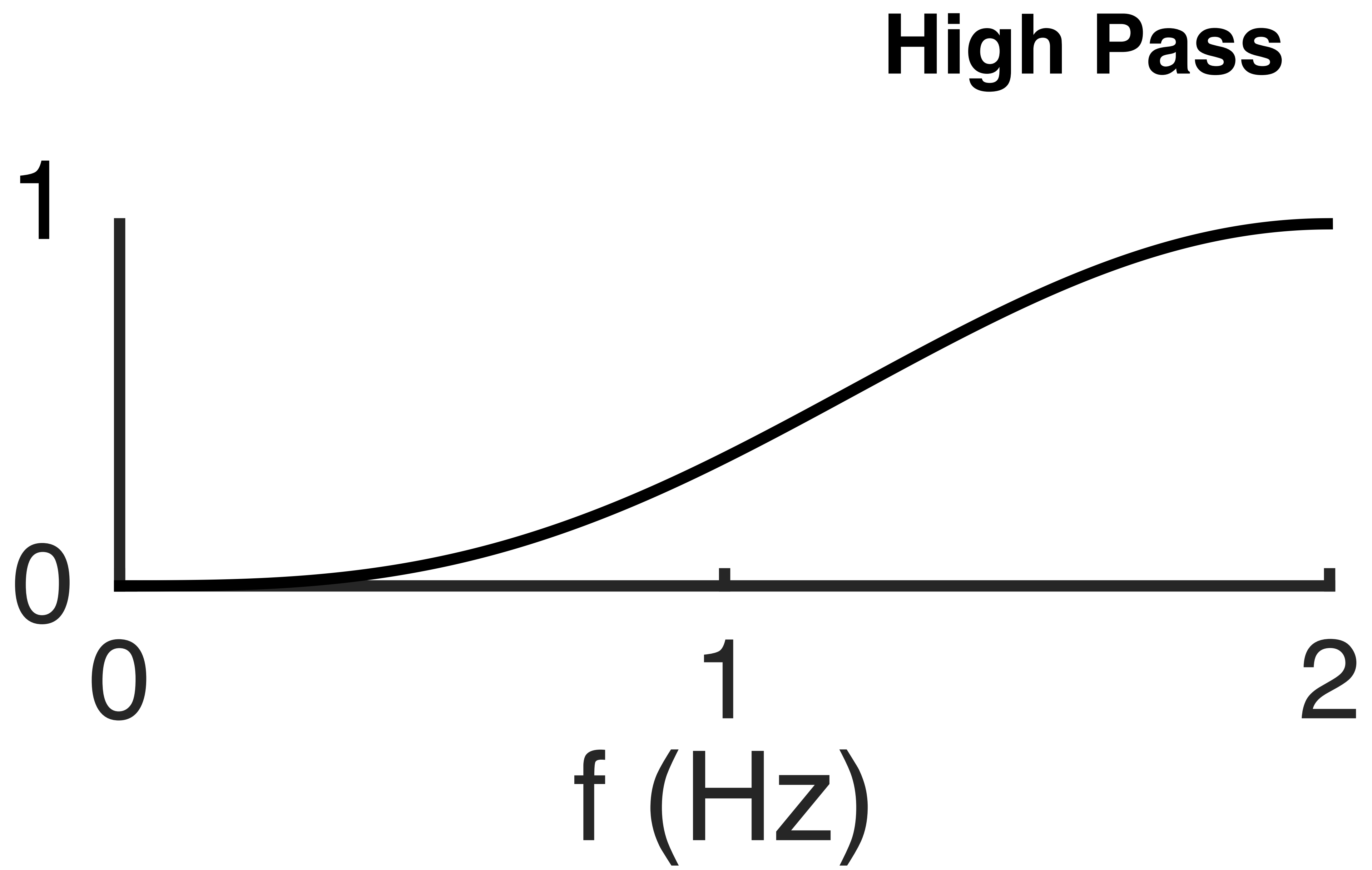

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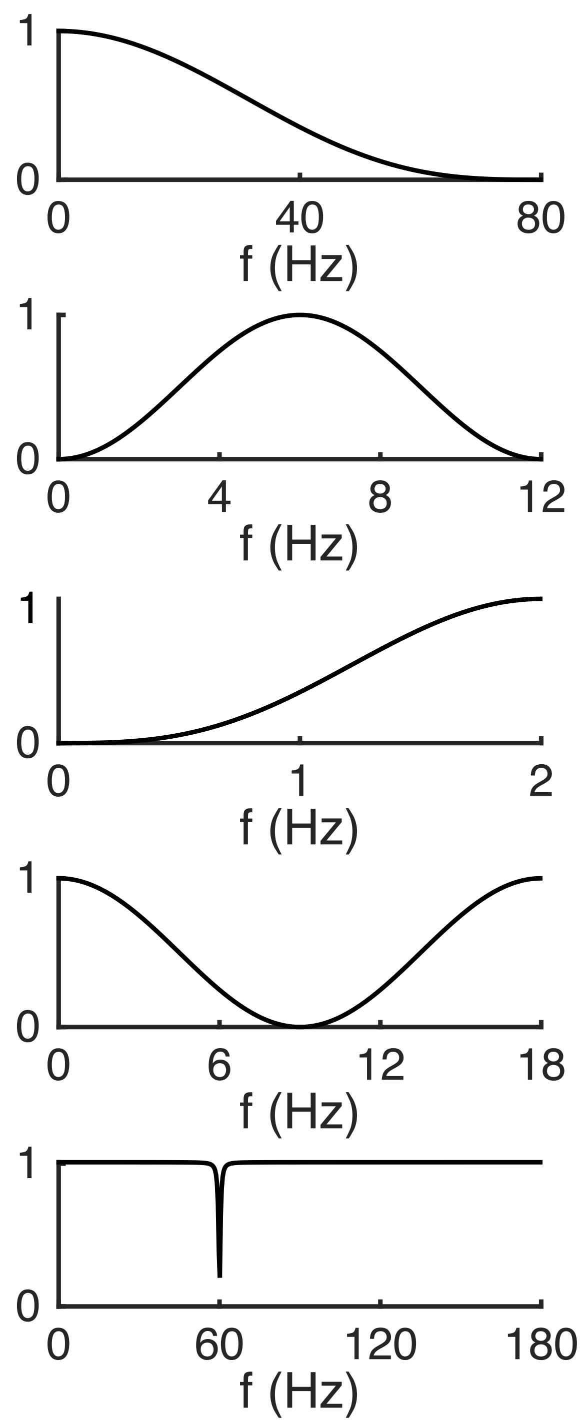

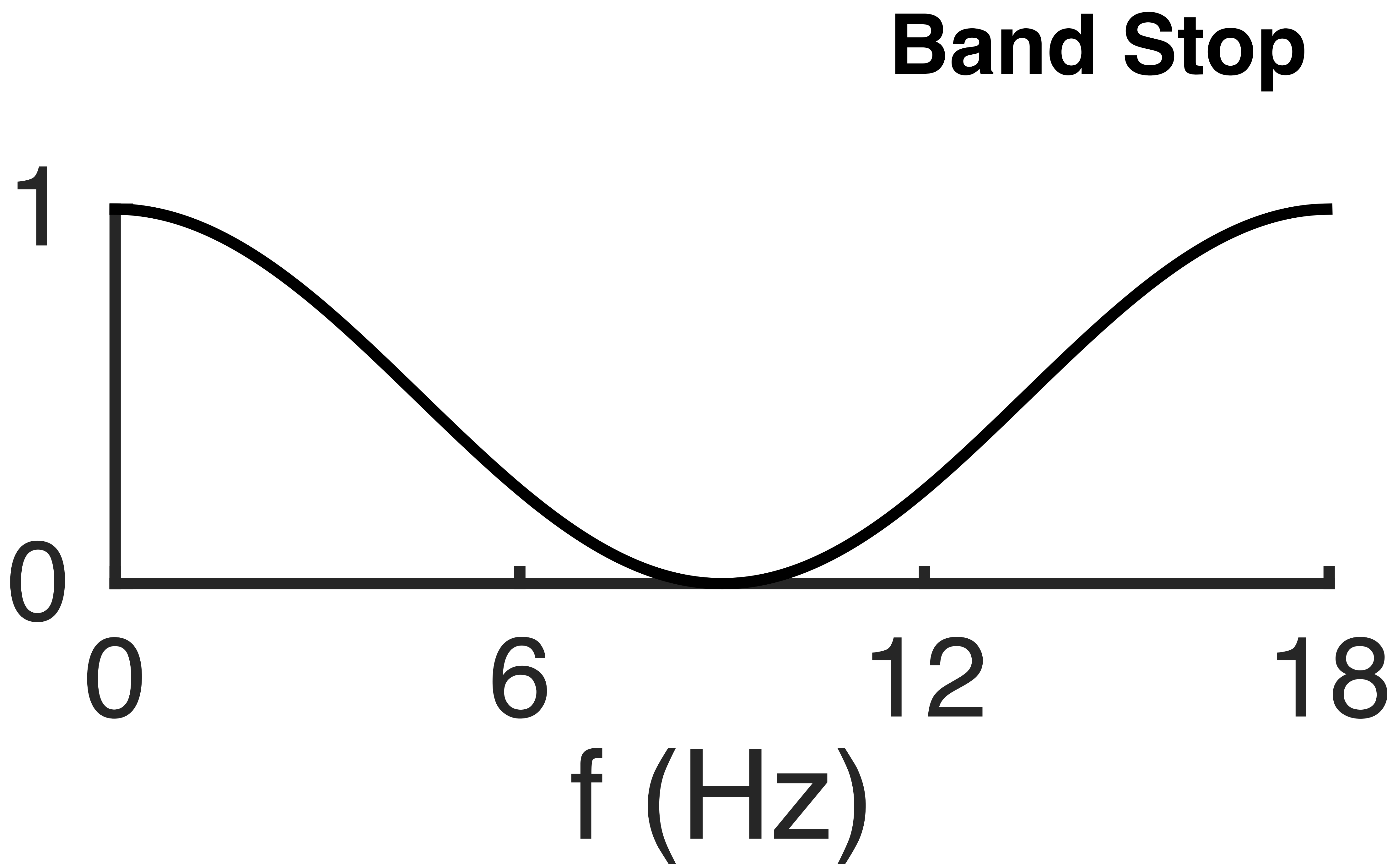

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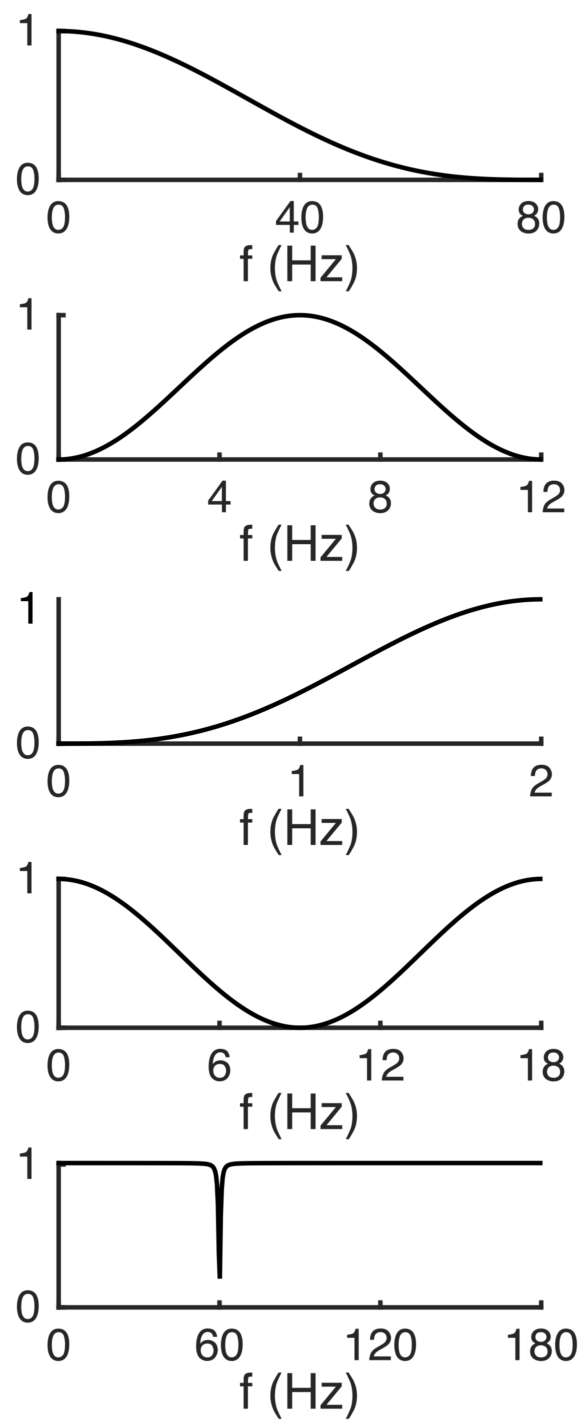

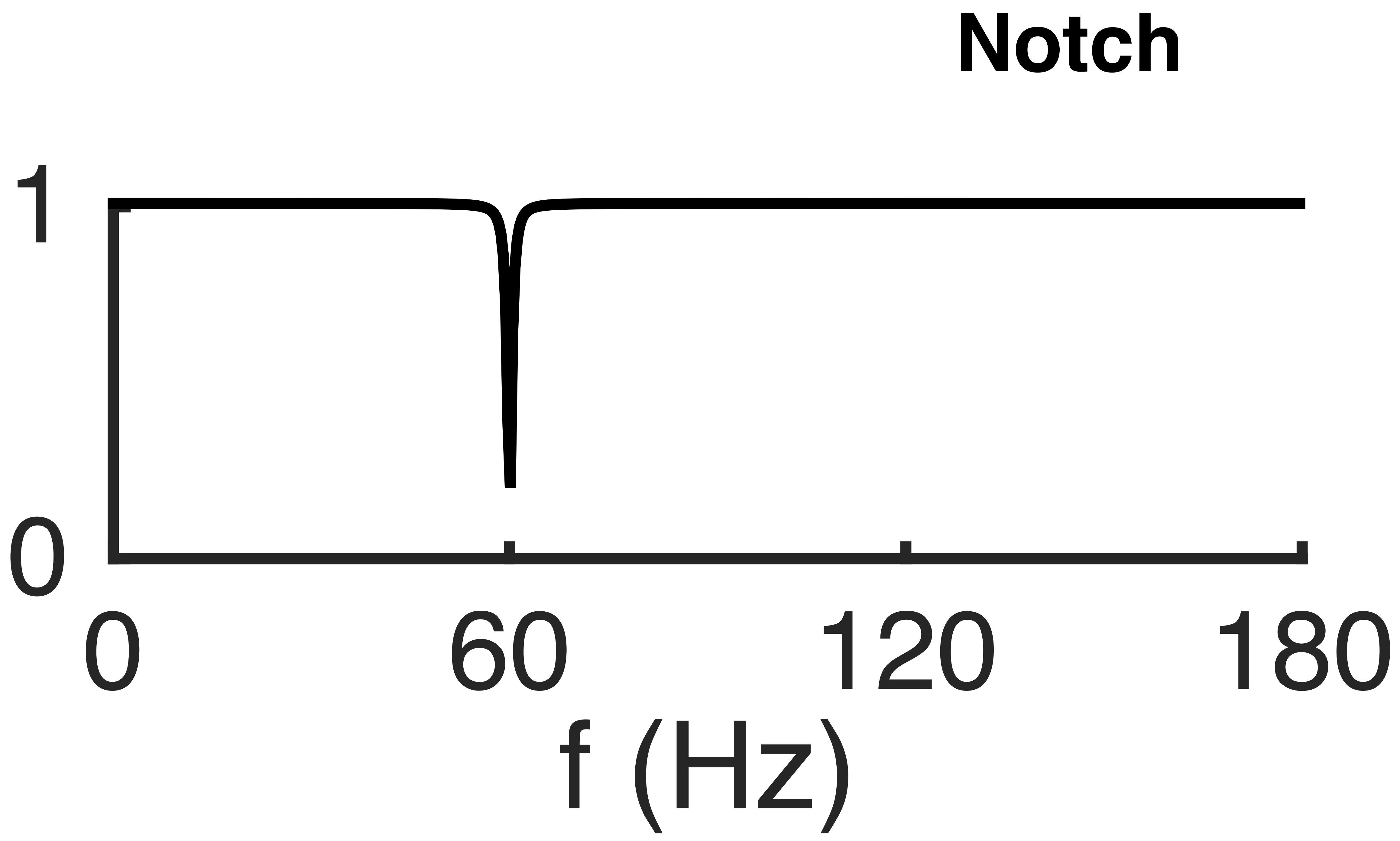

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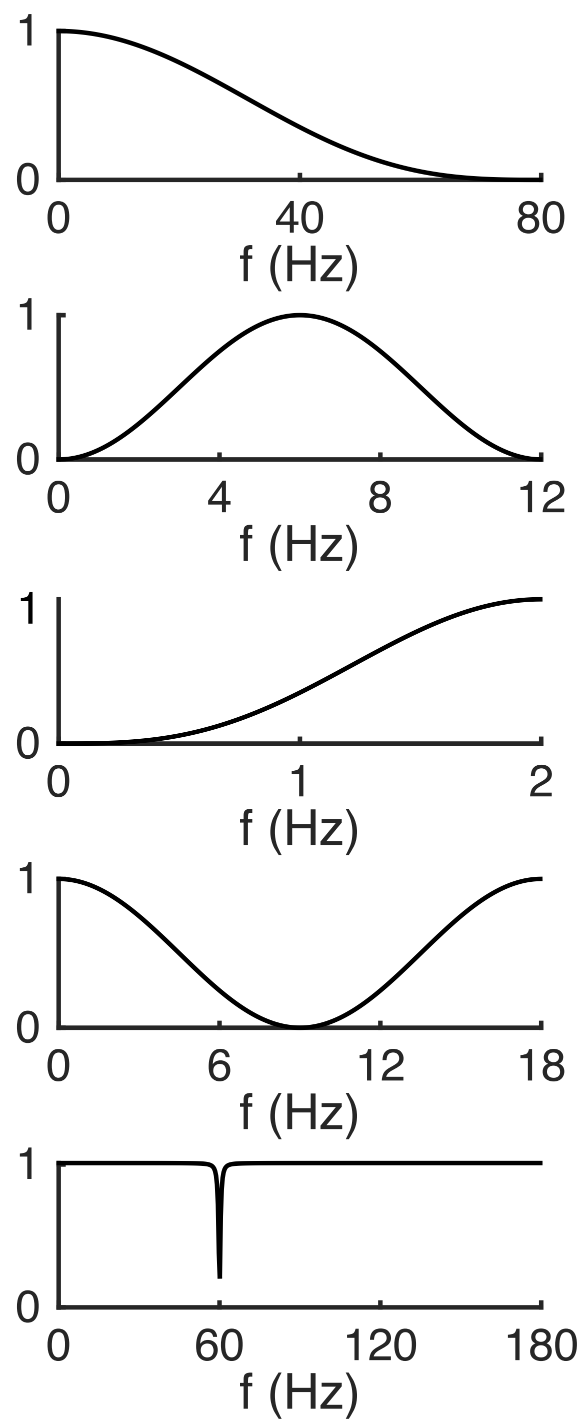

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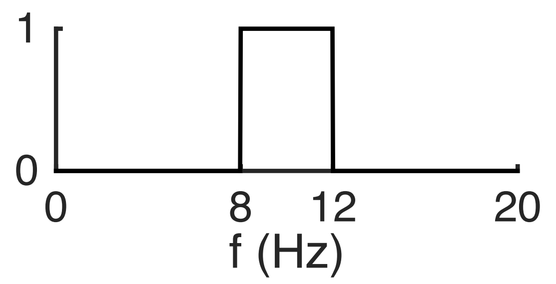
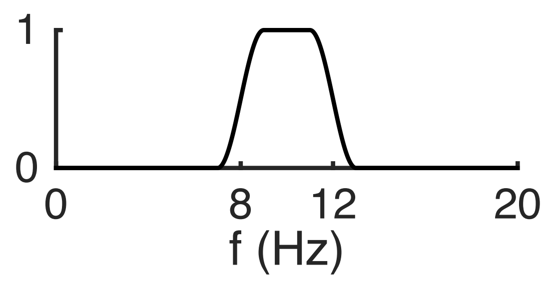
Filters: How Selective?

- *How sharp a transition?*

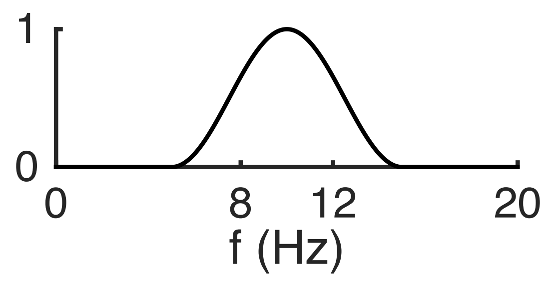
"Ideal" Filter



Sharp Transition



Soft Transition



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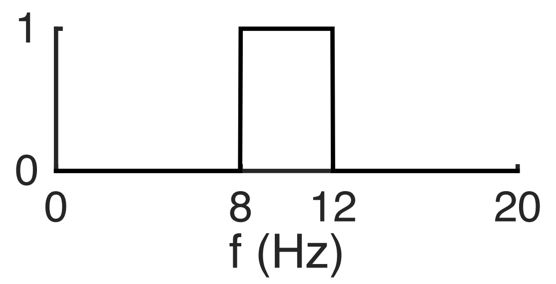

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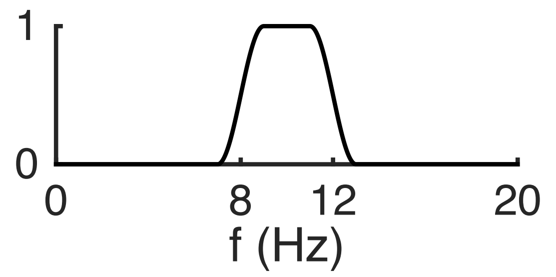
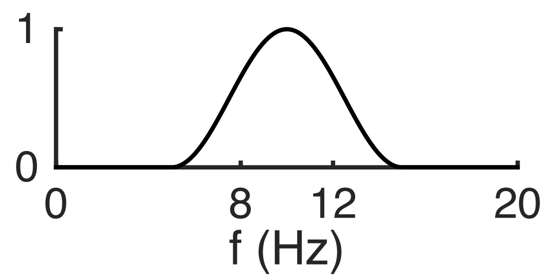
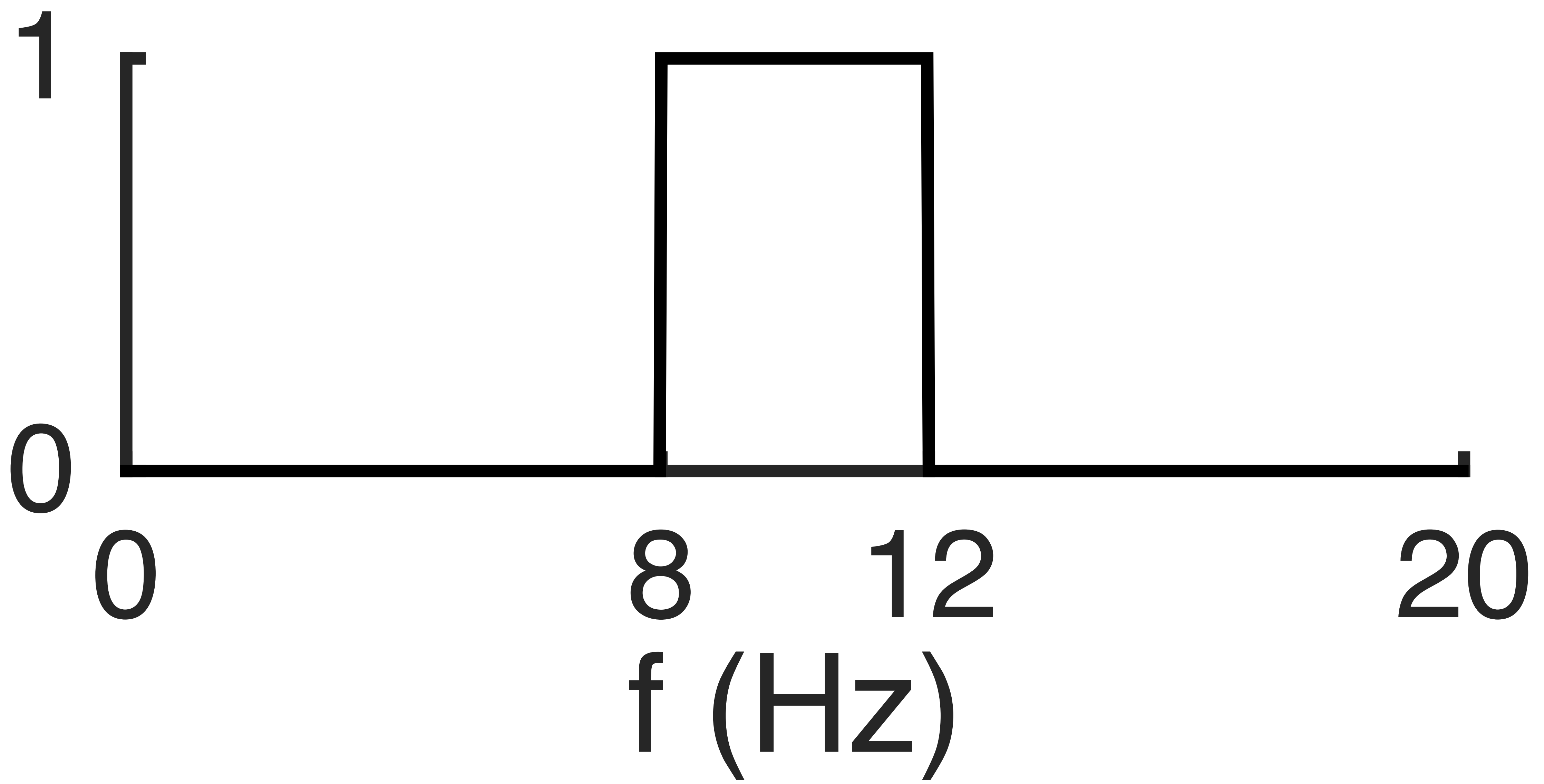


Soft Transition



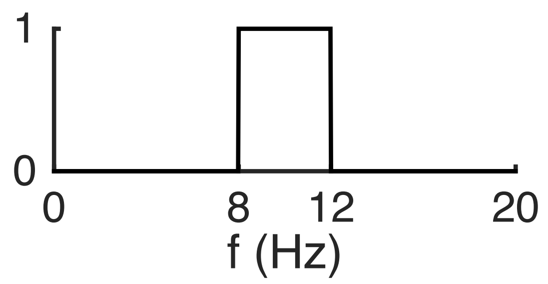
“Ideal” Filter



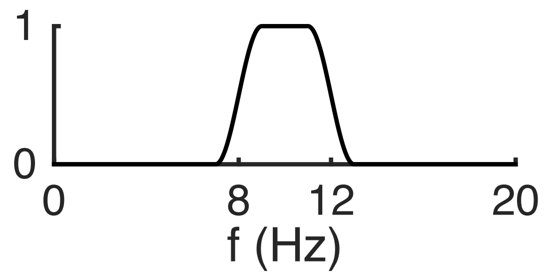
Filters: How Selective?

- *How sharp a transition?*

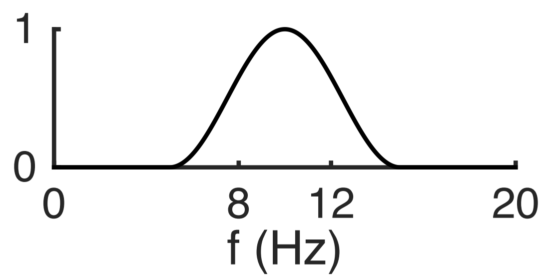
“Ideal” Filter



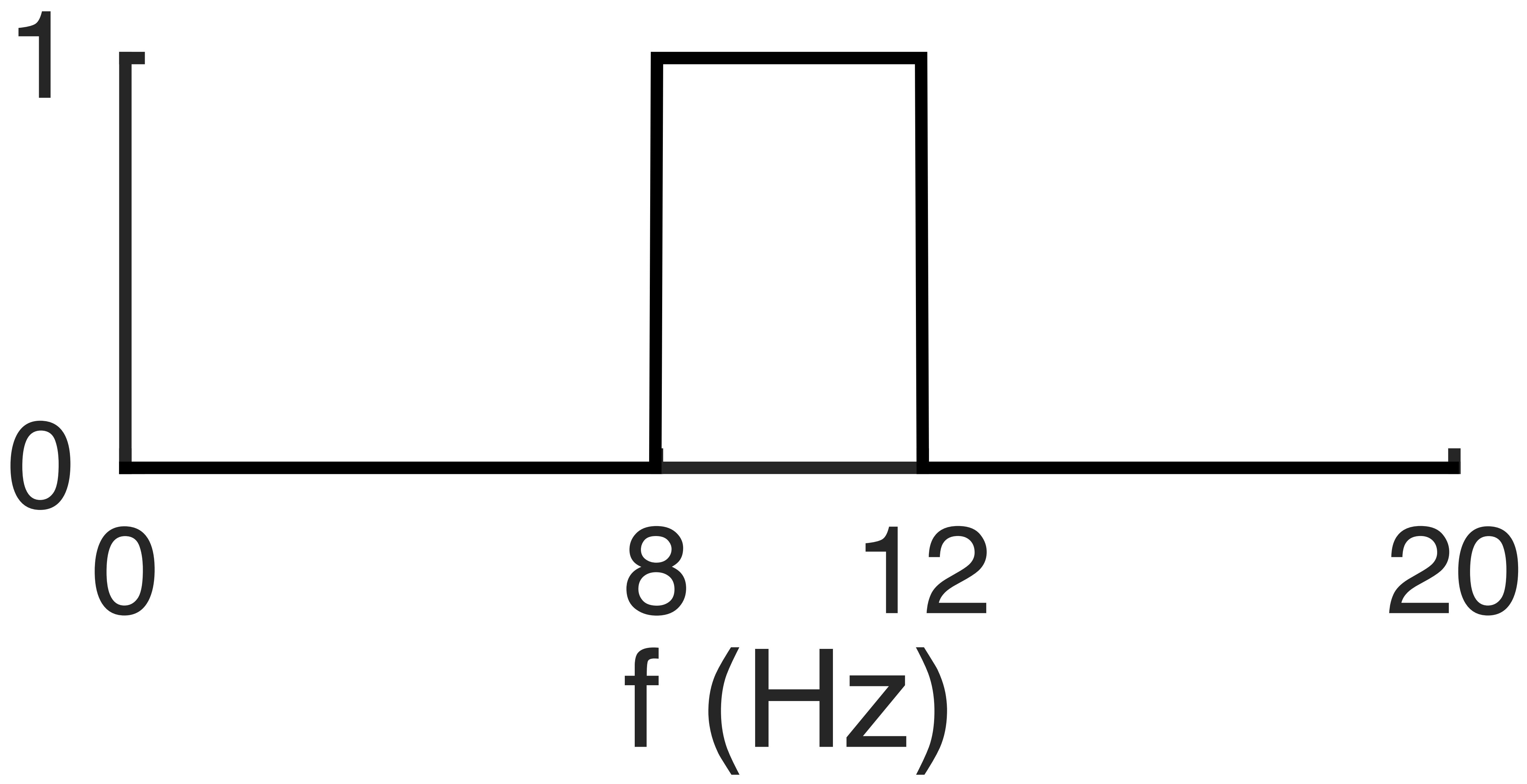
Sharp Transition



Soft Transition



“Ideal” Filter



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- *How sharp a transition?*

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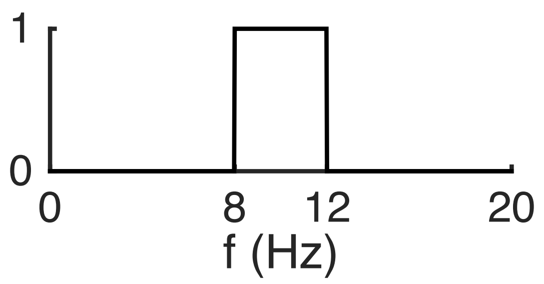
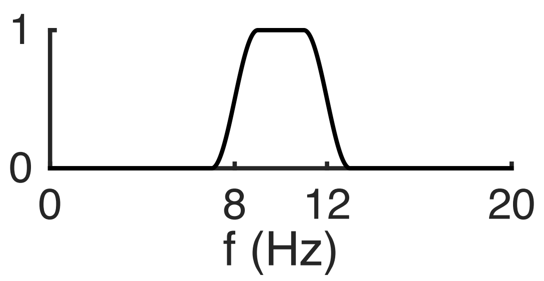
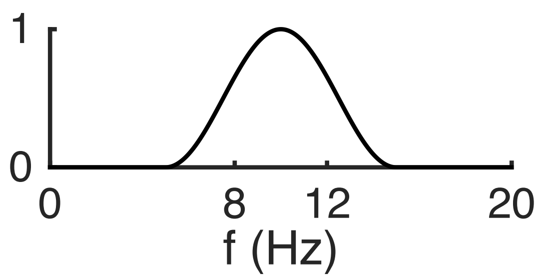


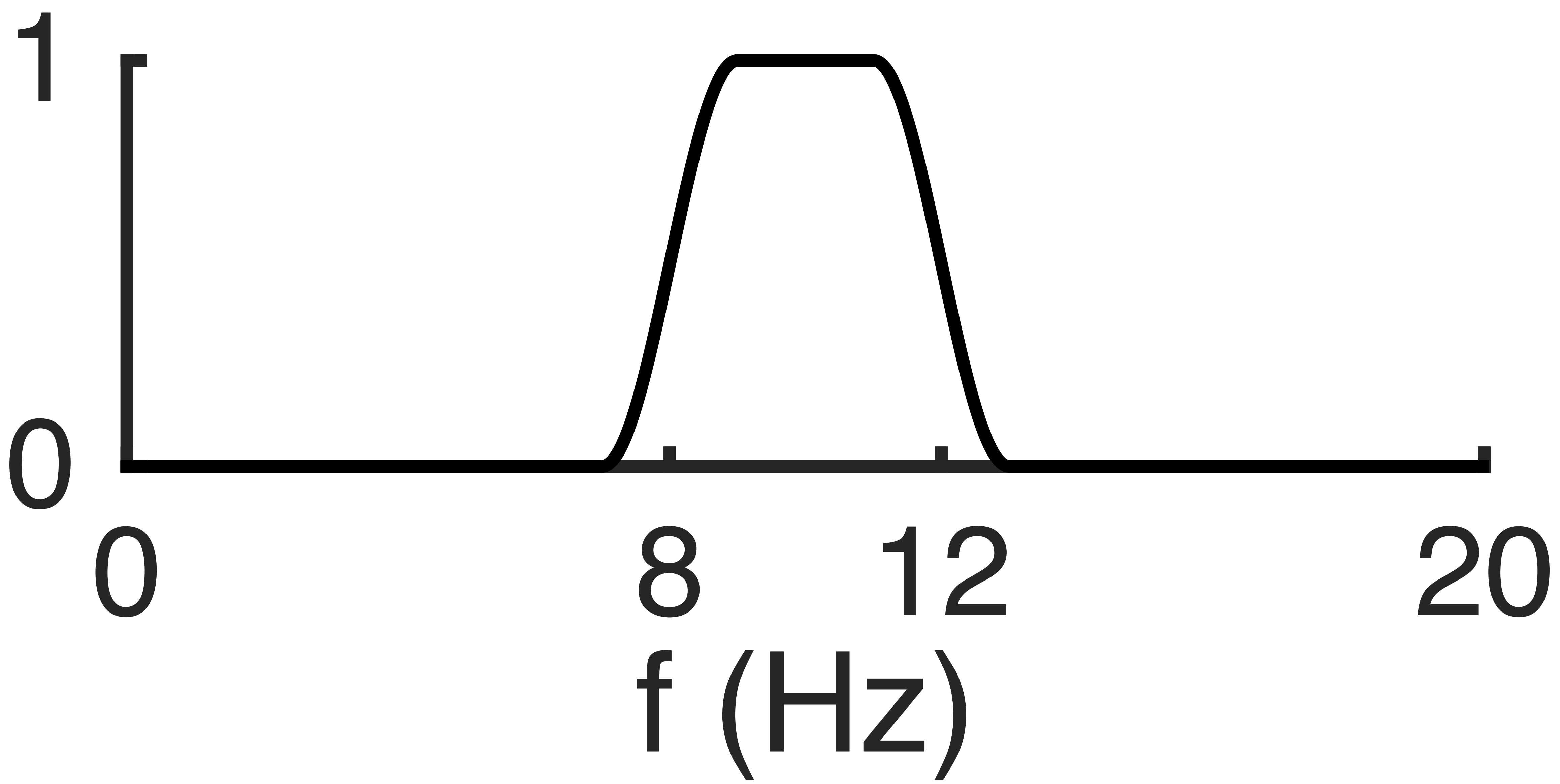
Sharp Transition



Soft Transition



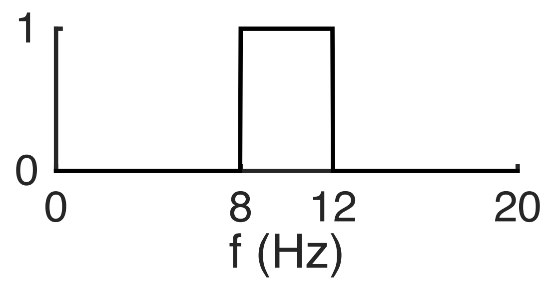
Sharp Transition



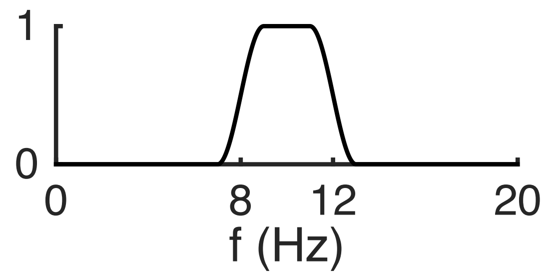
Filters: How Selective?

- *How sharp a transition?*

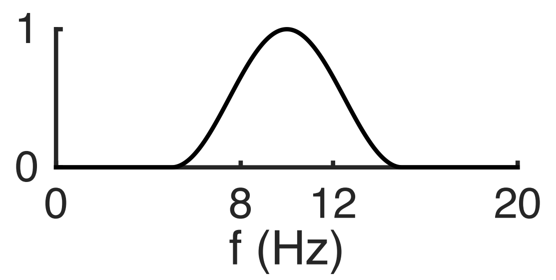
“Ideal” Filter



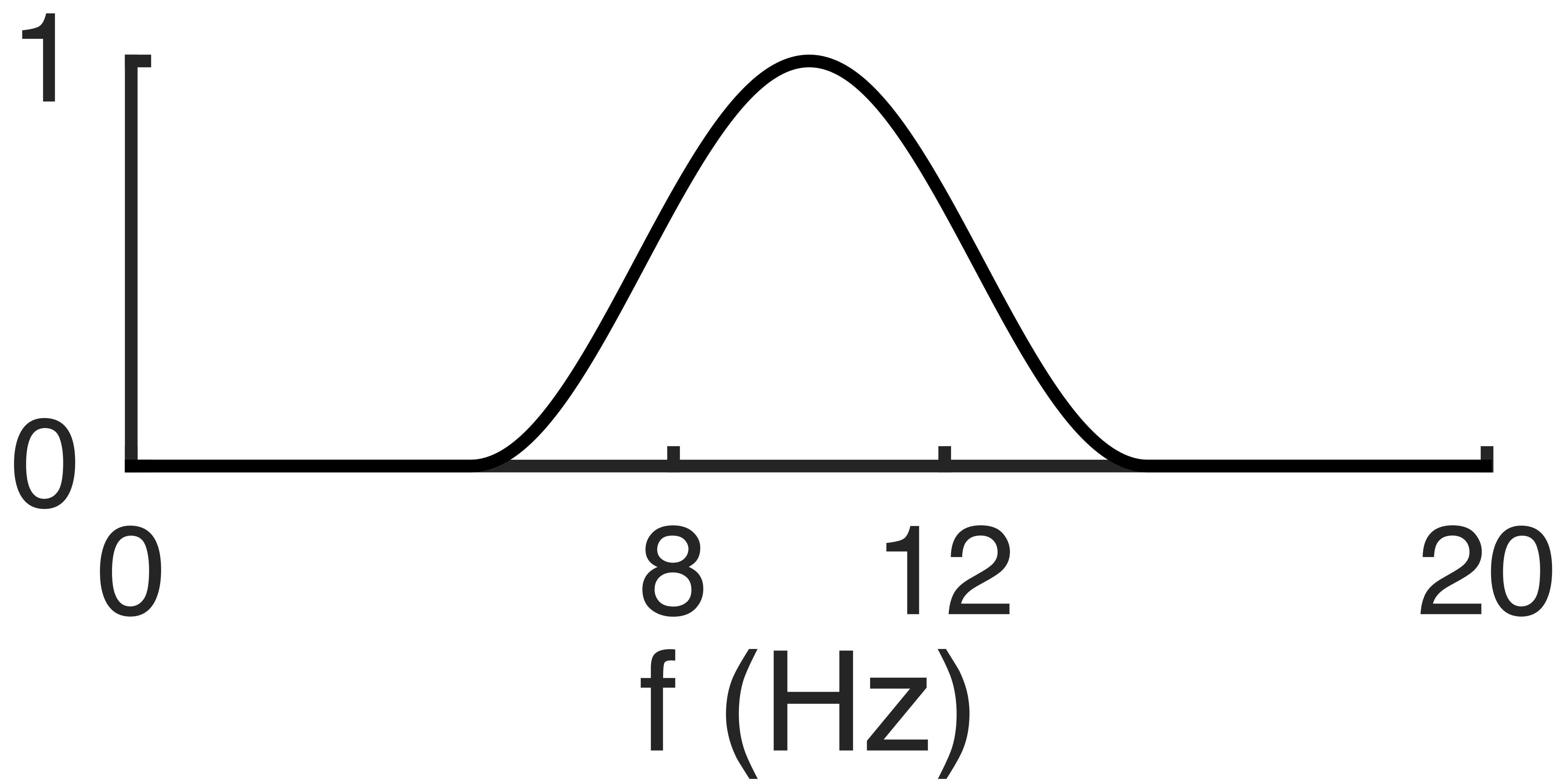
Sharp Transition



Soft Transition



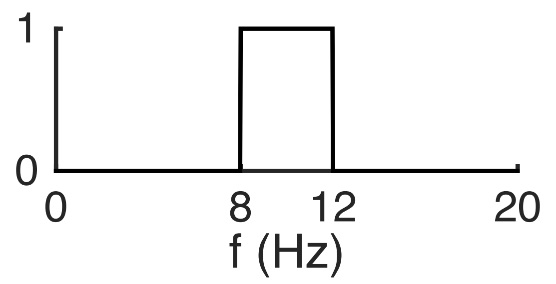
Soft Transition



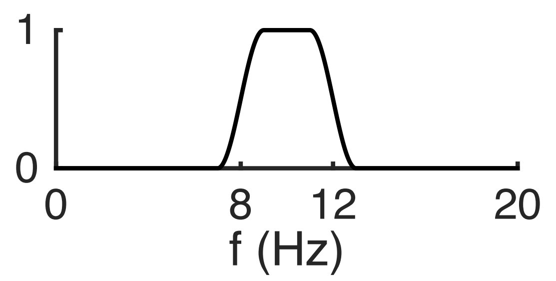
Filters: How Selective?

- *How sharp a transition?*

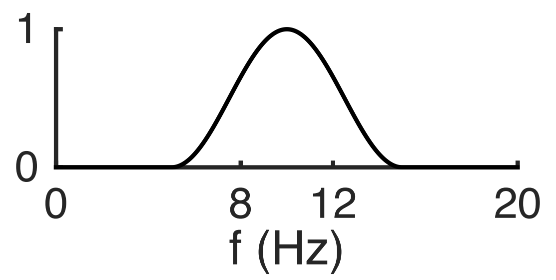
"Ideal" Filter



Sharp Transition



Soft Transition



Filters: How Do They Work?

Output of Filter:

- Linear Combination of *Input Signal* and ***Earlier Versions*** *of the Input Signal*
- Linear Combination of *Input Signal* and ***Earlier Versions*** of the ***Output*** *Signal*
- Linear Combination of *Input Signal* and ***Earlier Versions*** of both the ***Input and Output*** *Signals*

Examples:

$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t-\Delta t]$$

$$y[t] = \frac{1}{10}x[t] - \frac{9}{10}y[t-\Delta t]$$

$$y[t] = x[t] - x[t-\Delta t] + x[t-2\Delta t] + \frac{99}{100}y[t-\Delta t] - \left(\frac{99}{100}\right)^2 y[t-2\Delta t]$$

Example: Two-Point Moving Average

$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t - \Delta t]$$

What to Expect:

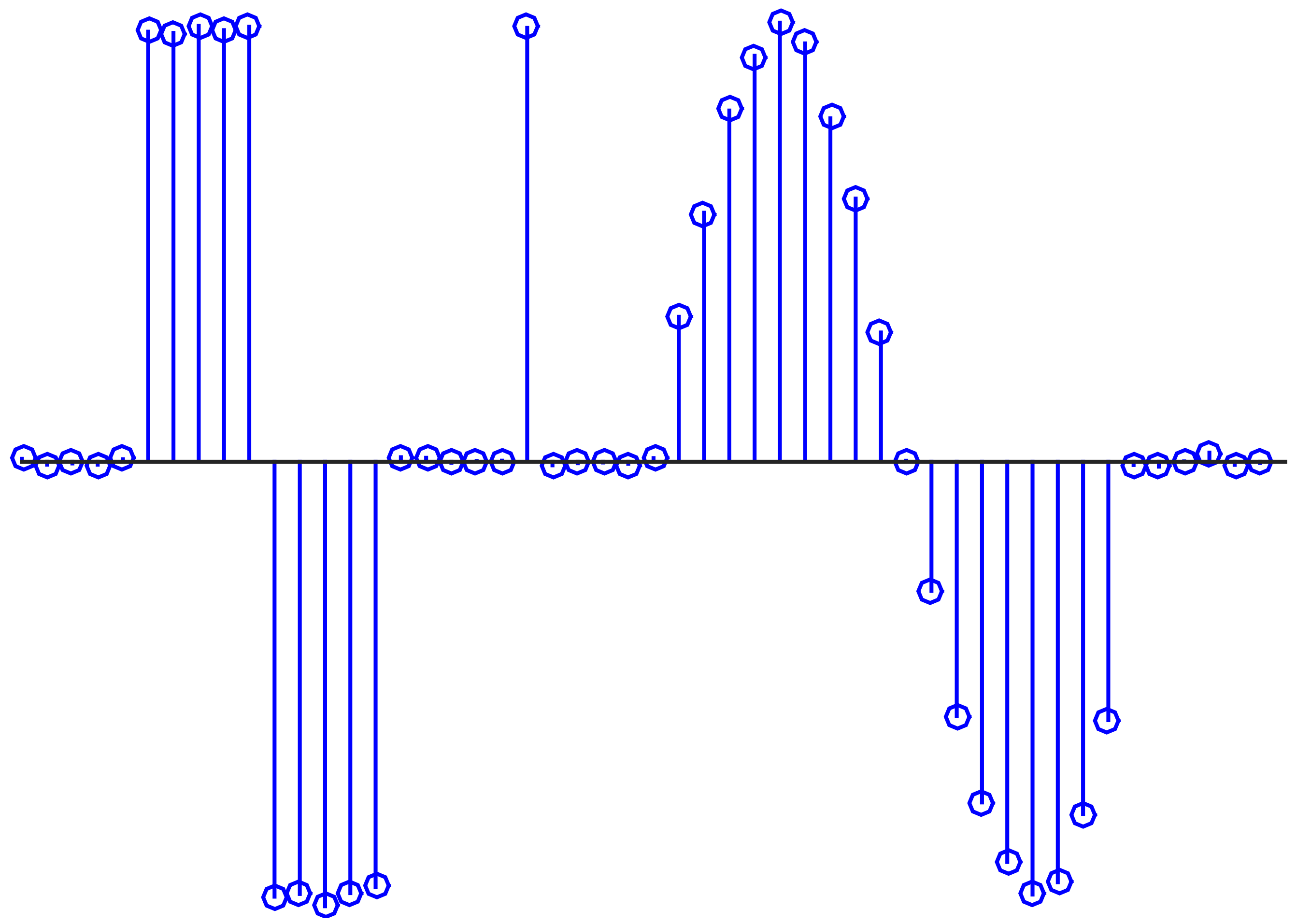
- Smooth over rough patches
- Soften sudden changes
- Leave slowly varying signals largely unchanged
- Low Pass Filter?

Example: Two-Point Moving Average

$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t - \Delta t]$$

Example: Two-Point Moving Average

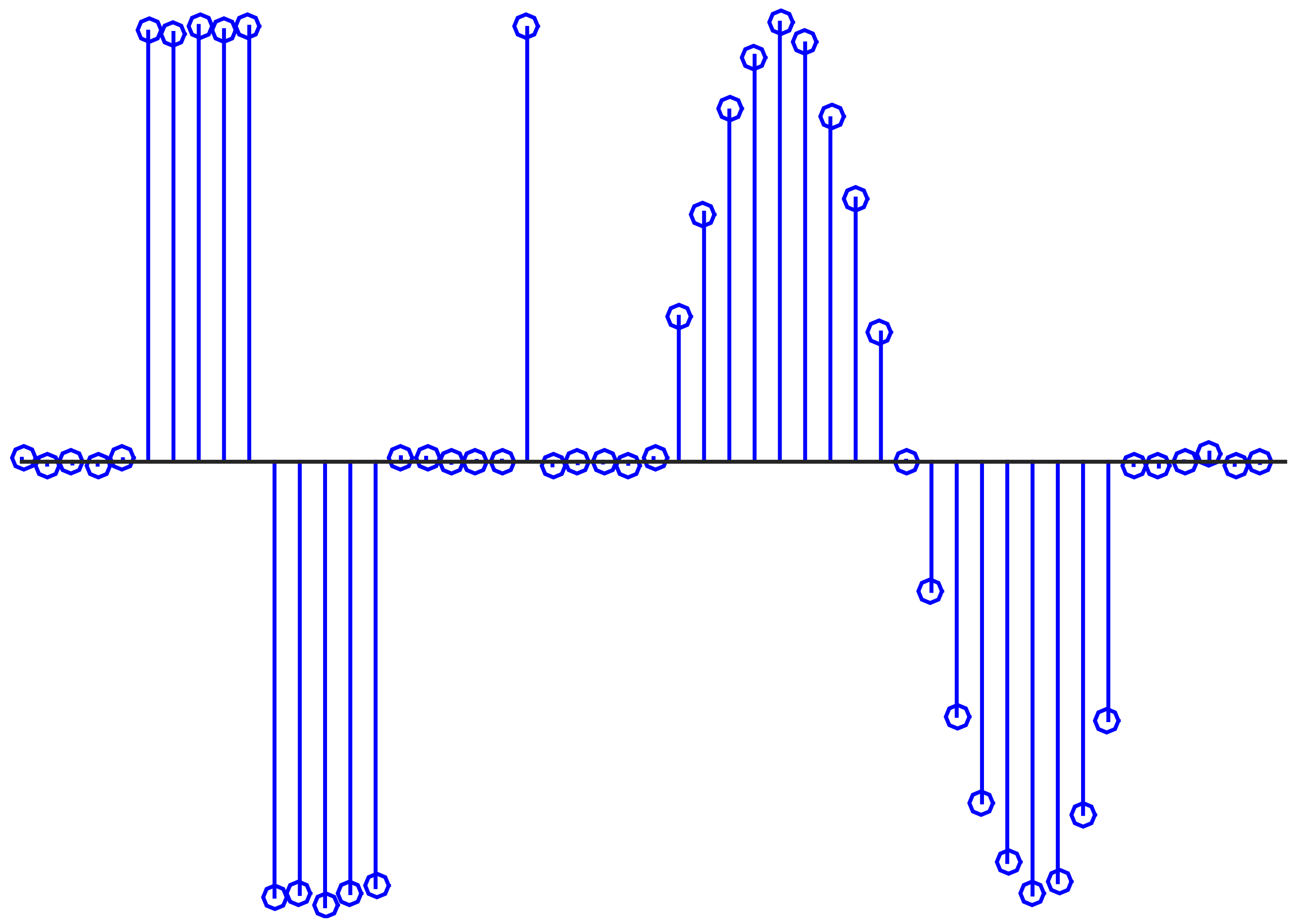
$x[t]$



$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t - \Delta t]$$

Example: Two-Point Moving Average

$$x[t]$$



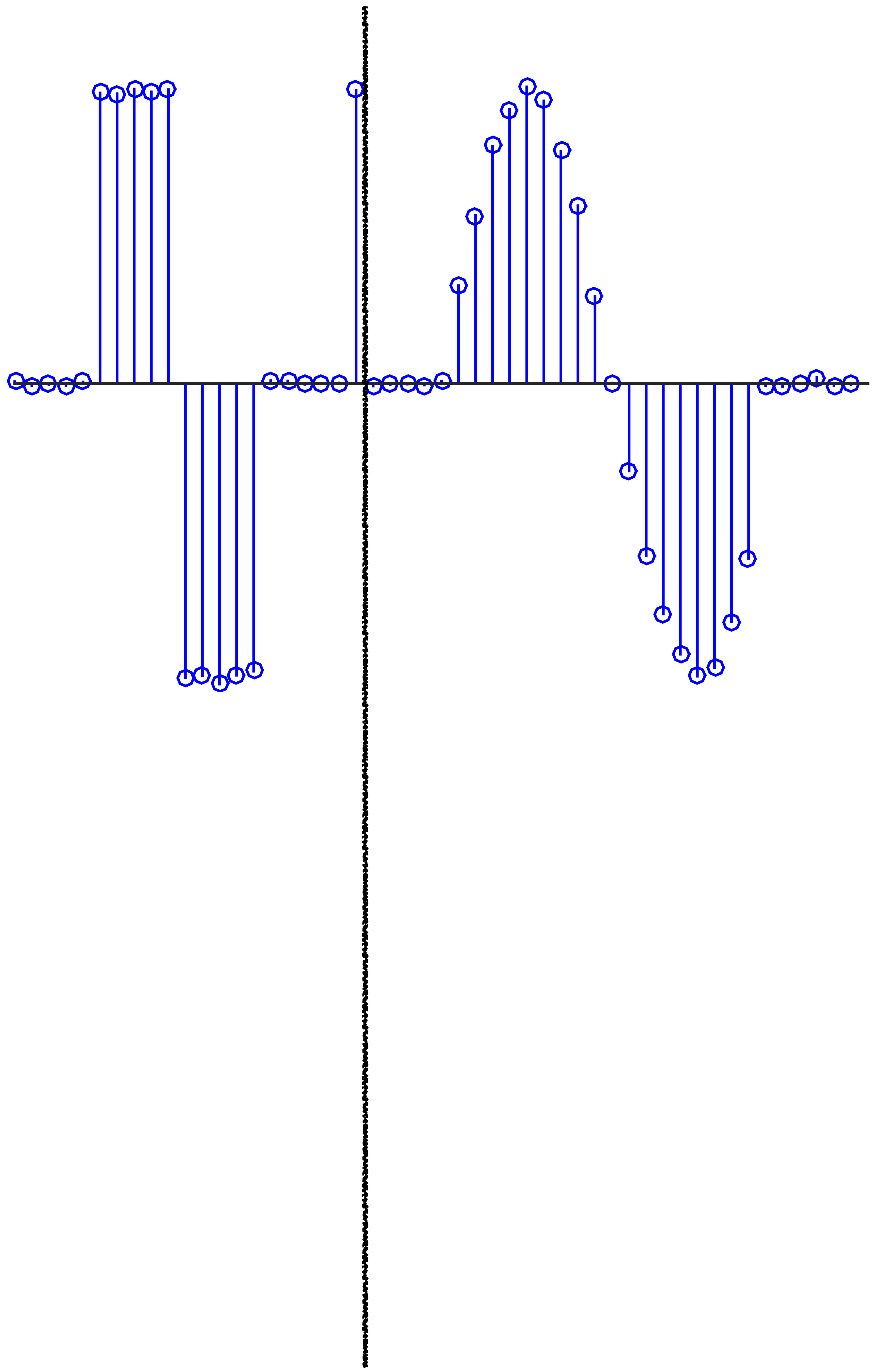
$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t - \Delta t]$$

$$x[t - \Delta t]$$

Example: Two-Point Moving Average

$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t - \Delta t]$$

$x[t]$

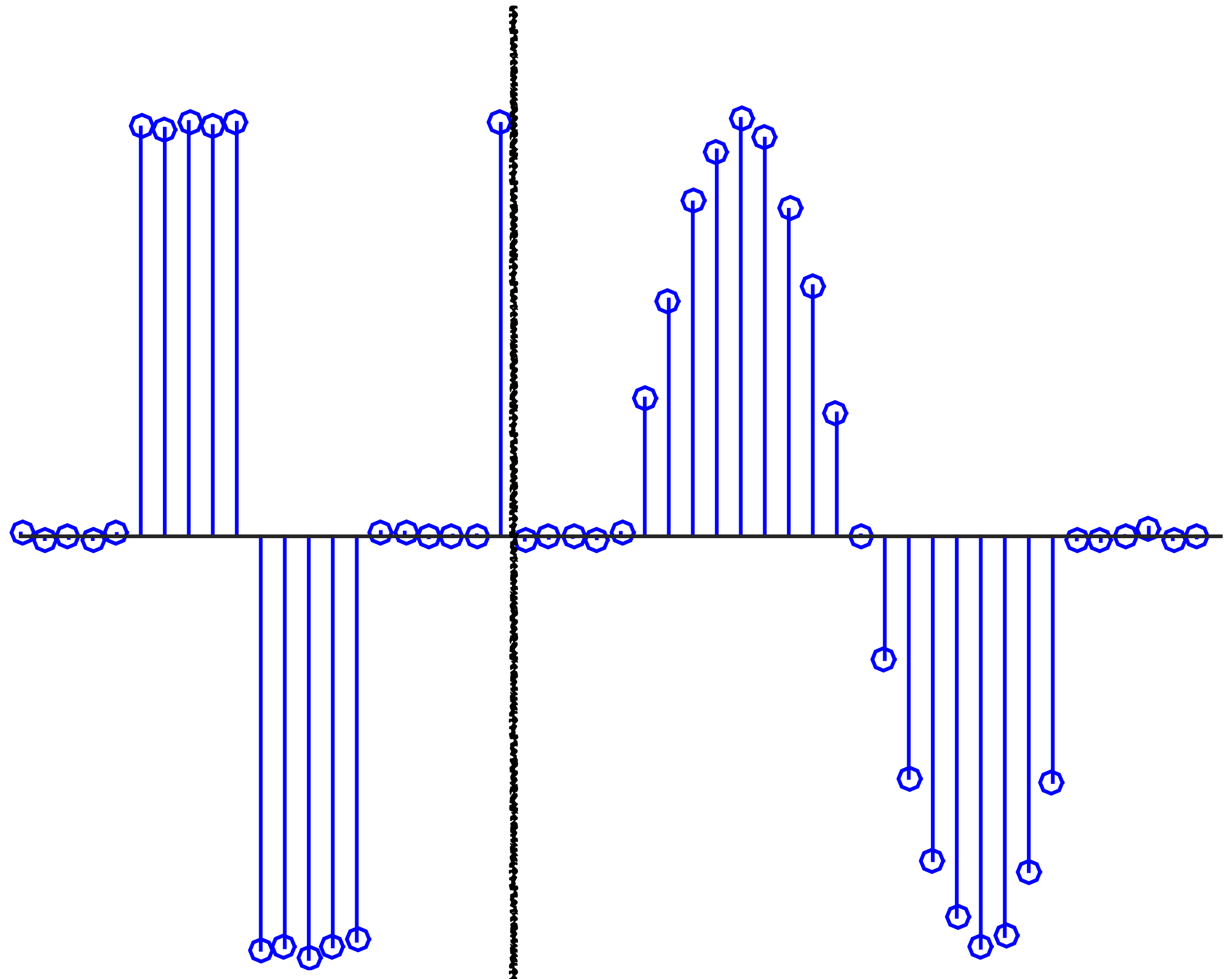


$x[t - \Delta t]$

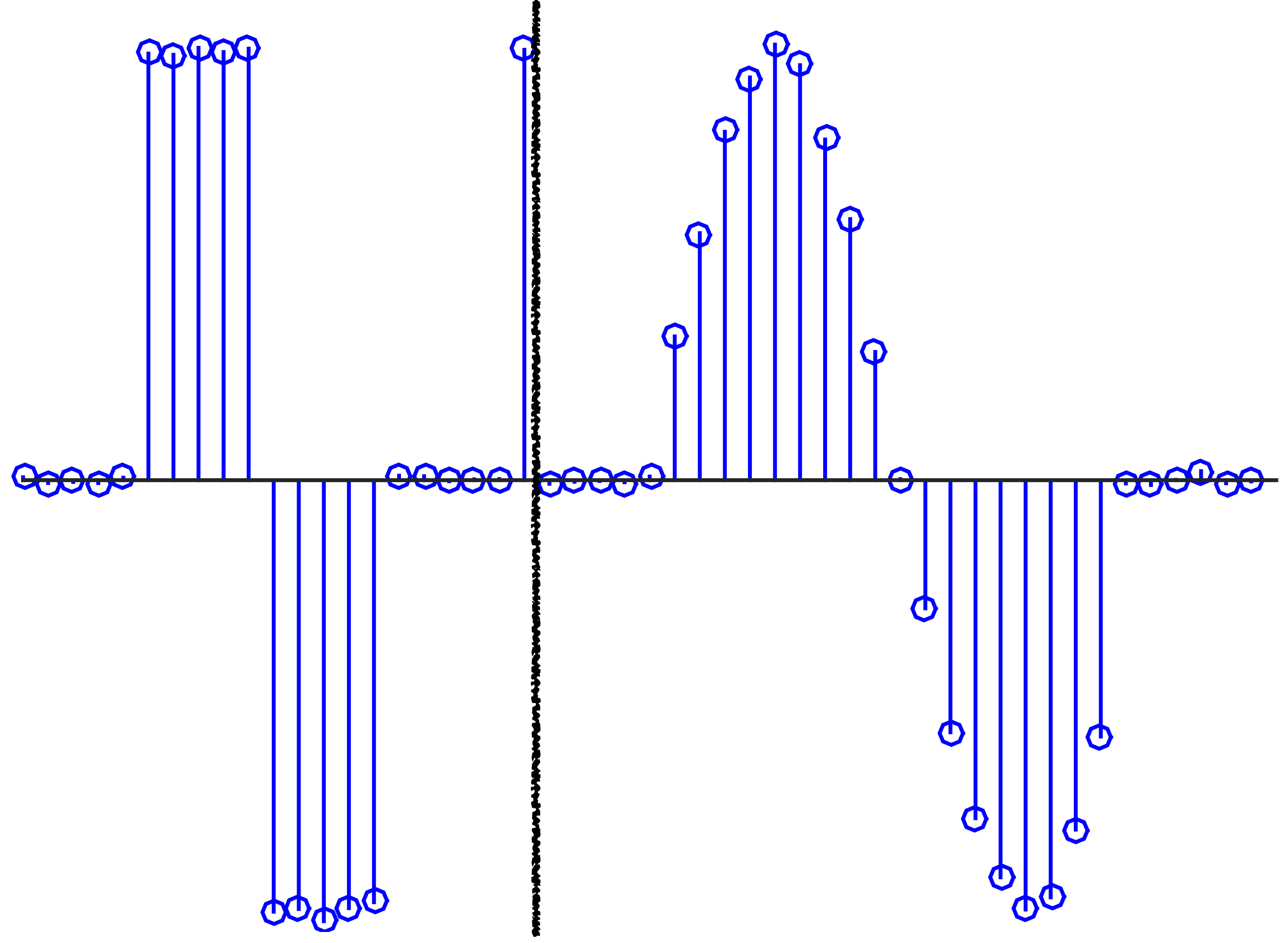
Example: Two-Point Moving Average

$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t-\Delta t]$$

$x[t]$



$x[t-\Delta t]$

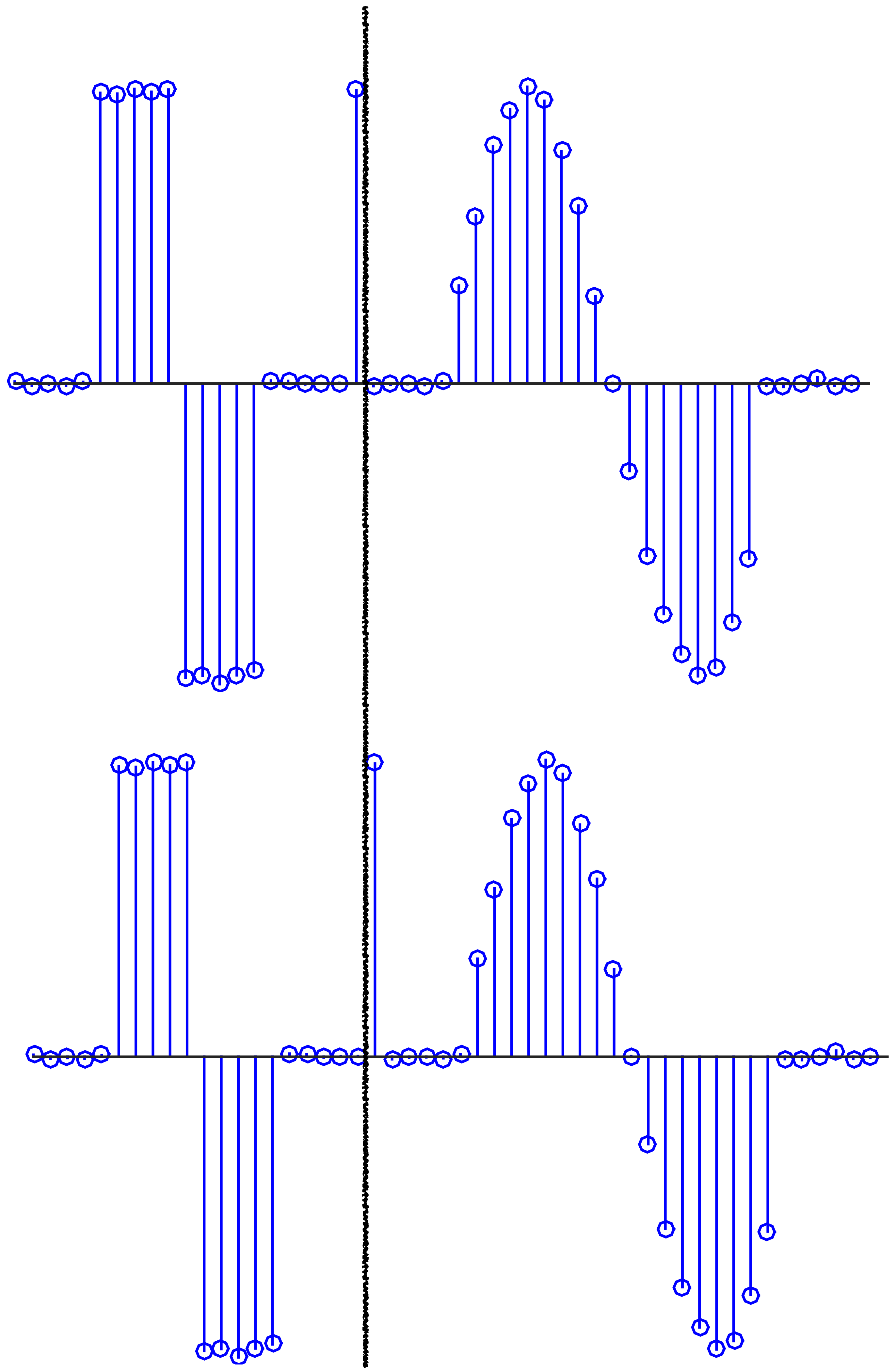


Example: Two-Point Moving Average

$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t - \Delta t]$$

$x[t]$

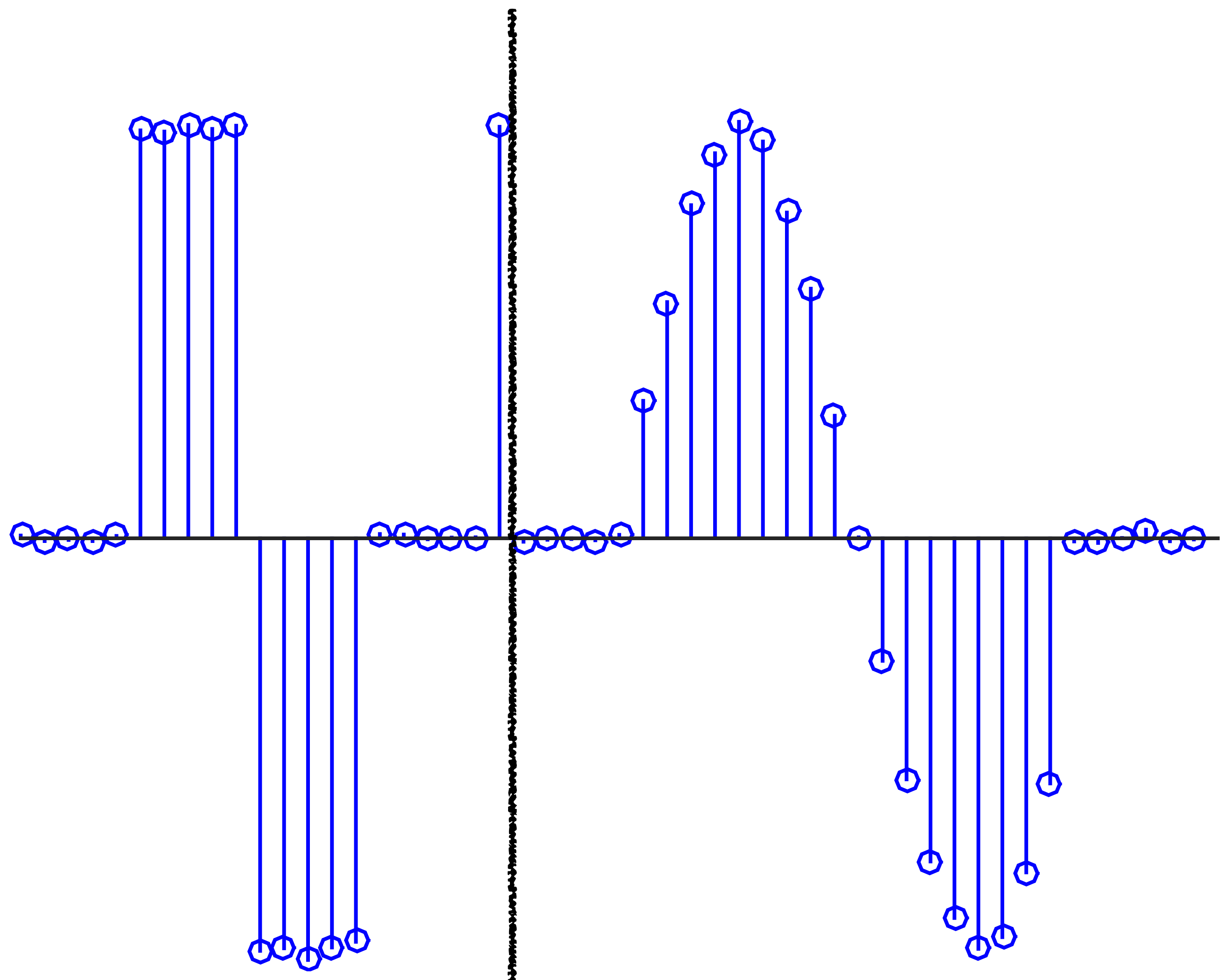
$x[t - \Delta t]$



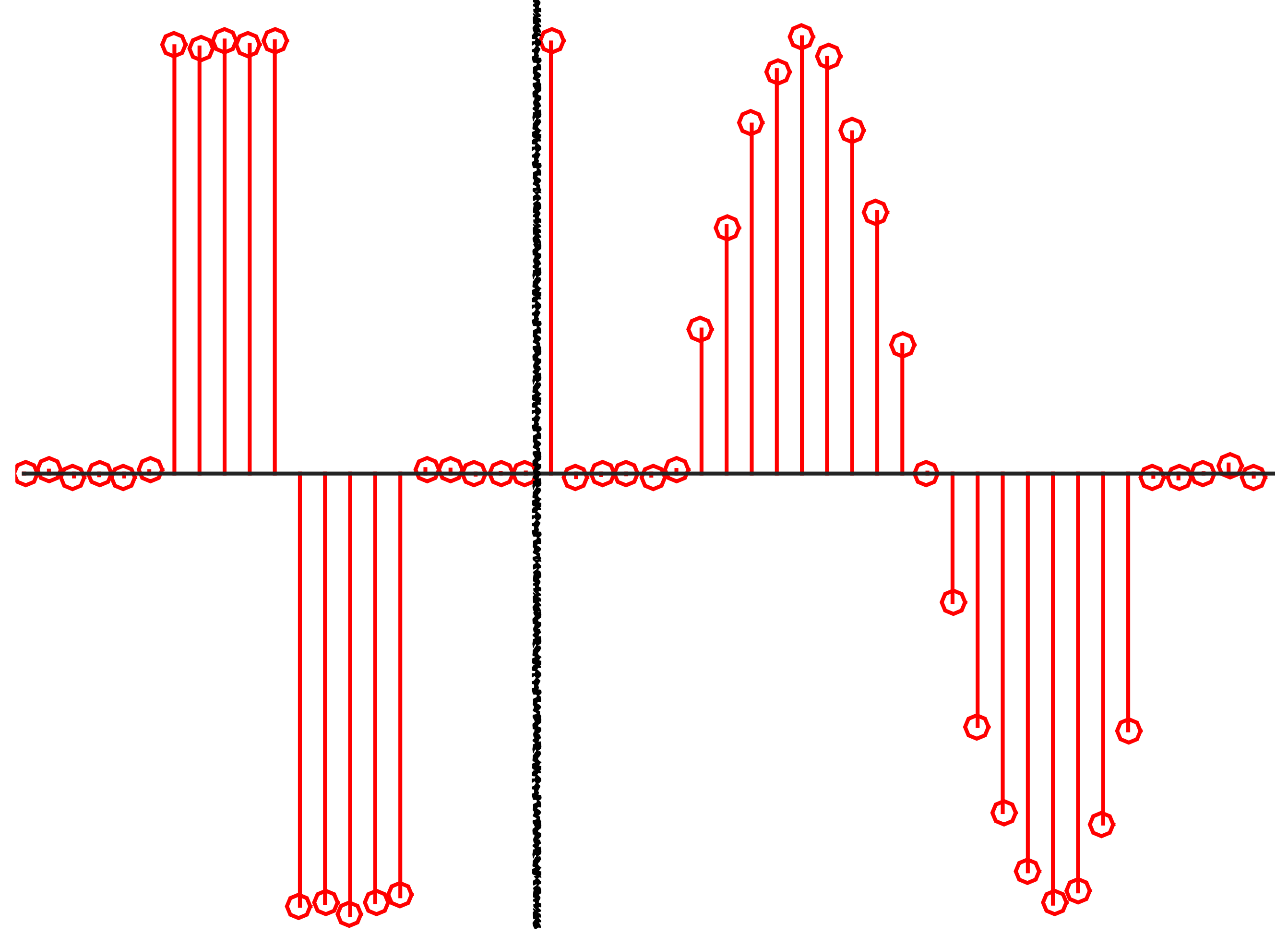
Example: Two-Point Moving Average

$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t - \Delta t]$$

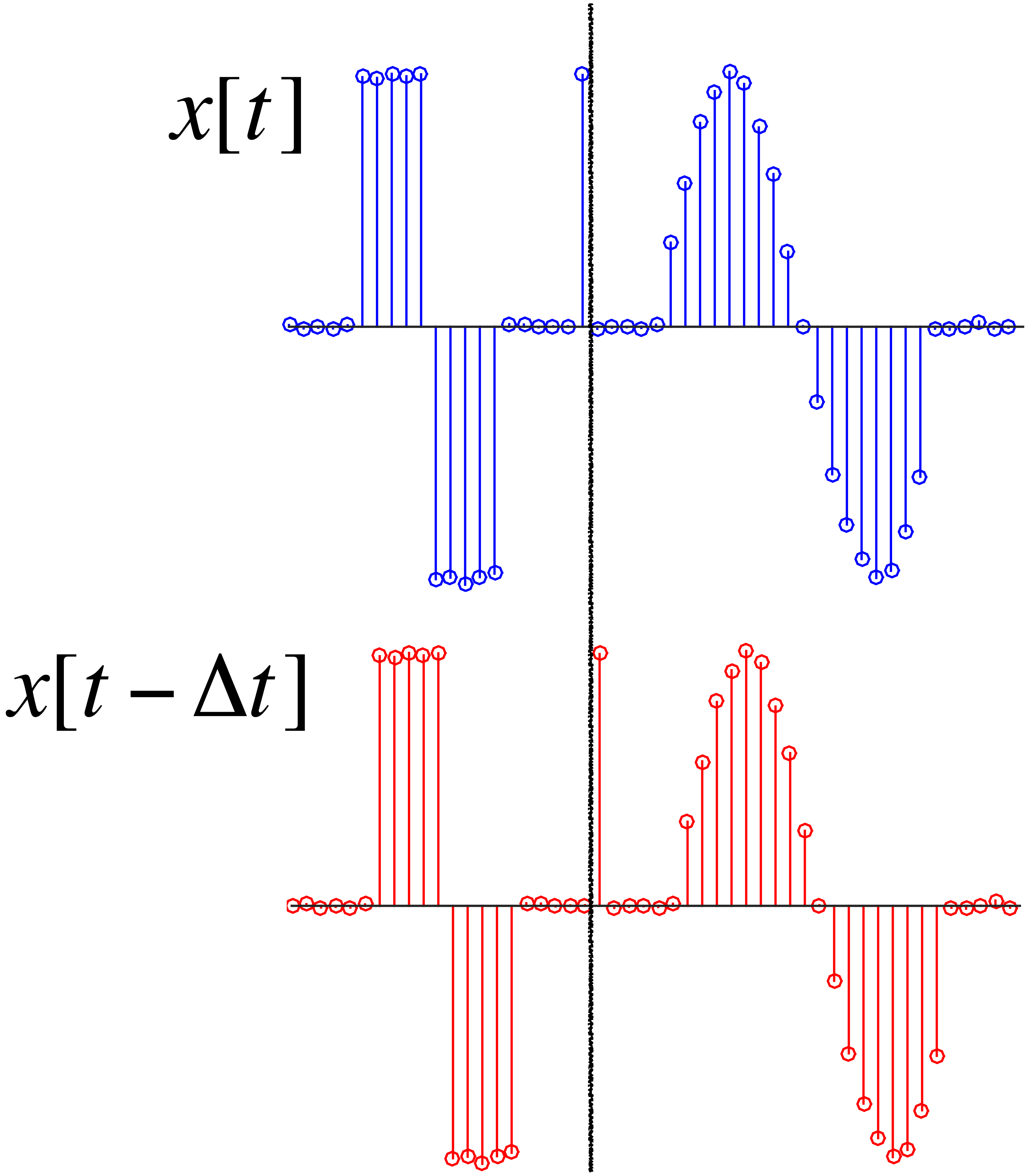
$x[t]$



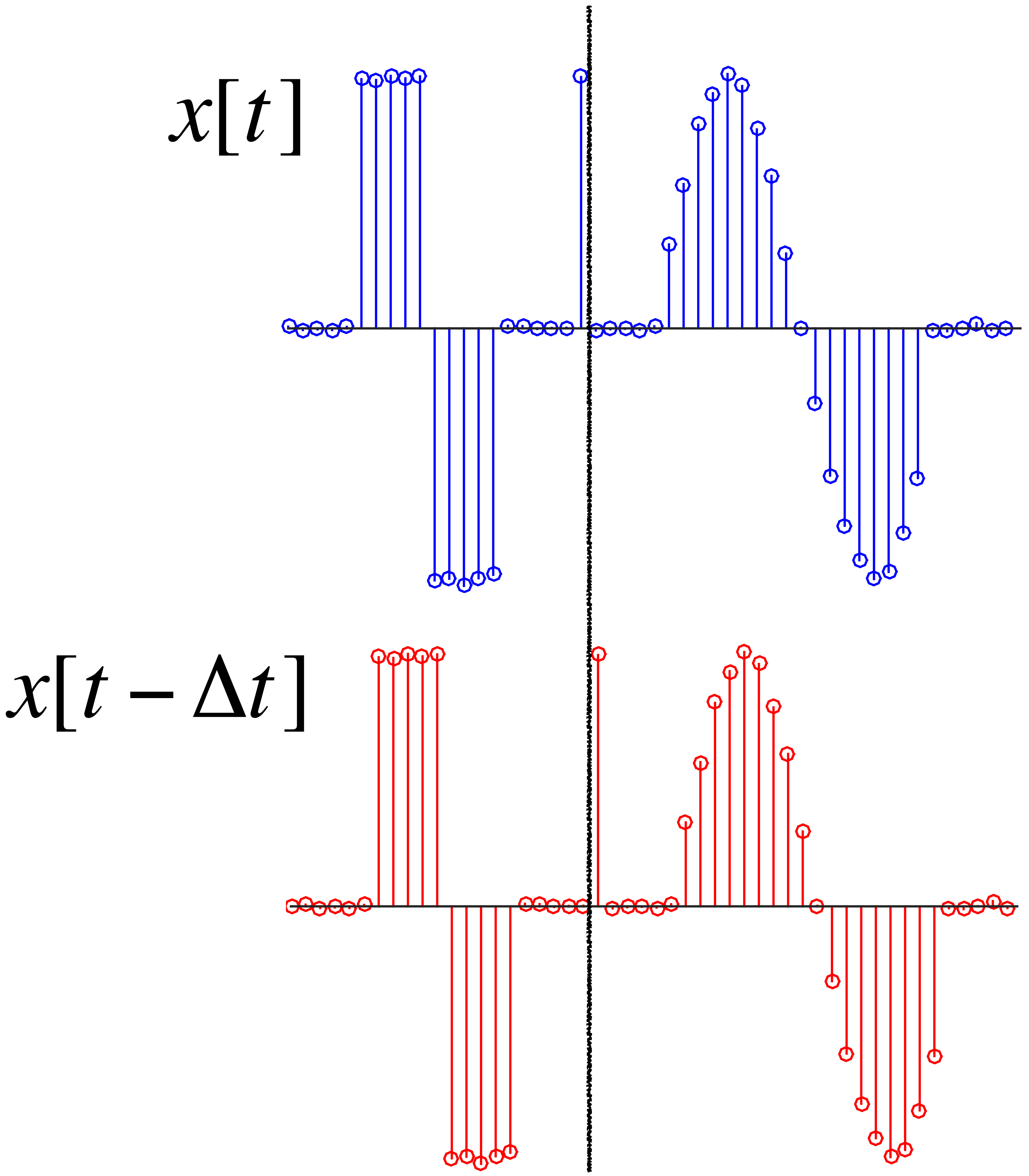
$x[t - \Delta t]$

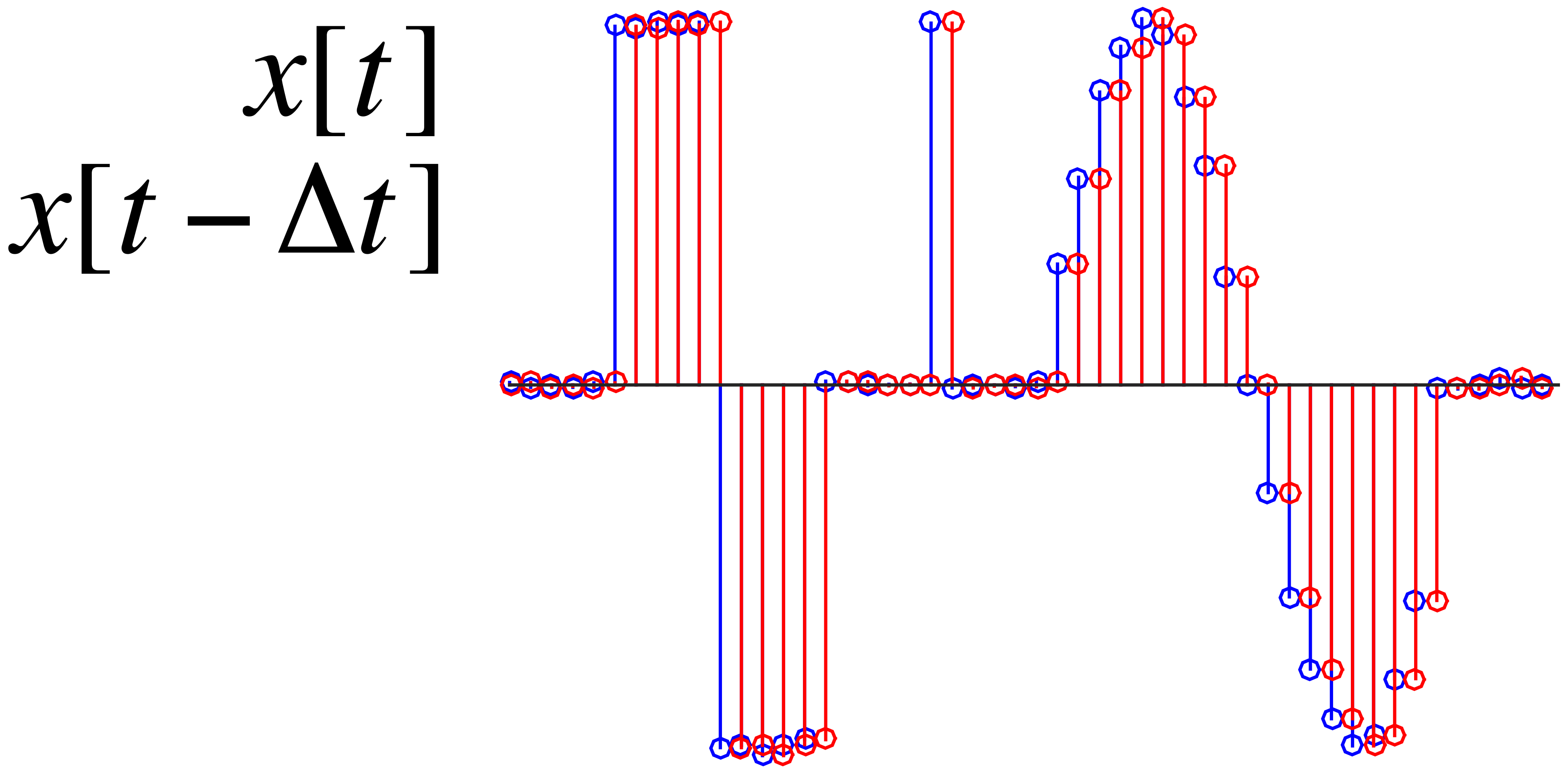


Example: Two-Point Moving Average



Example: Two-Point Moving Average

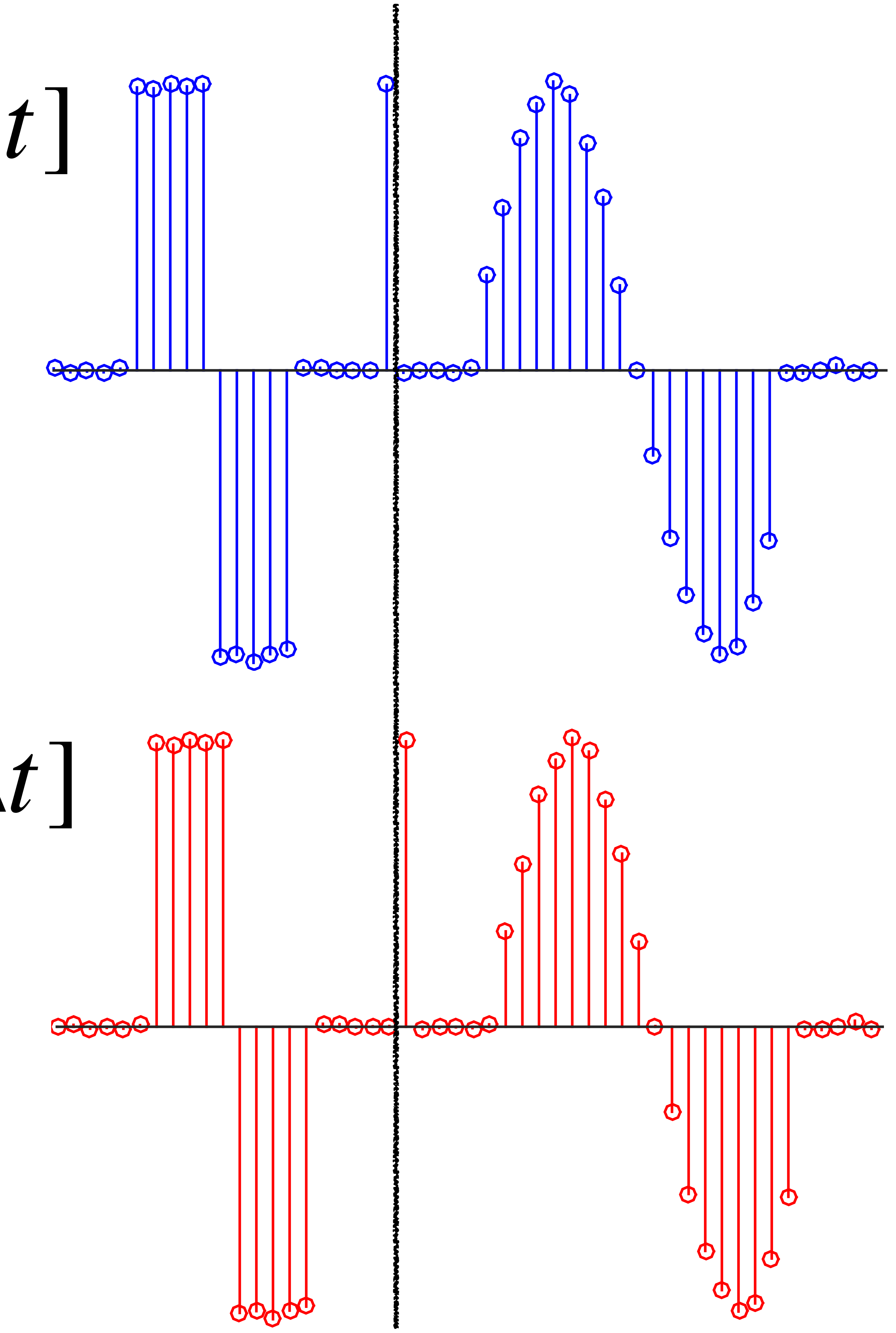




Example: Two-Point Moving Average

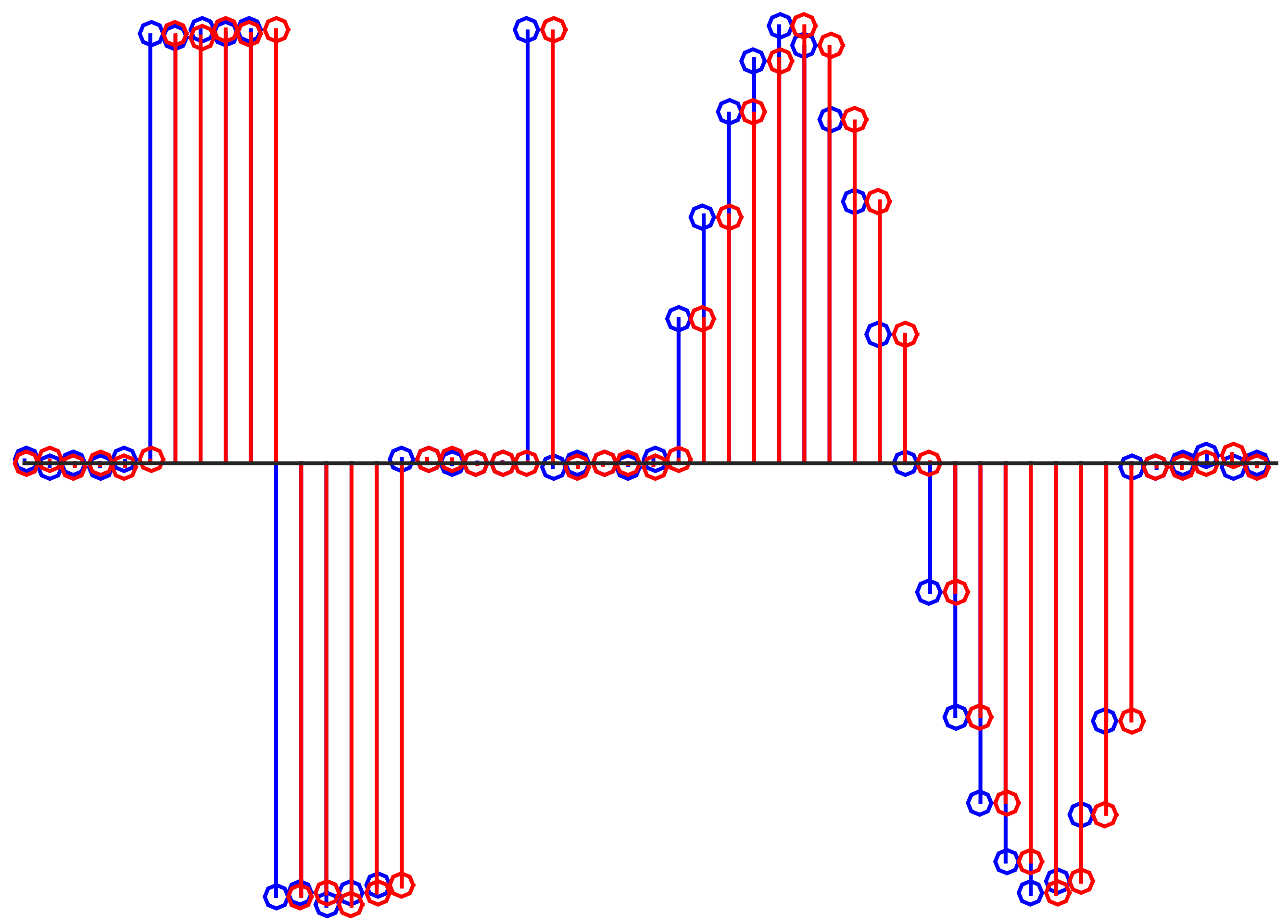
$x[t]$

$x[t-\Delta t]$



$x[t]$

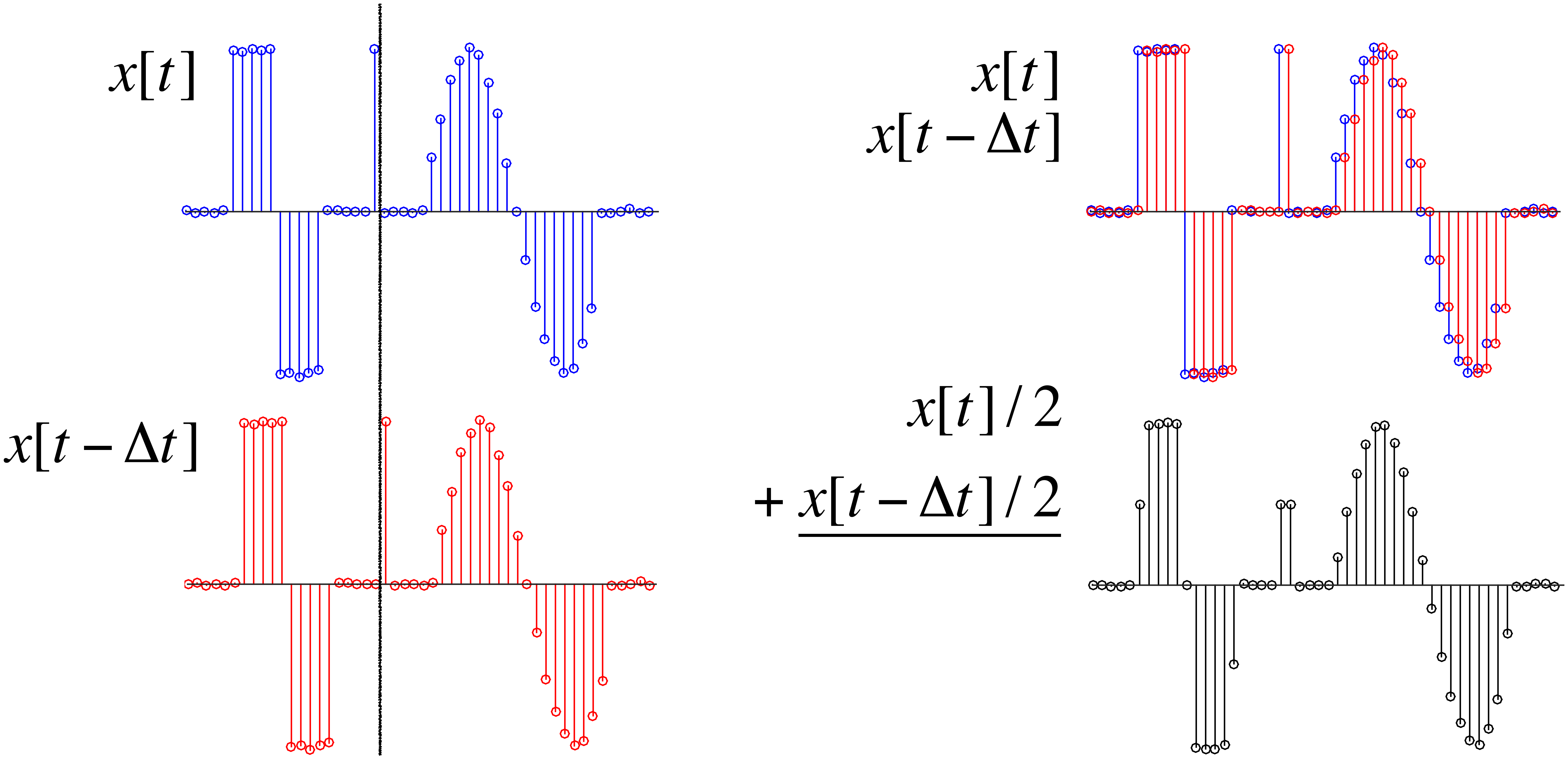
$x[t-\Delta t]$



$$x[t]/2$$

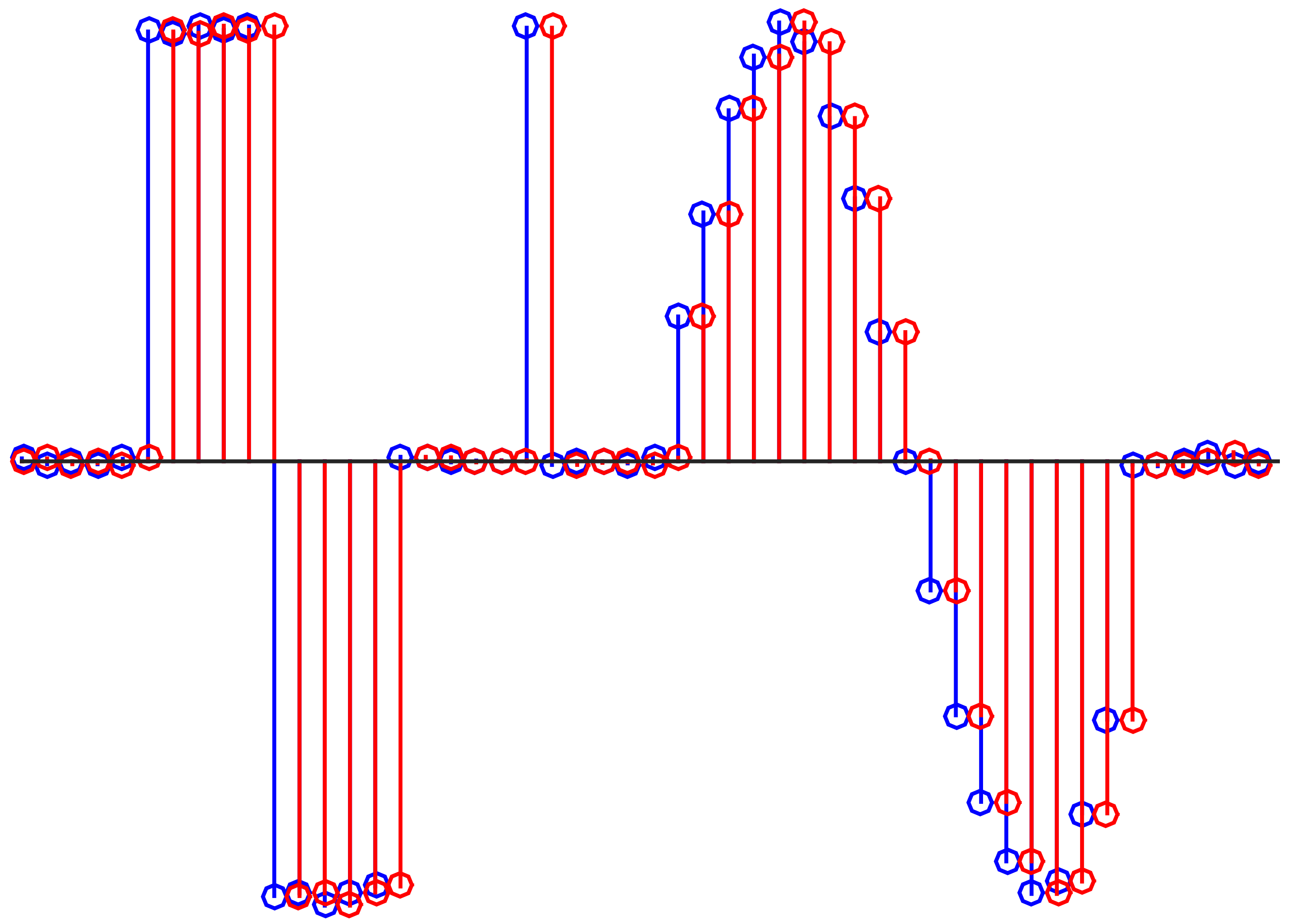
$$+\ \underline{x[t-\Delta t]/2}$$

Example: Two-Point Moving Average

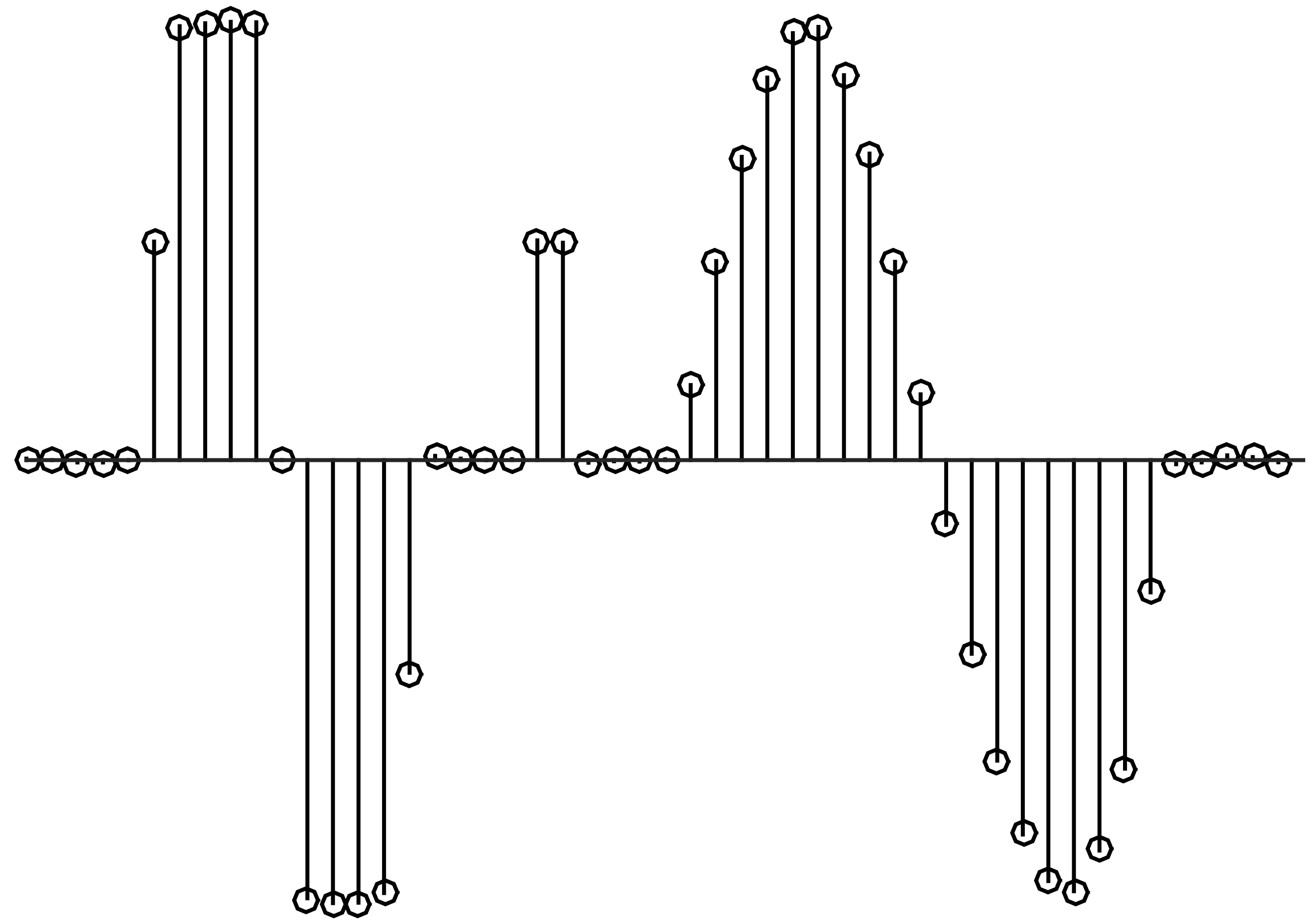


Example: Two-Point Moving Average

$x[t]$
$x[t-\Delta t]$



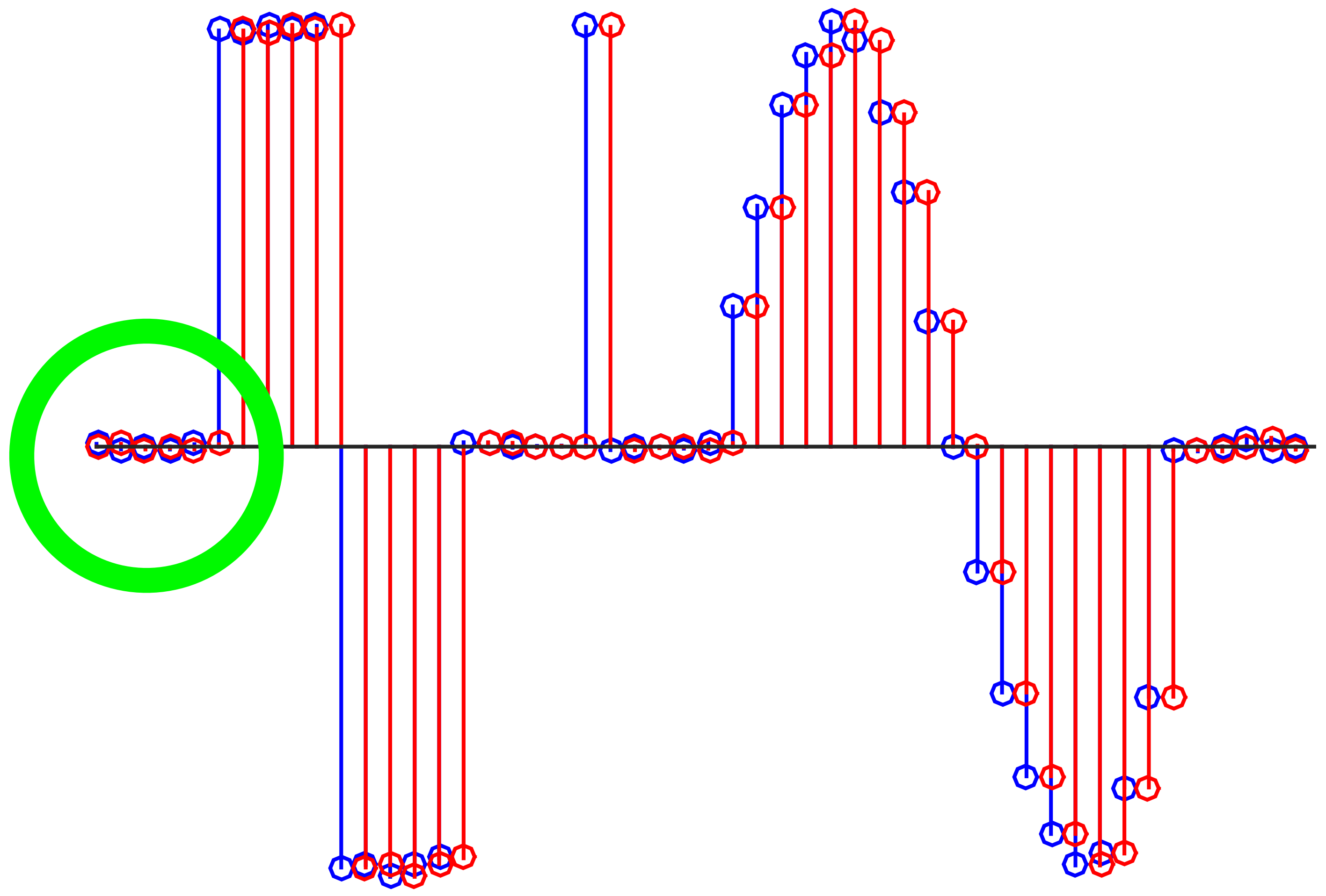
$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t-\Delta t]$$



Example: Two-Point Moving Average

$x[t]$

$x[t-\Delta t]$



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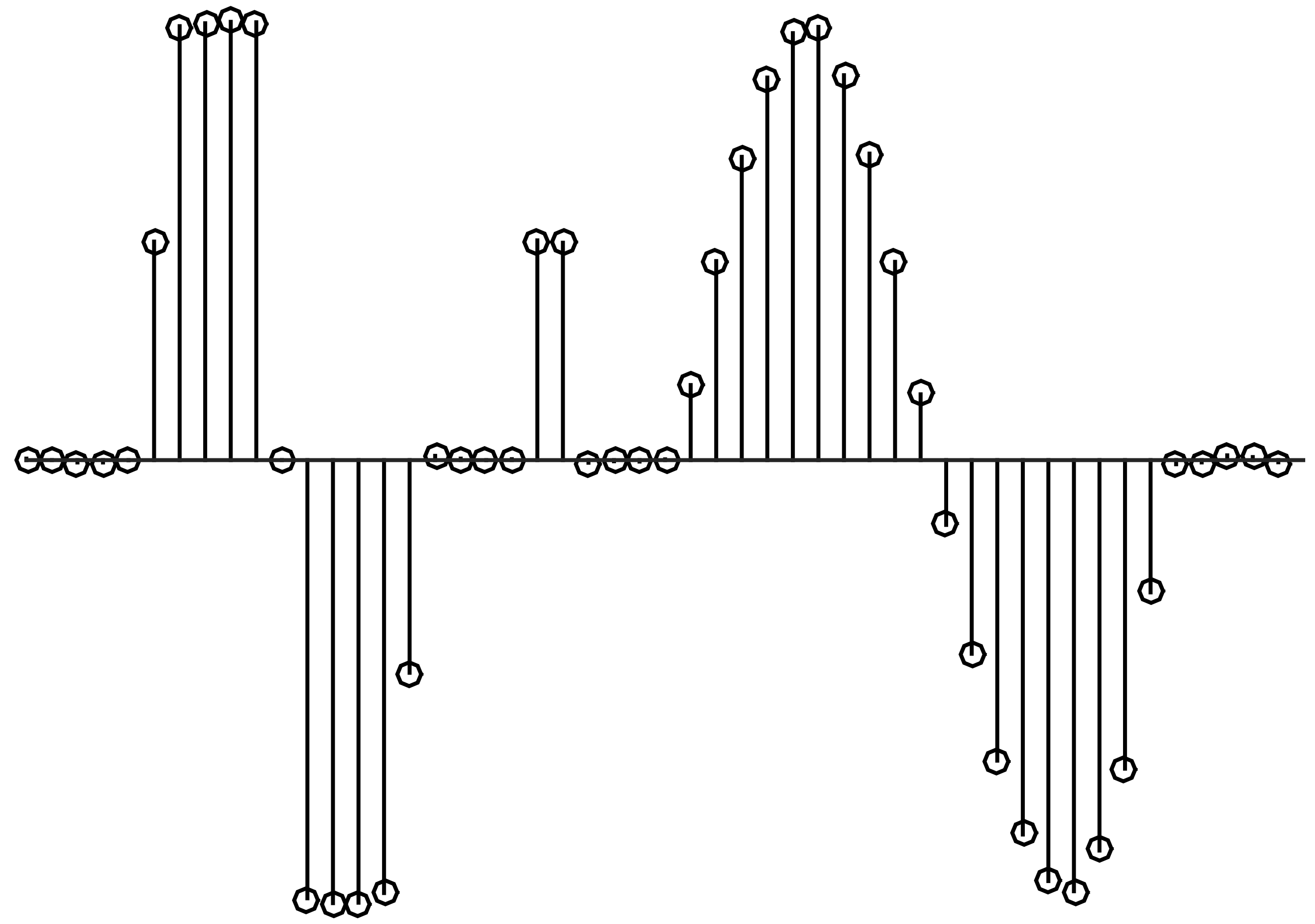
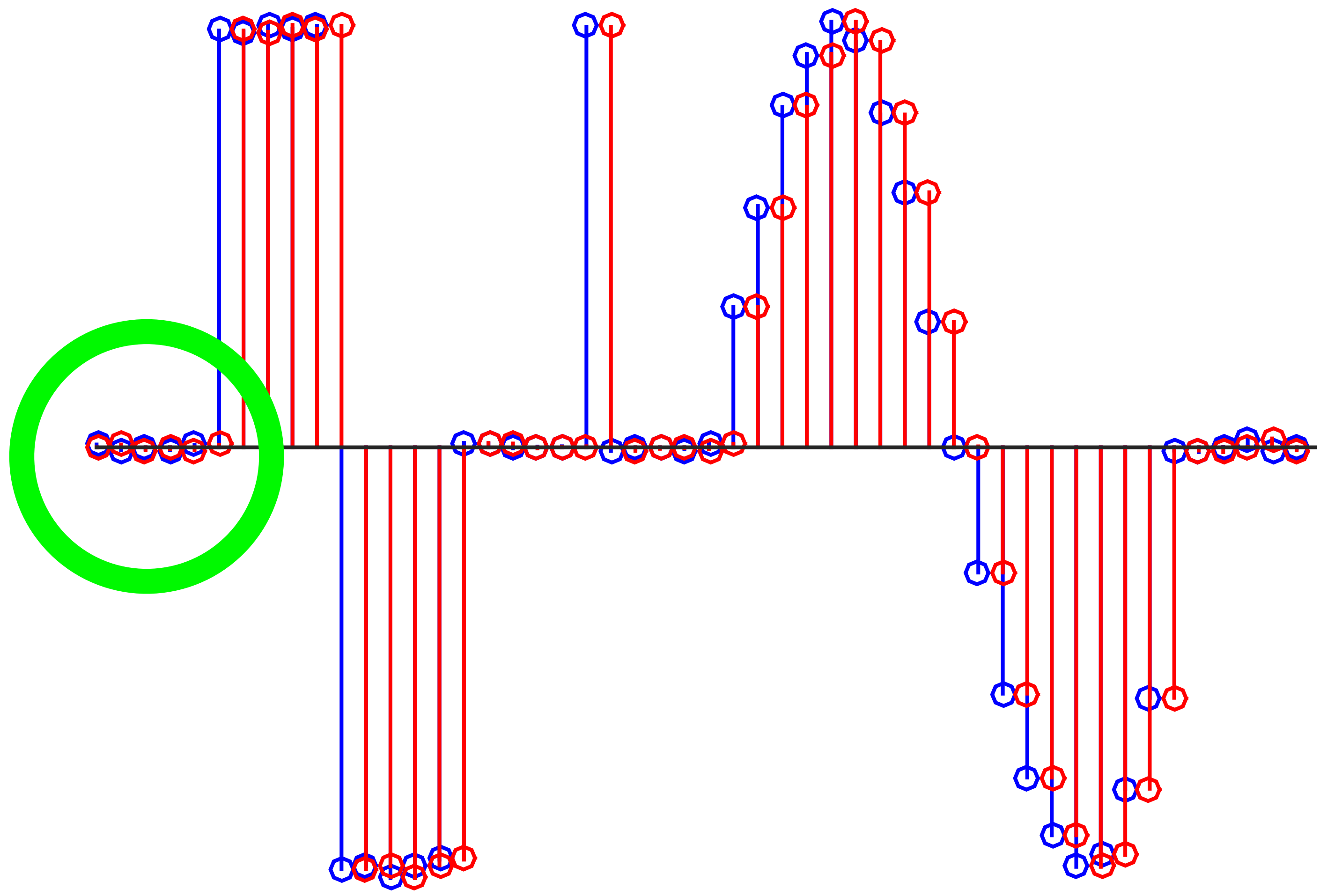


Example: Two-Point Moving Average

$x[t]$
$x[t-\Delta t]$



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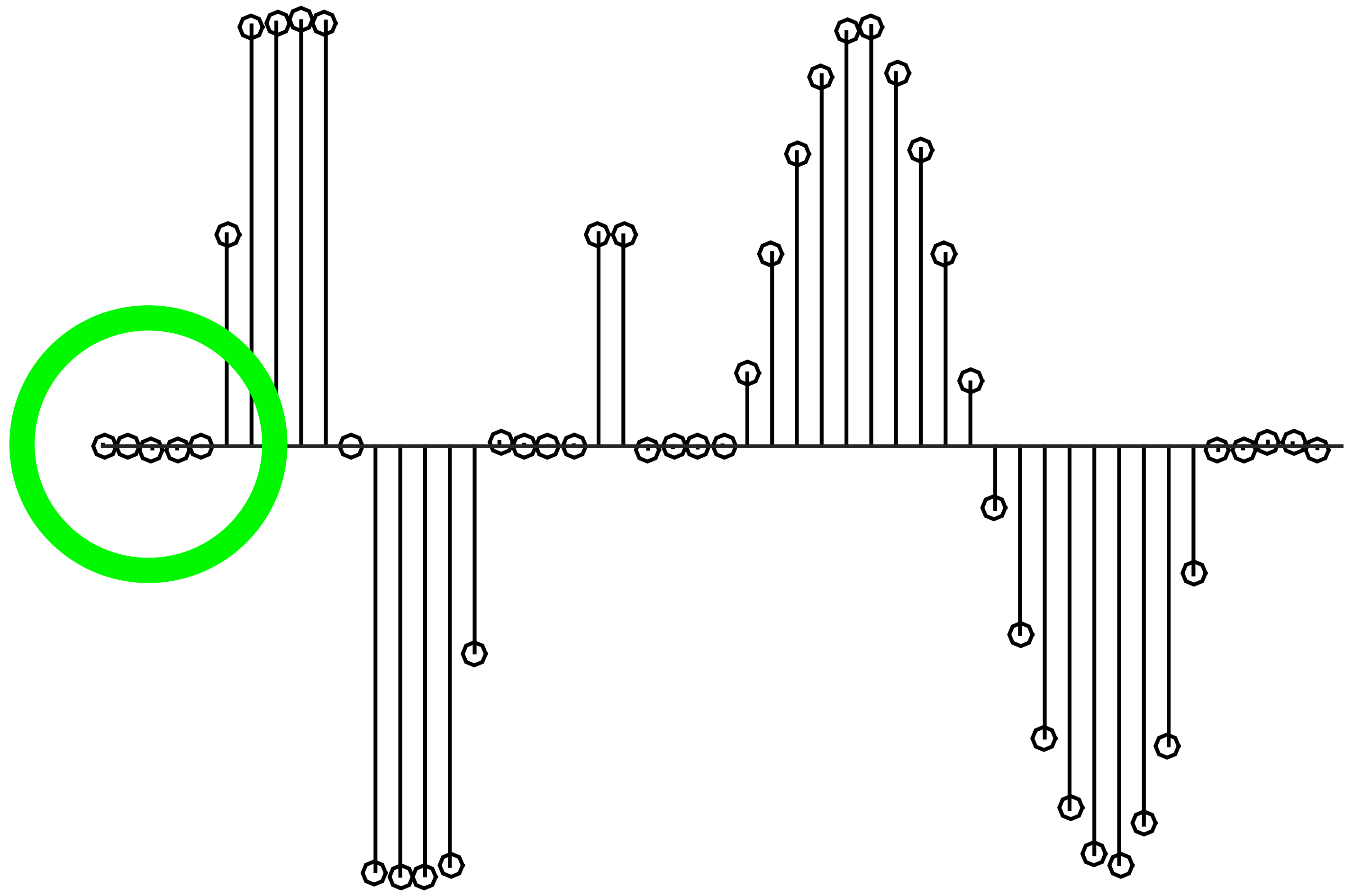
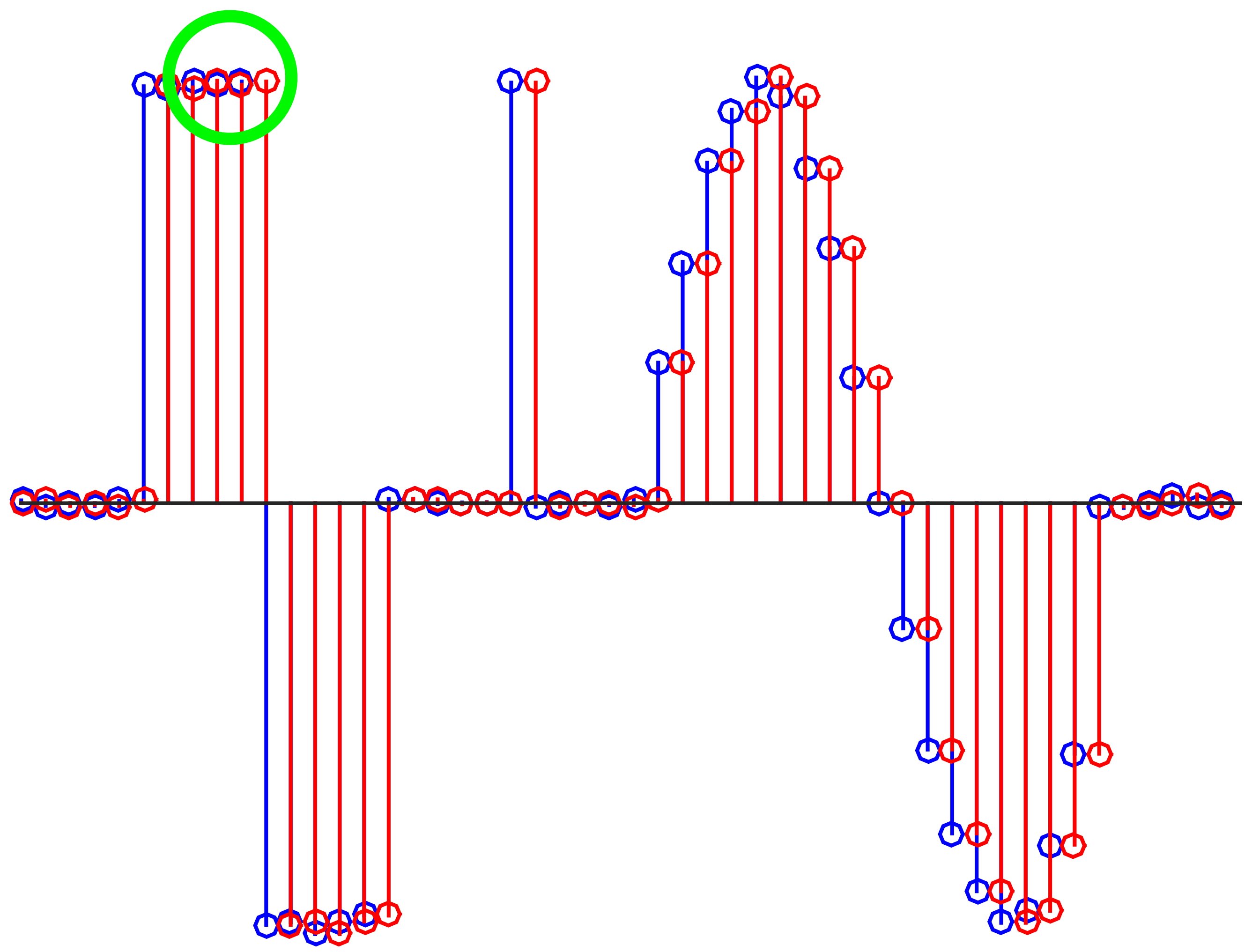


Example: Two-Point Moving Average

$x[t]$
$x[t-\Delta t]$



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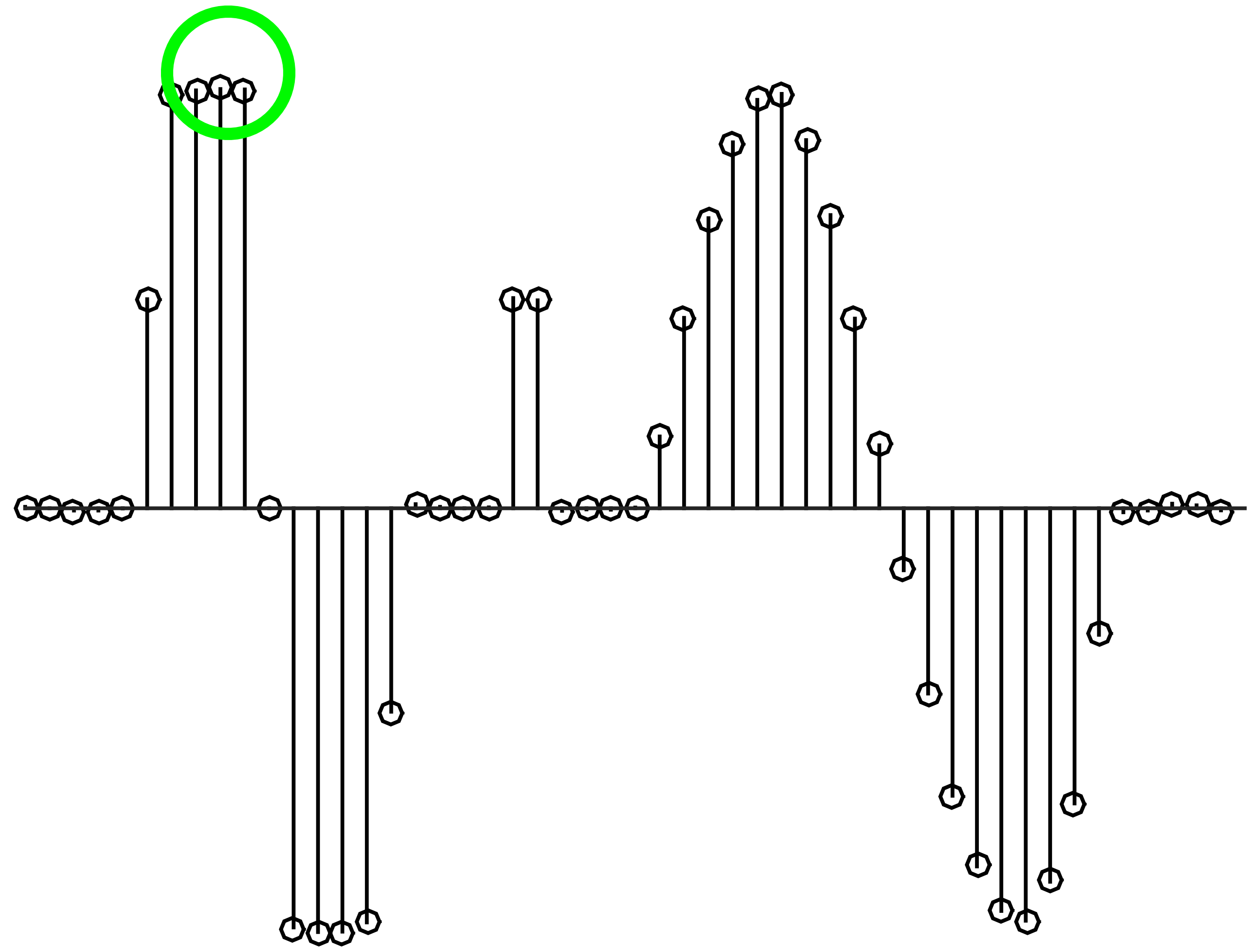
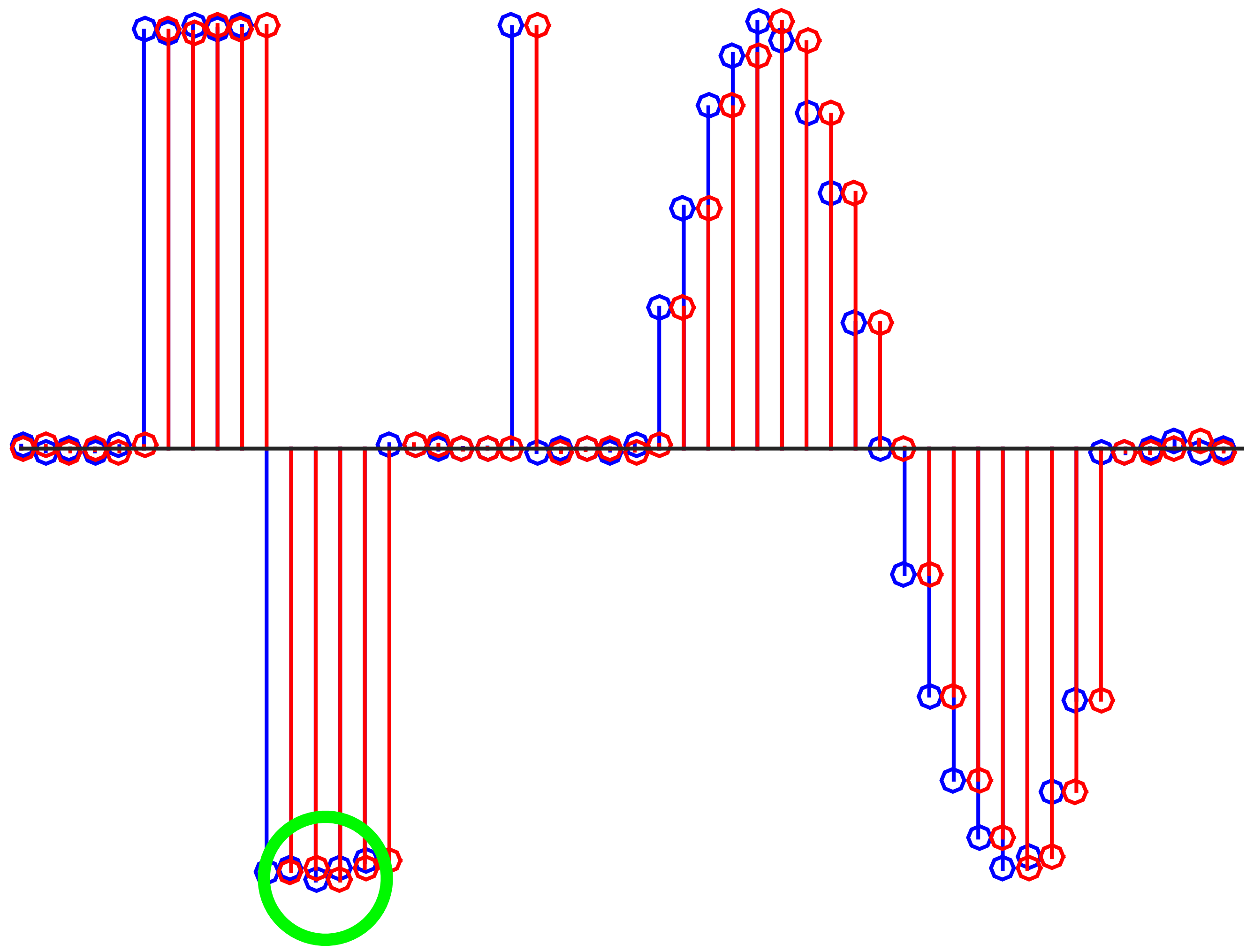
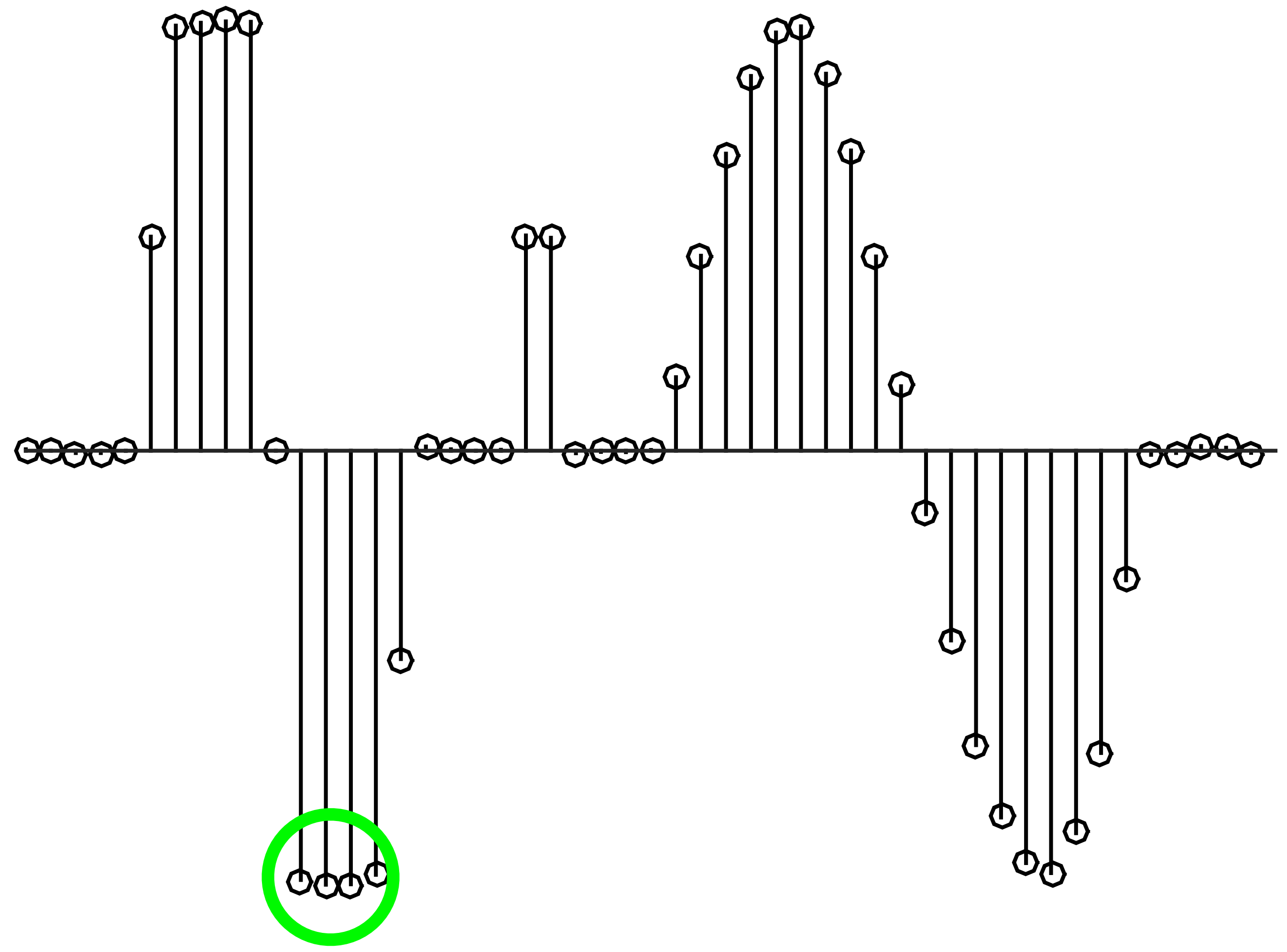


Example: Two-Point Moving Average

$x[t]$
$x[t-\Delta t]$

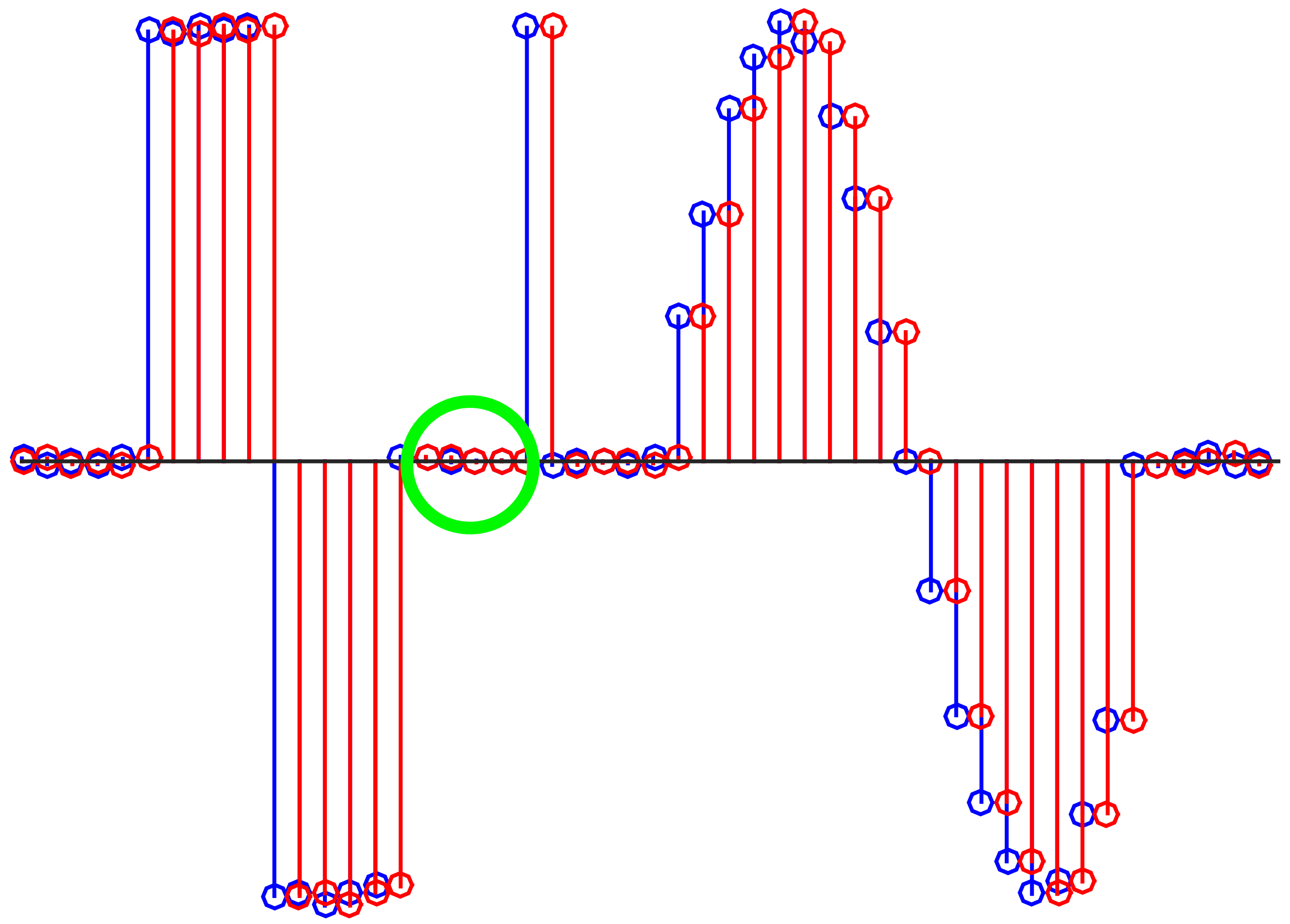


$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t-\Delta t]$$



Example: Two-Point Moving Average

$$x[t]$$
$$x[t-\Delta t]$$



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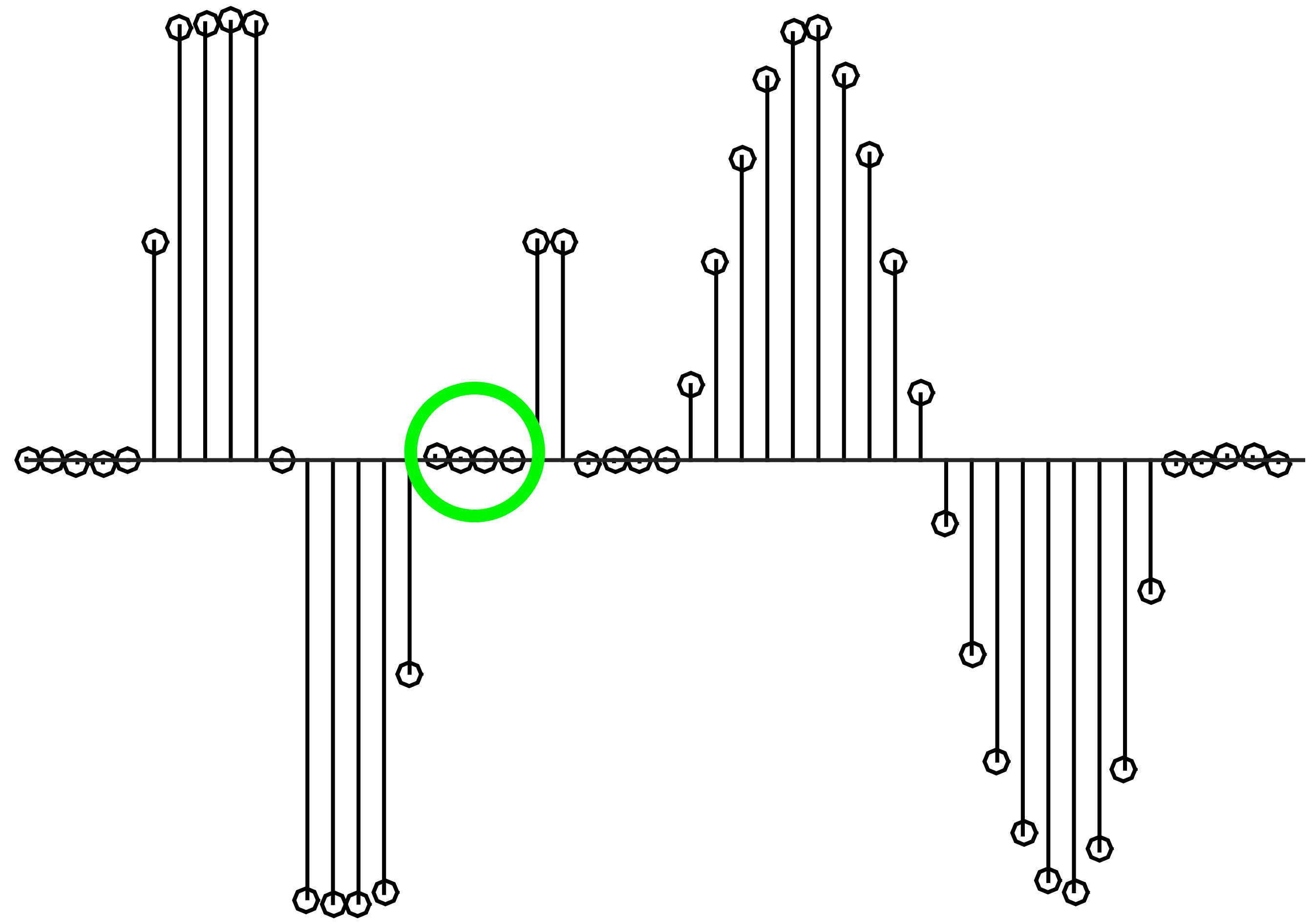
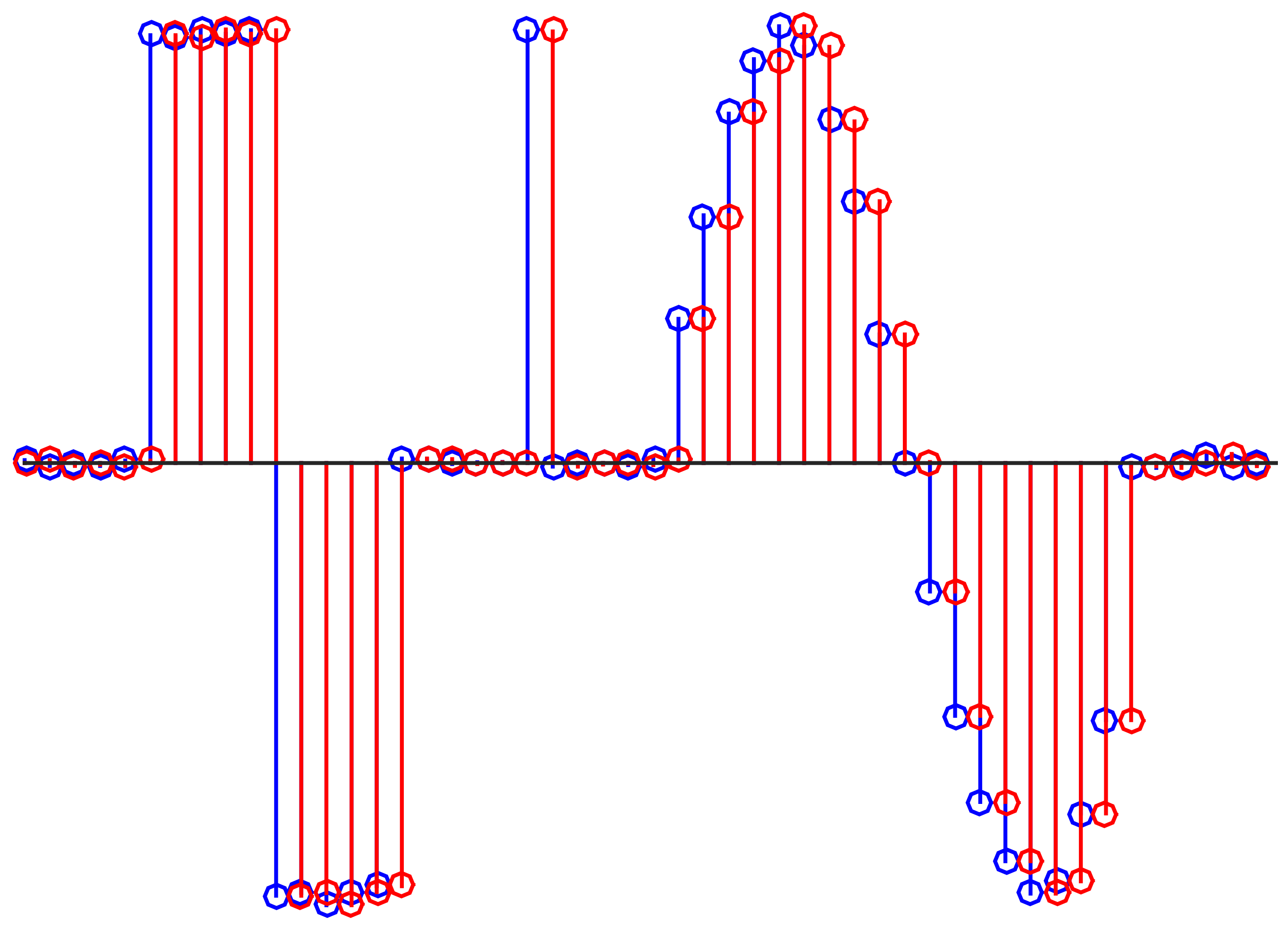
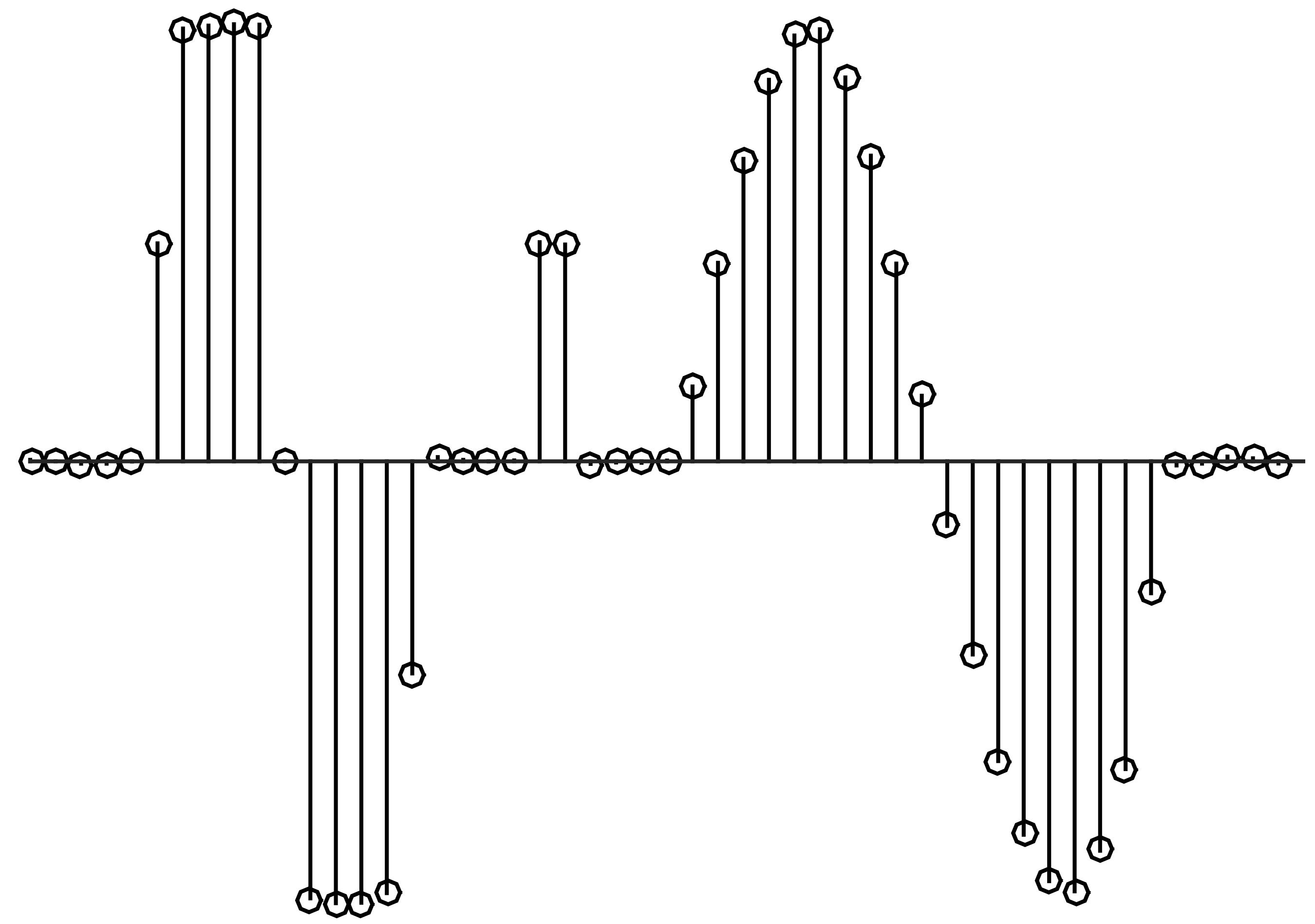


Example: Two-Point Moving Average

$x[t]$
$x[t - \Delta t]$

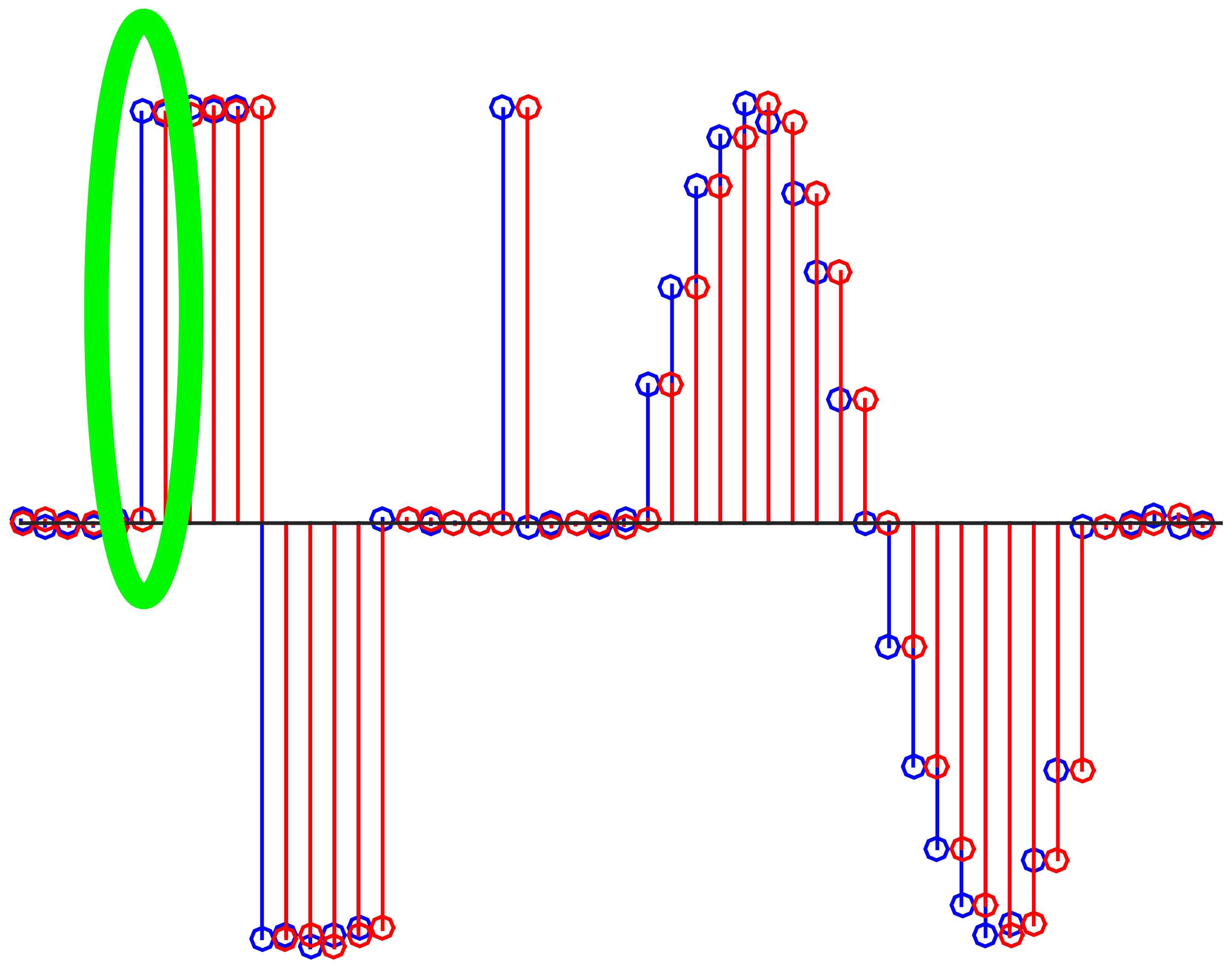


$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t - \Delta t]$$



Example: Two-Point Moving Average

$x[t]$
$x[t-\Delta t]$



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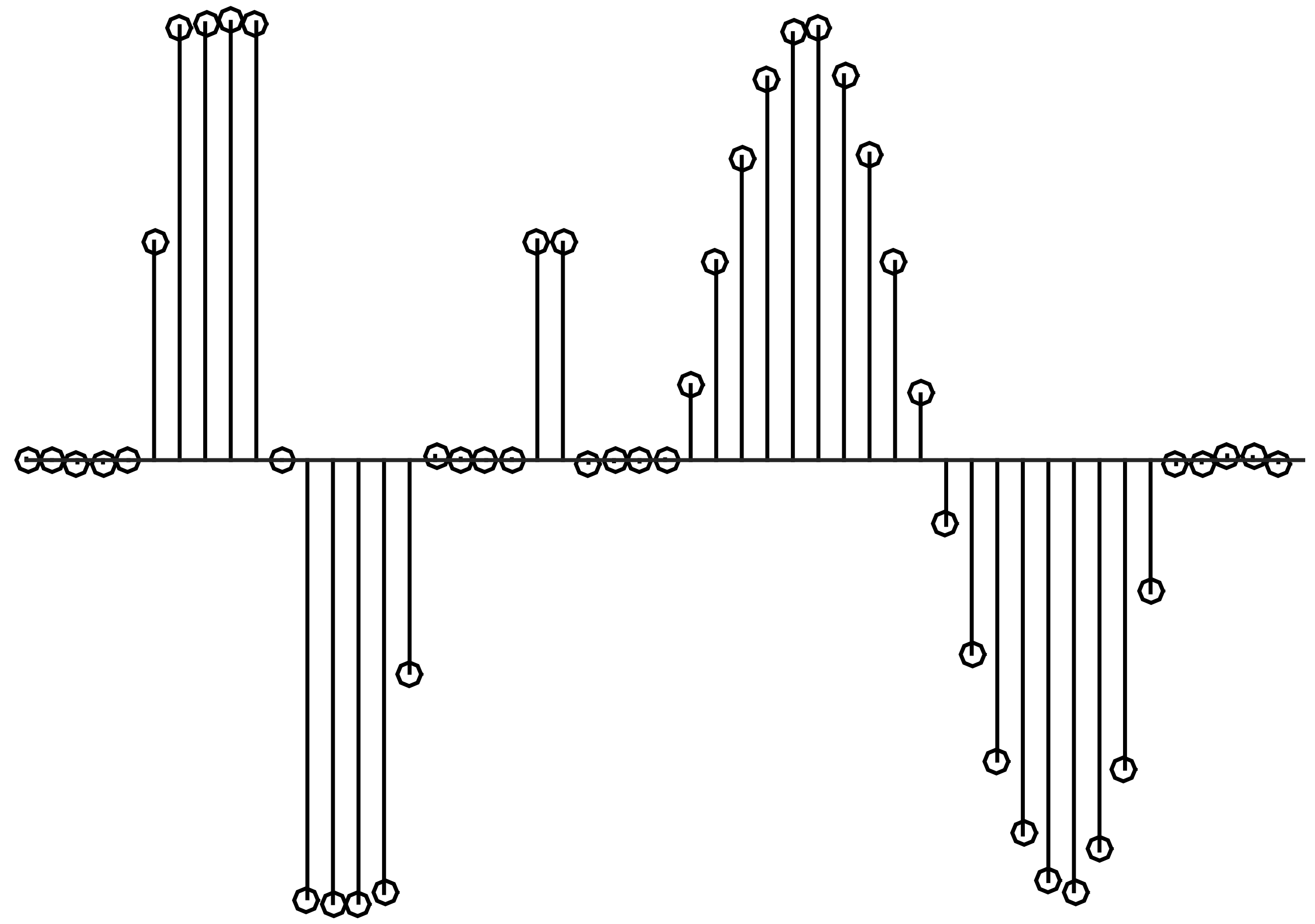
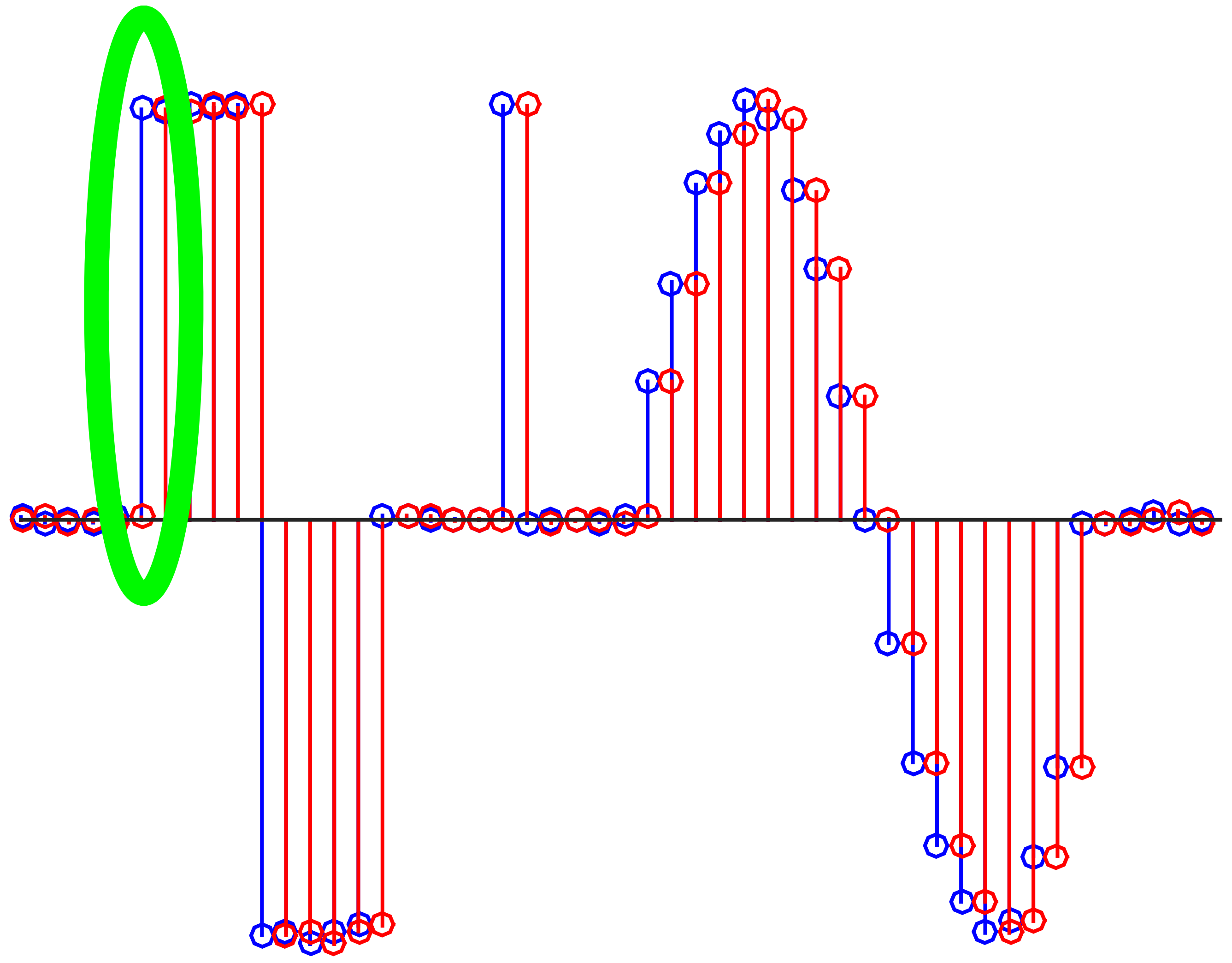


Example: Two-Point Moving Average

$x[t]$
$x[t-\Delta t]$



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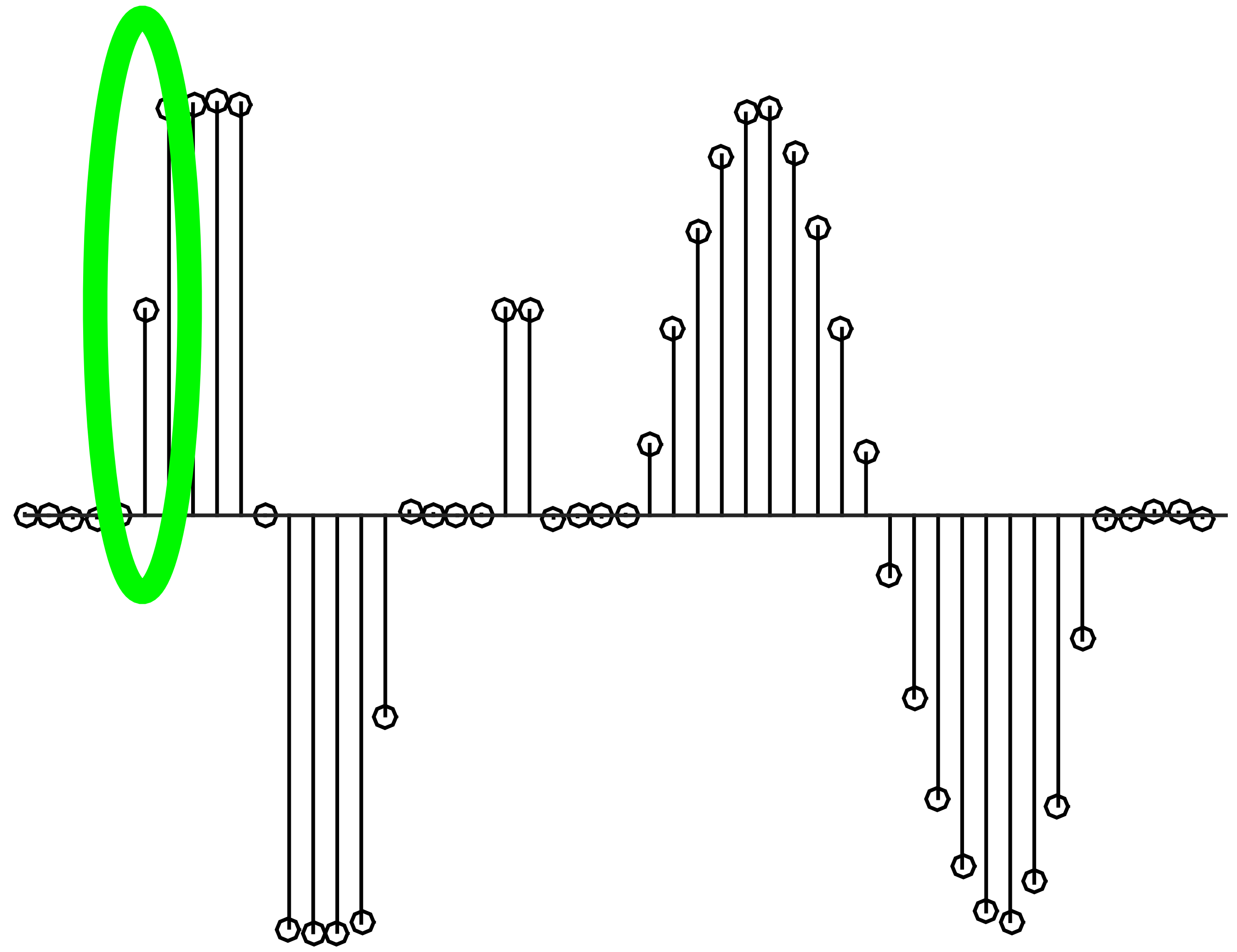
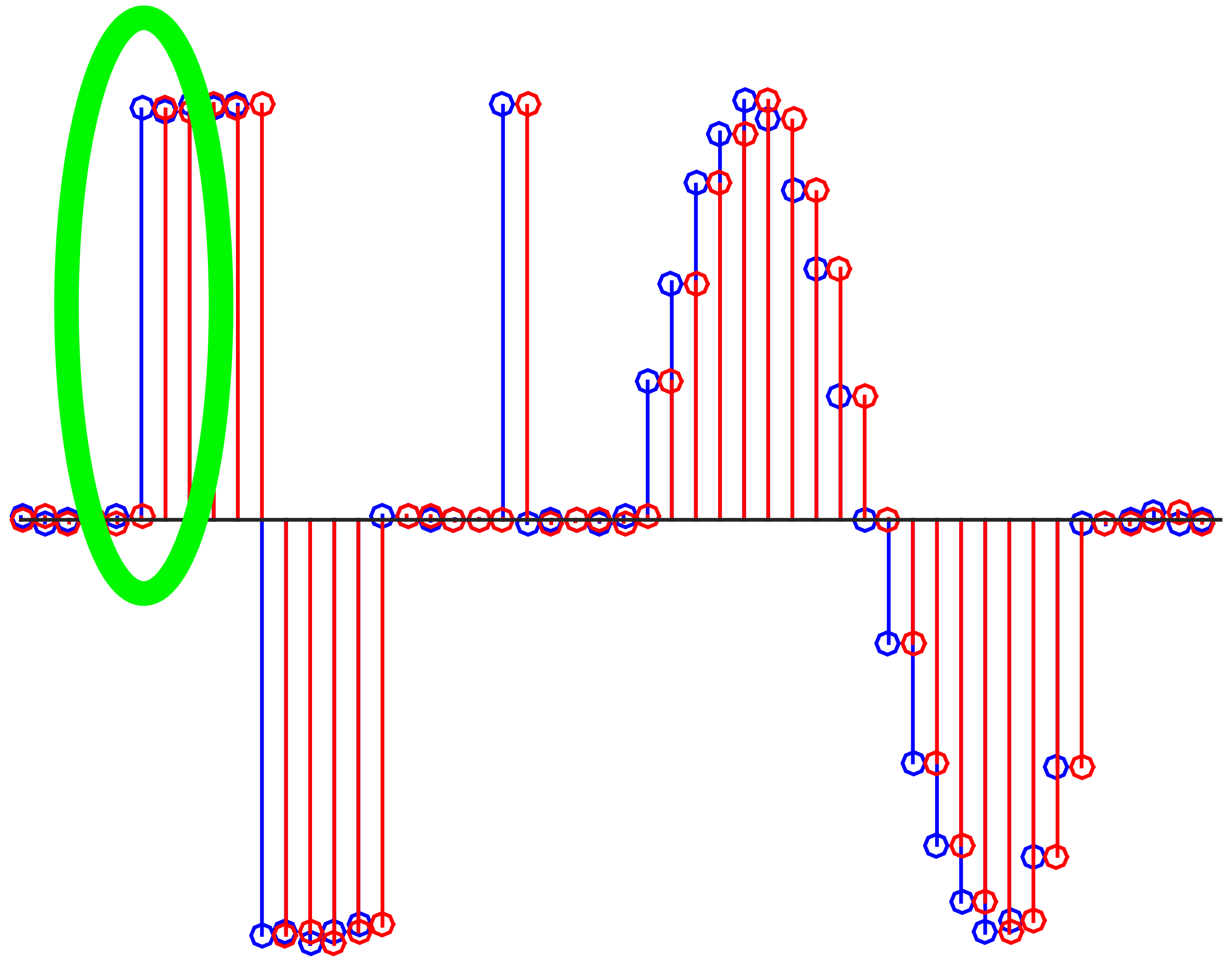
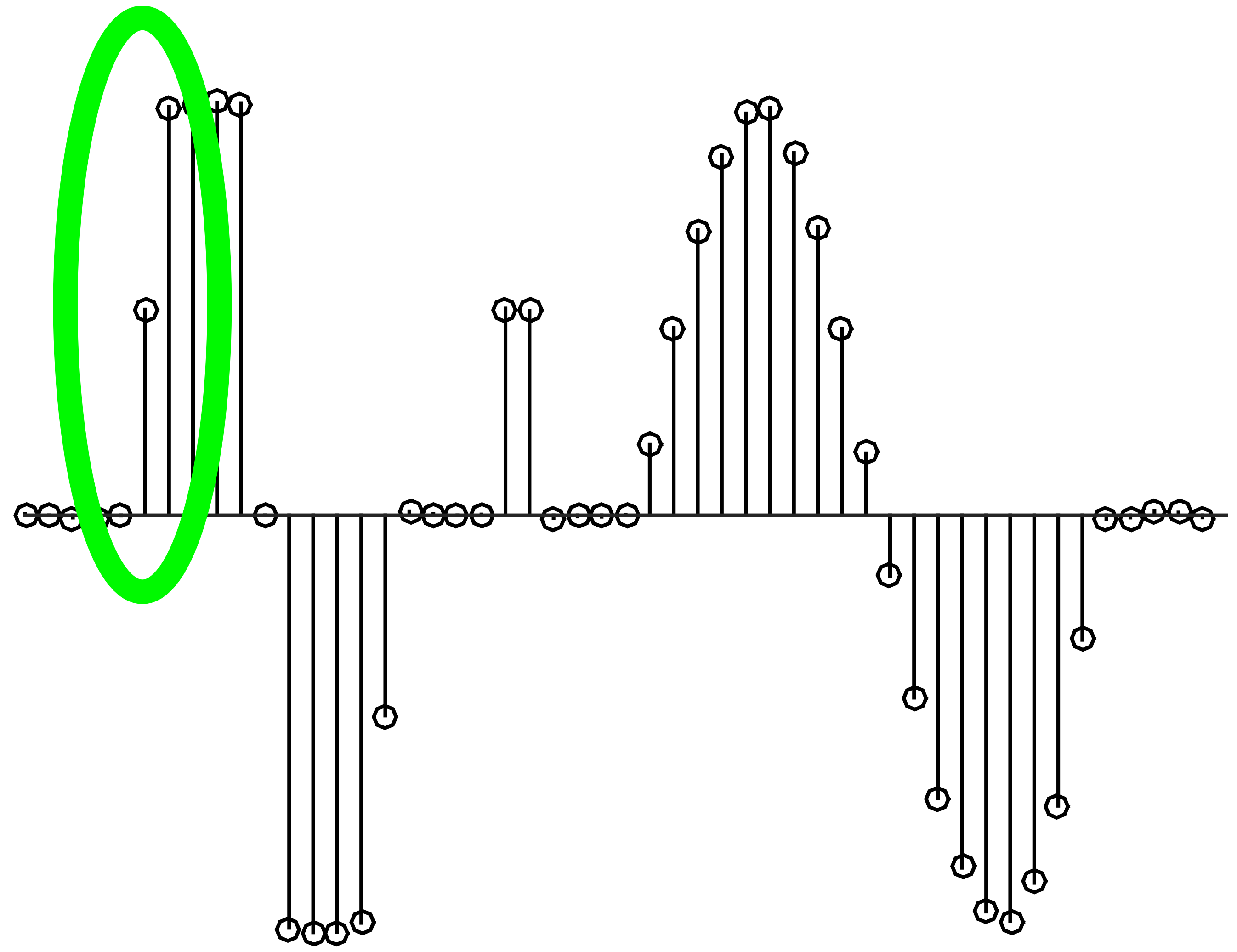


Example: Two-Point Moving Average

$x[t]$
$x[t-\Delta t]$



$$y[t]=\frac{1}{2}x[t]+\frac{1}{2}x[t-\Delta t]$$



Example: Two-Point Moving Average

$x[t]$
$x[t-\Delta t]$

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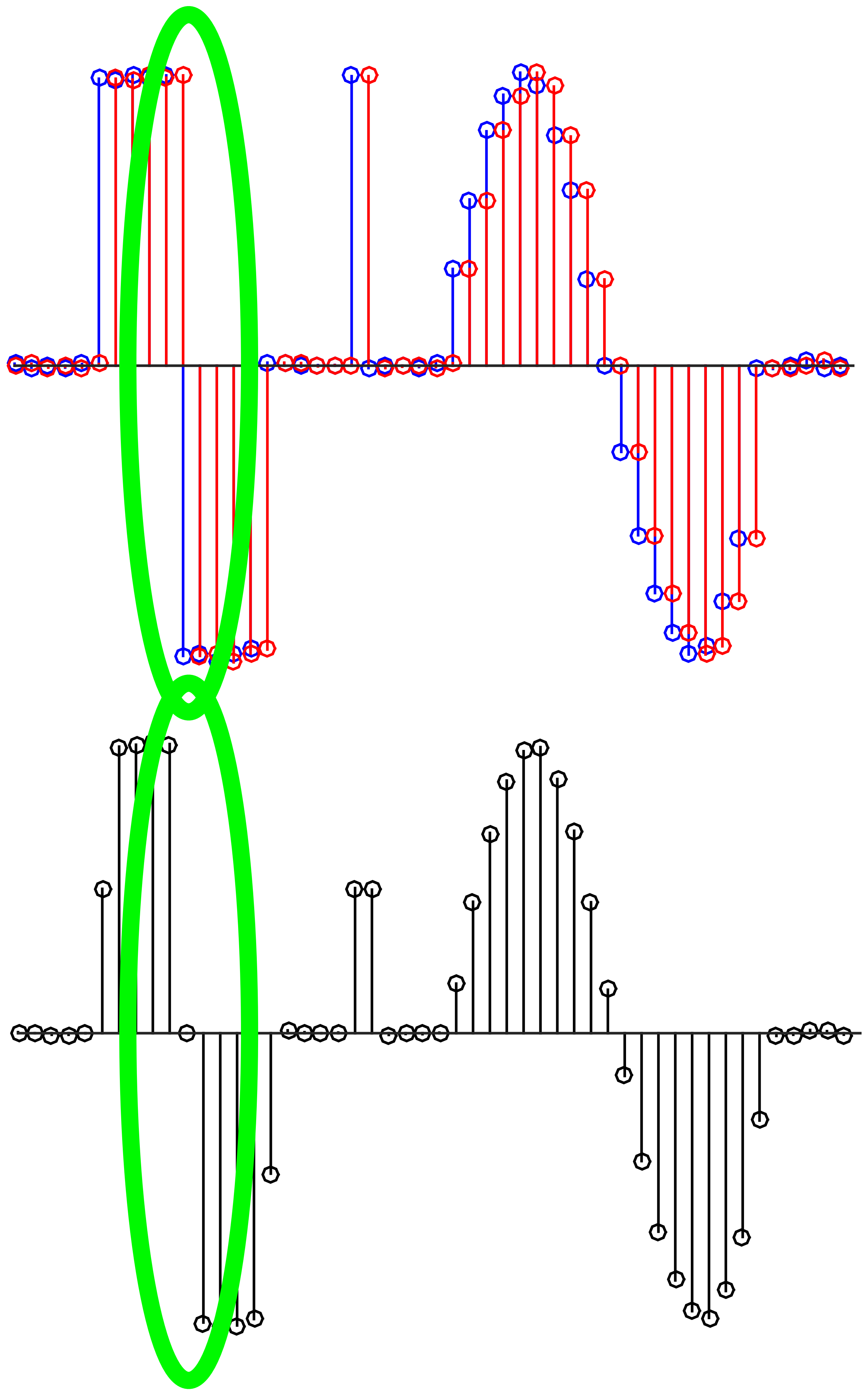
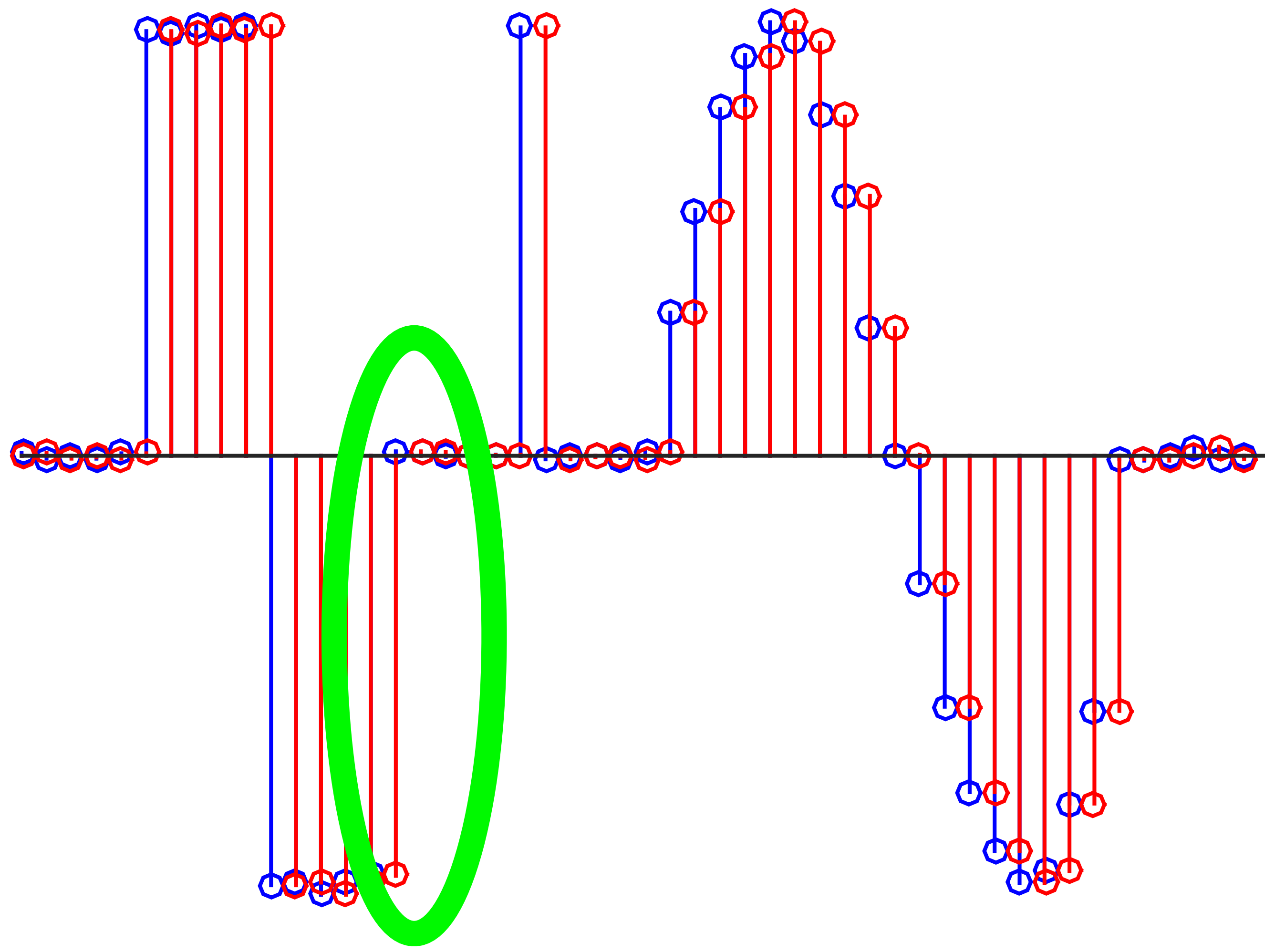
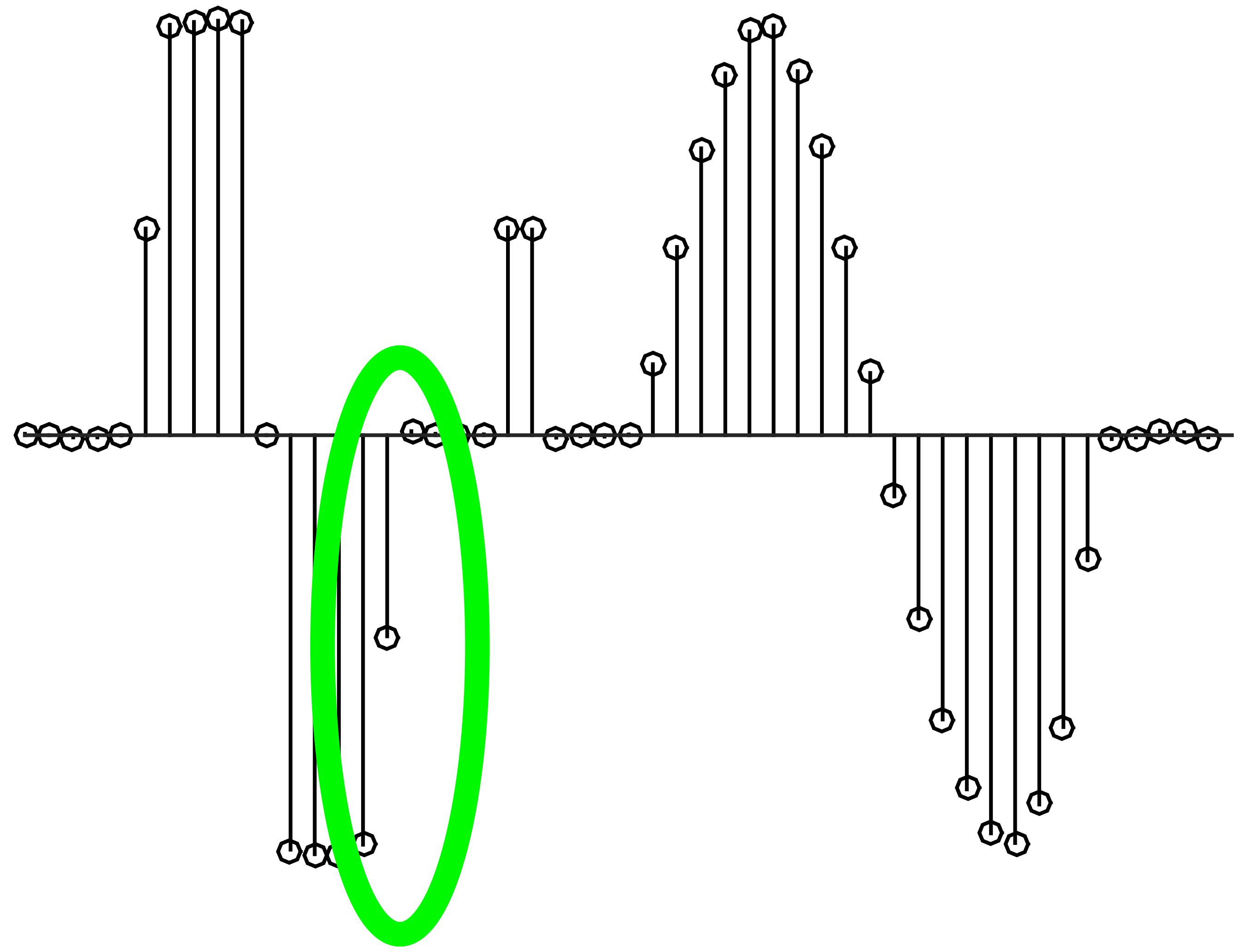


Example: Two-Point Moving Average

$x[t]$
$x[t - \Delta t]$



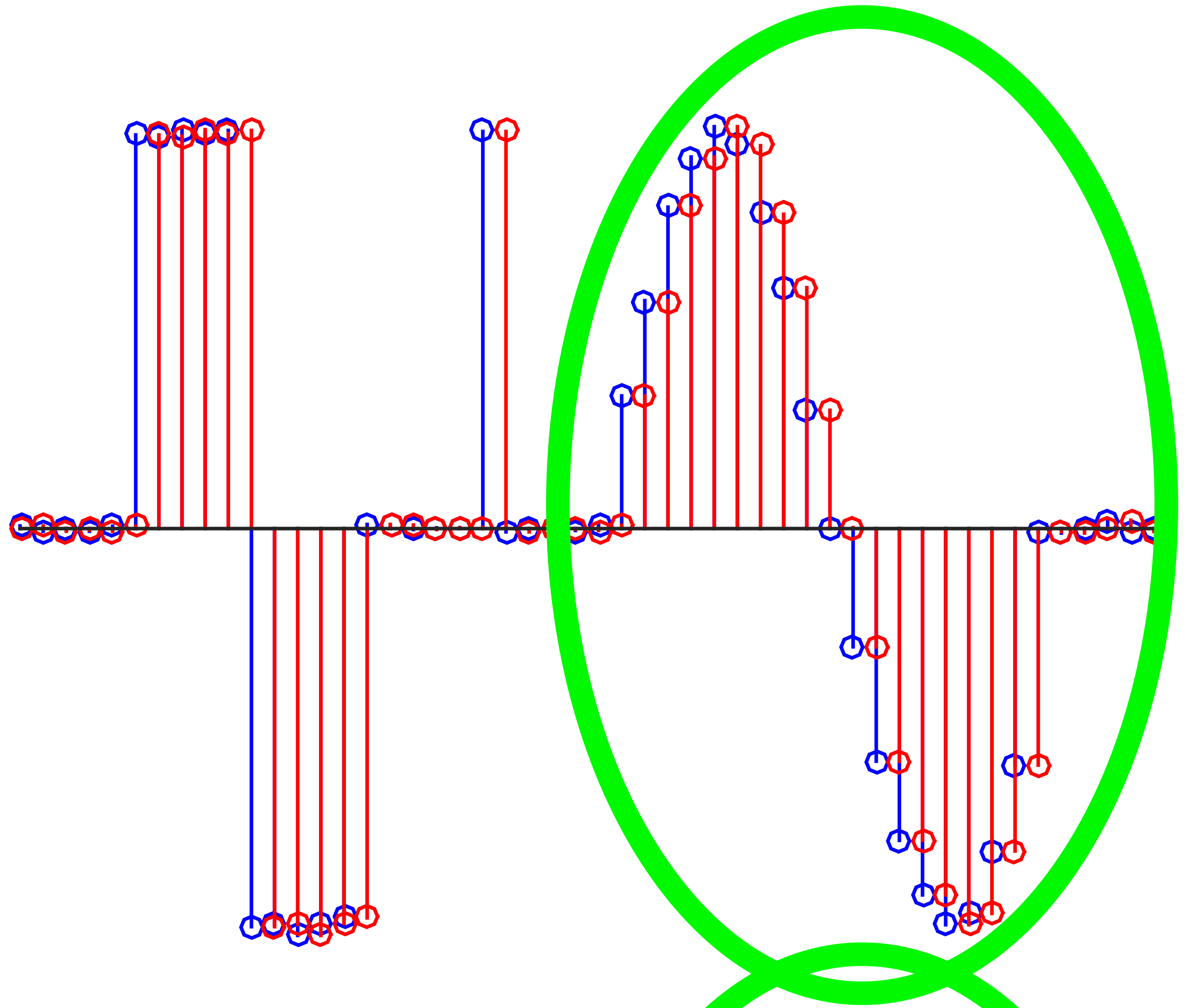
$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t - \Delta t]$$



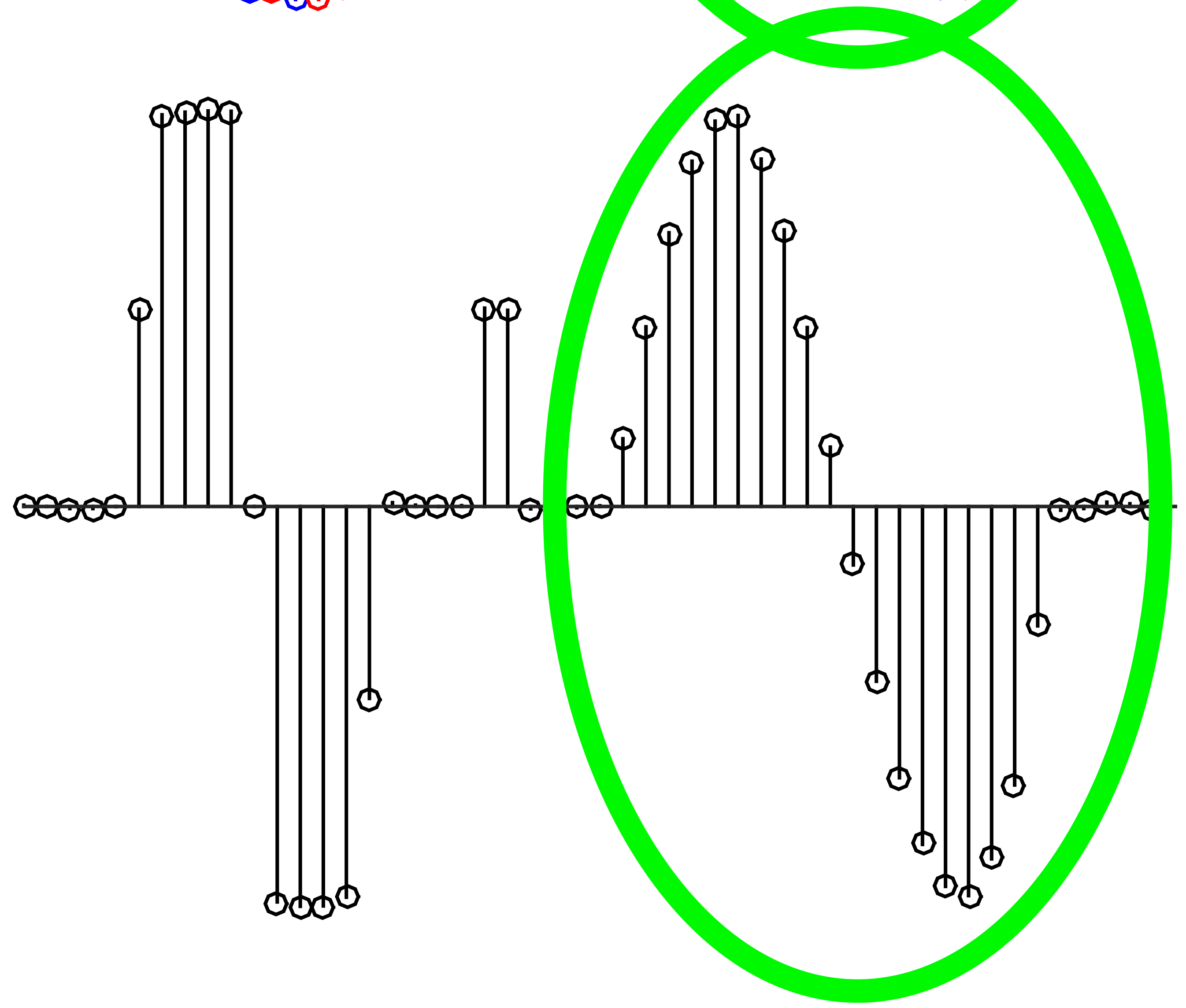
Example: Two-Point Moving Average

$x[t]$

$x[t - \Delta t]$

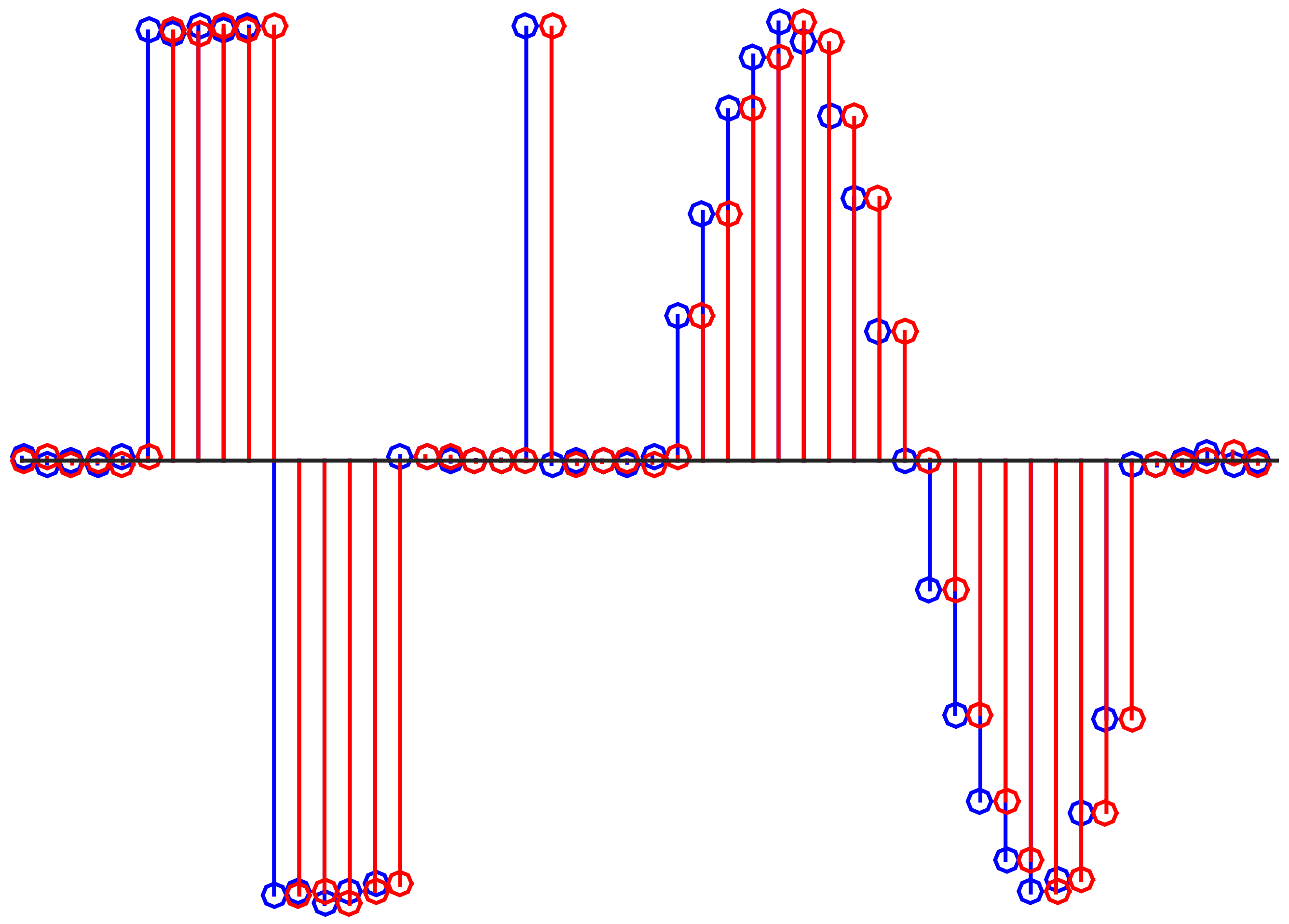


$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t - \Delta t]$$



Example: Two-Point Moving Average

$$x[t]$$
$$x[t-\Delta t]$$



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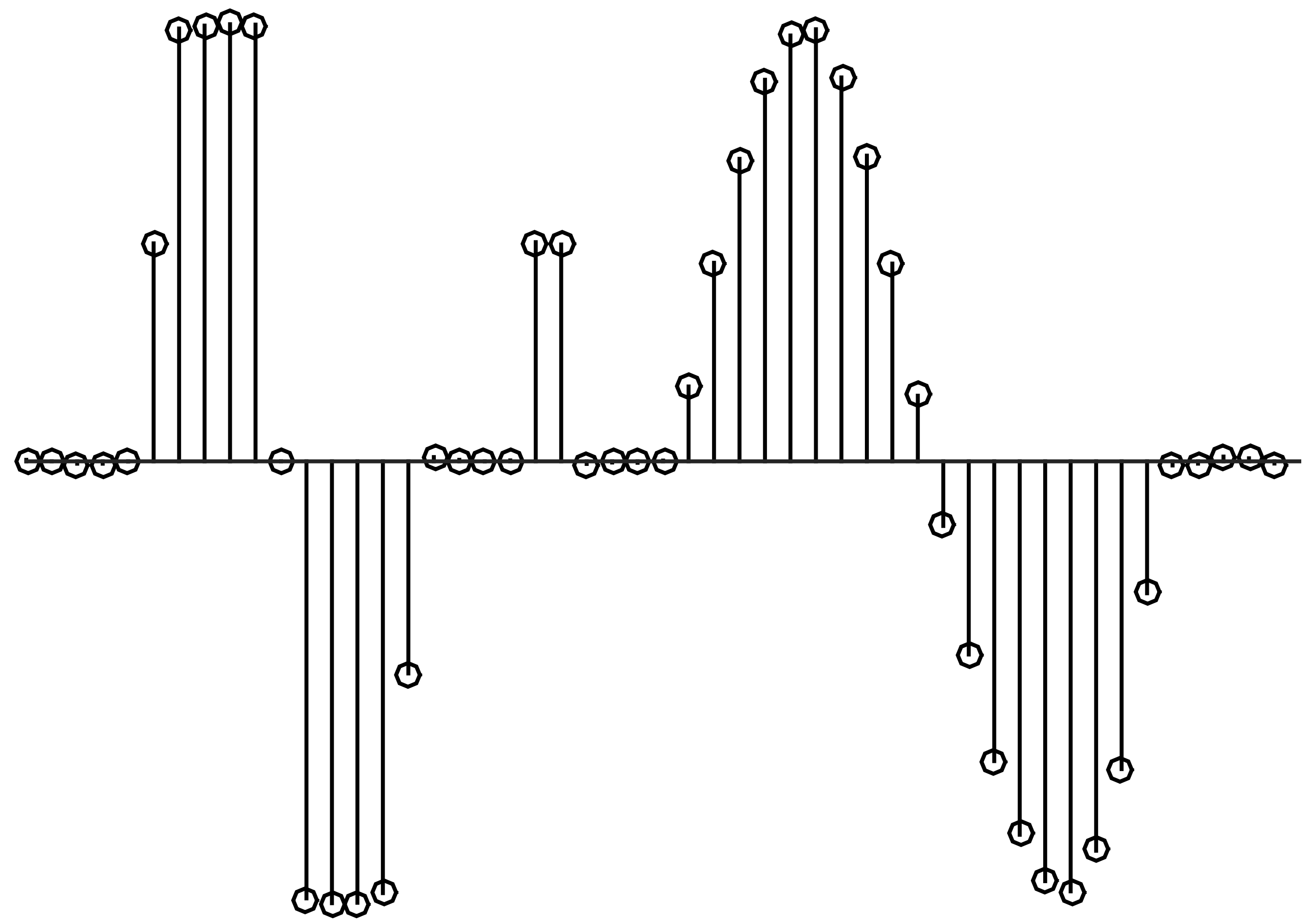
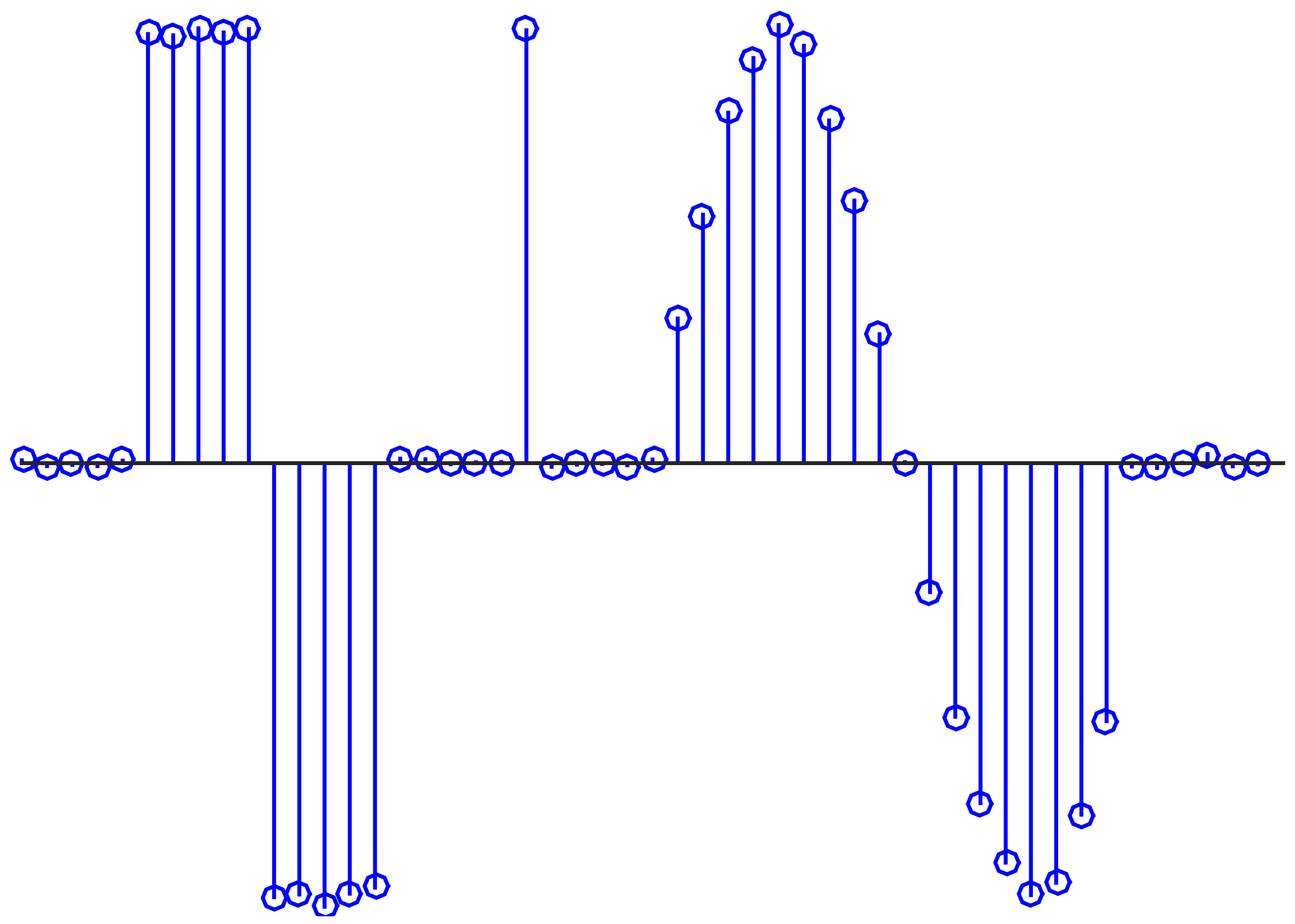


Example: Two-Point Moving Average

$$x[t]$$



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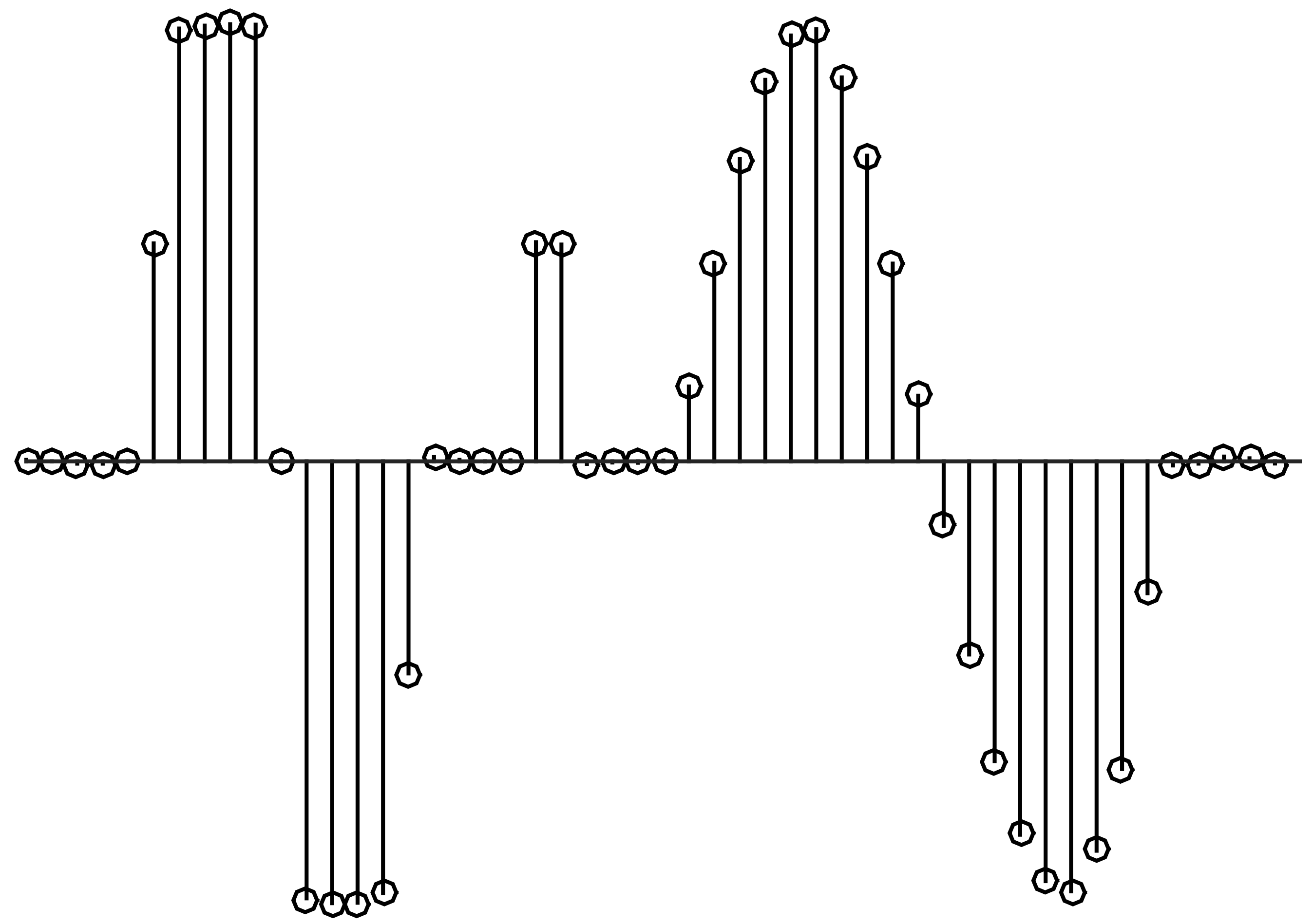
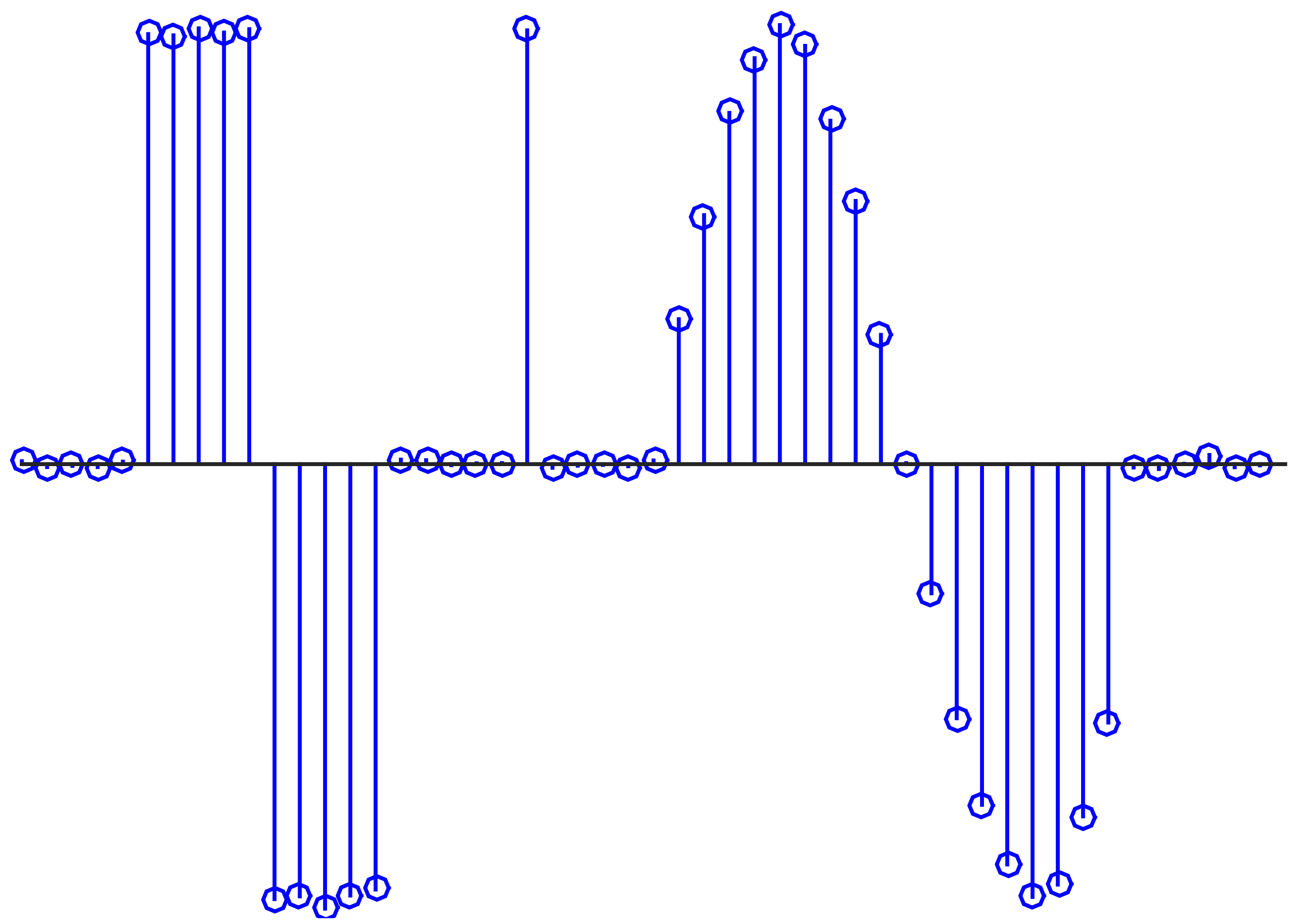


Results:

- Softens sudden changes
- Leaves slowly varying signals largely unchanged
- Slight delay in output relative to input
- Low Pass Filter?

Example: Two-Point Moving Average

$x[t]$



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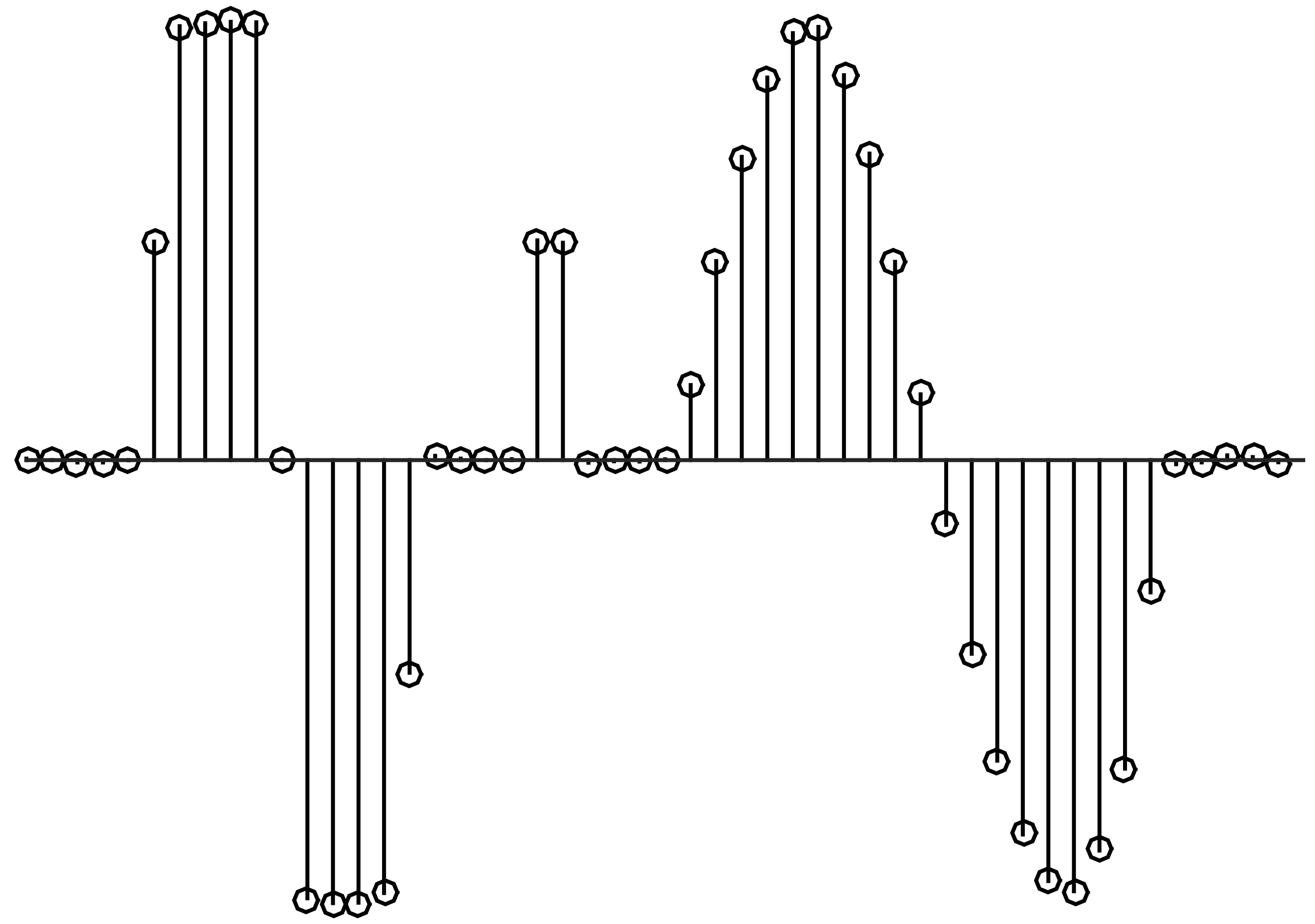

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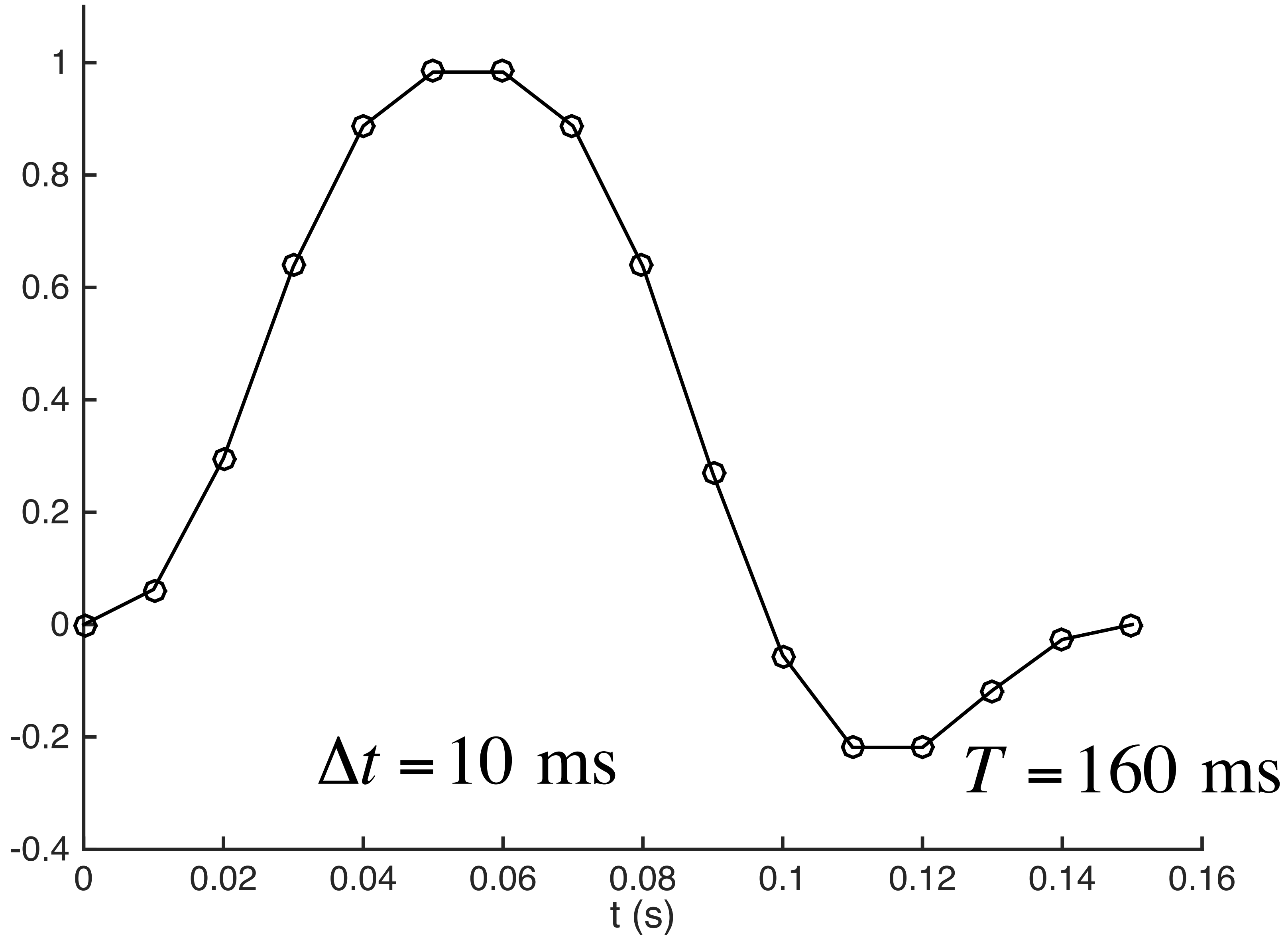
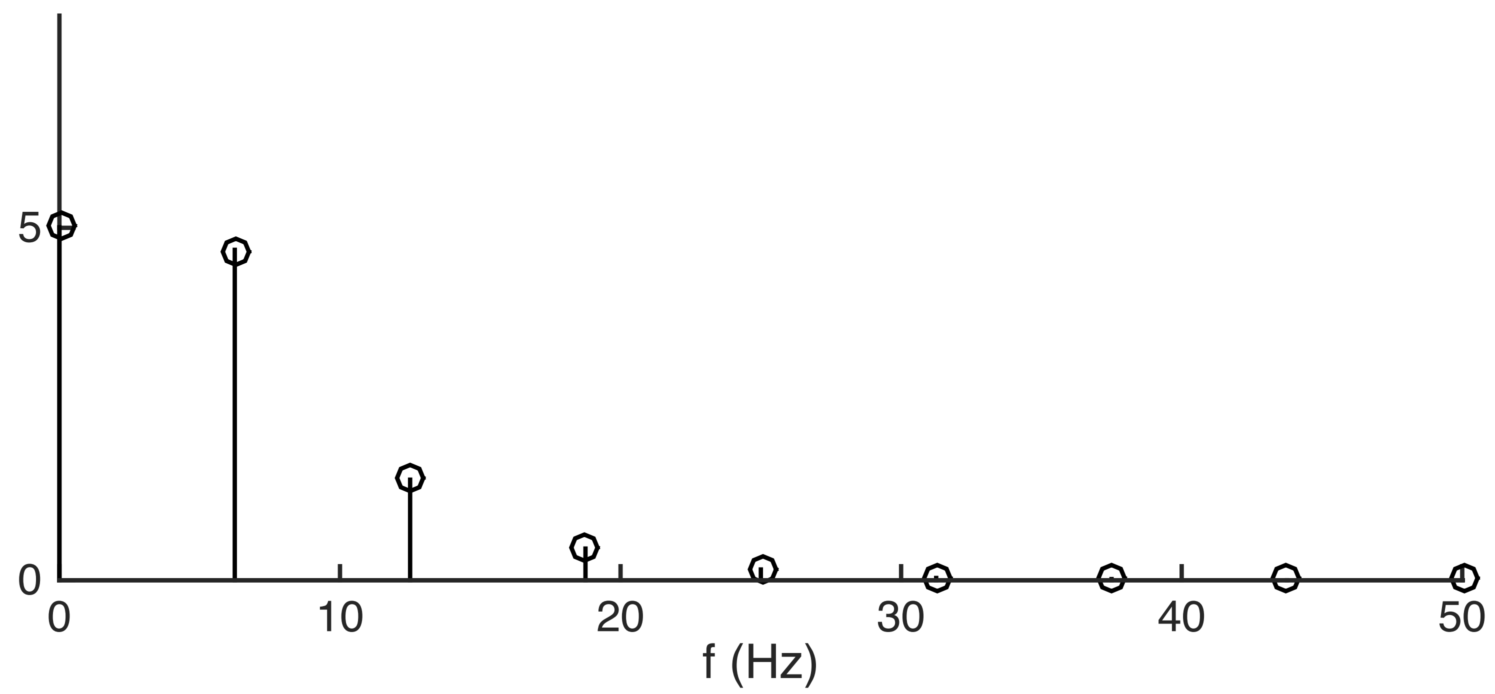
The Fourier Transform

- **Every** Time-Domain Signal can be Re-expressed as a Sum of Sinusoids/Oscillations
- # of time points = # of frequencies
- Reciprocal relationship: *time resolution* (Δt) & *frequency span* (f_s)
- Reciprocal relationship: *frequency resolution* (Δf) & *time span* (T)





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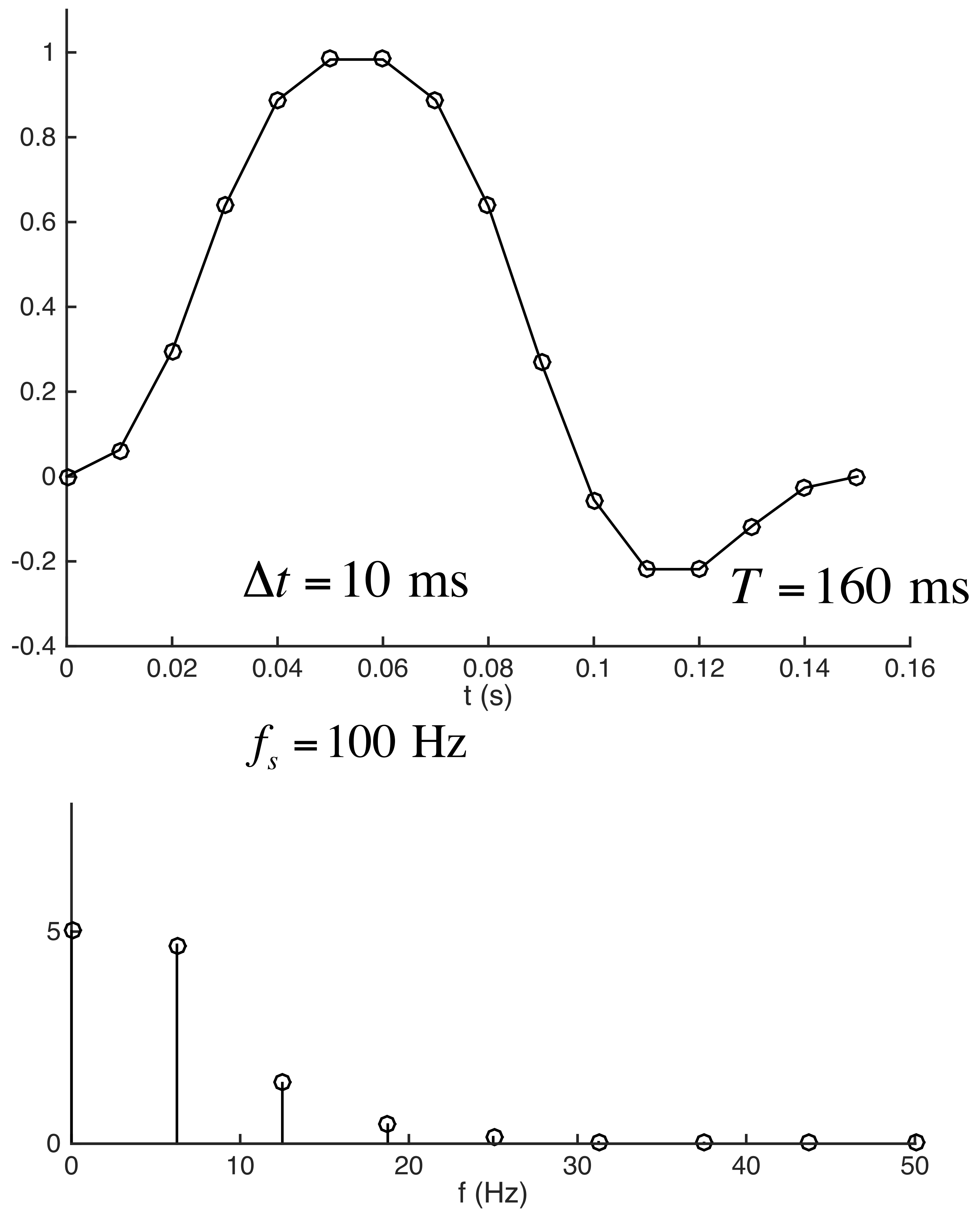

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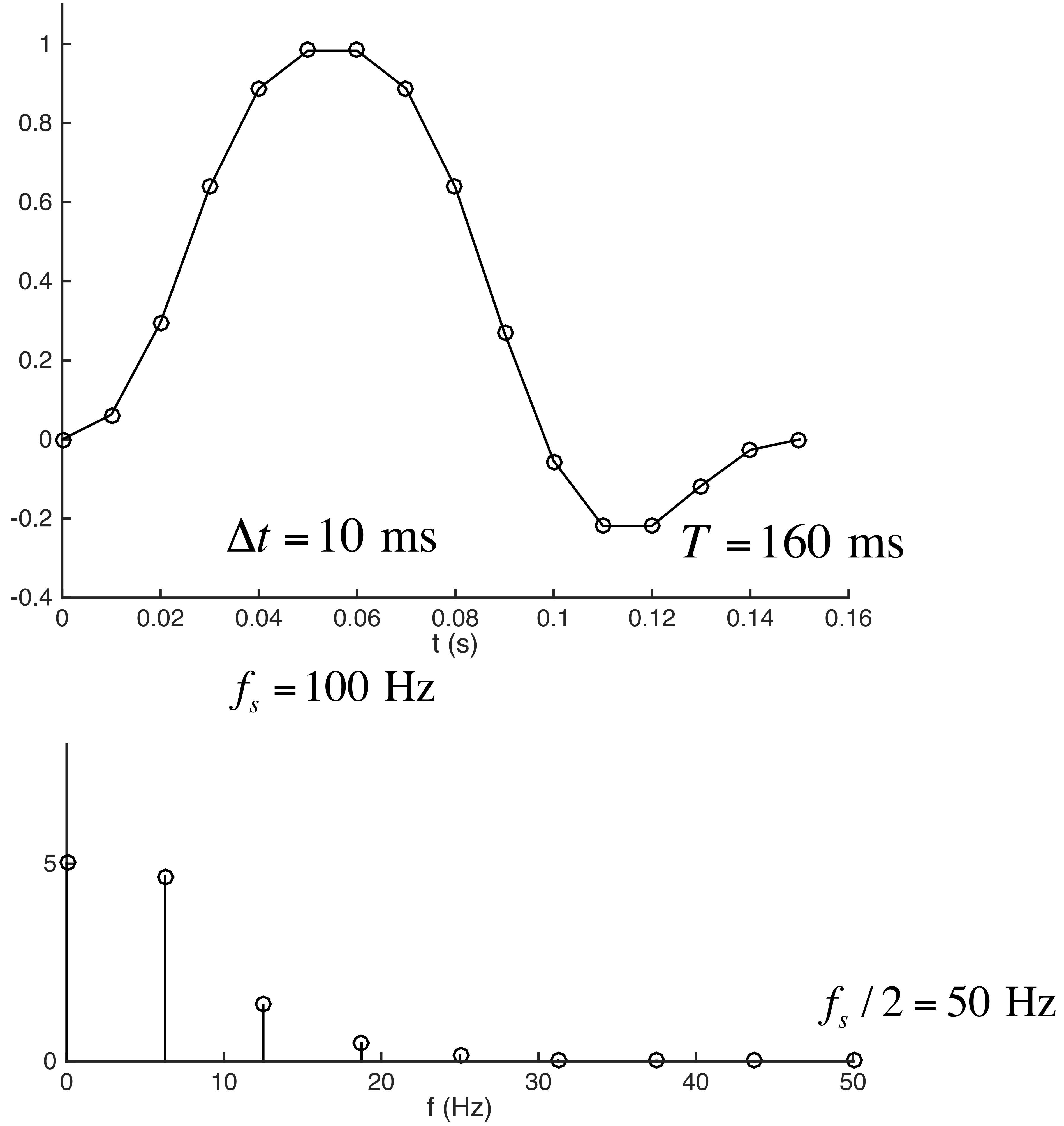

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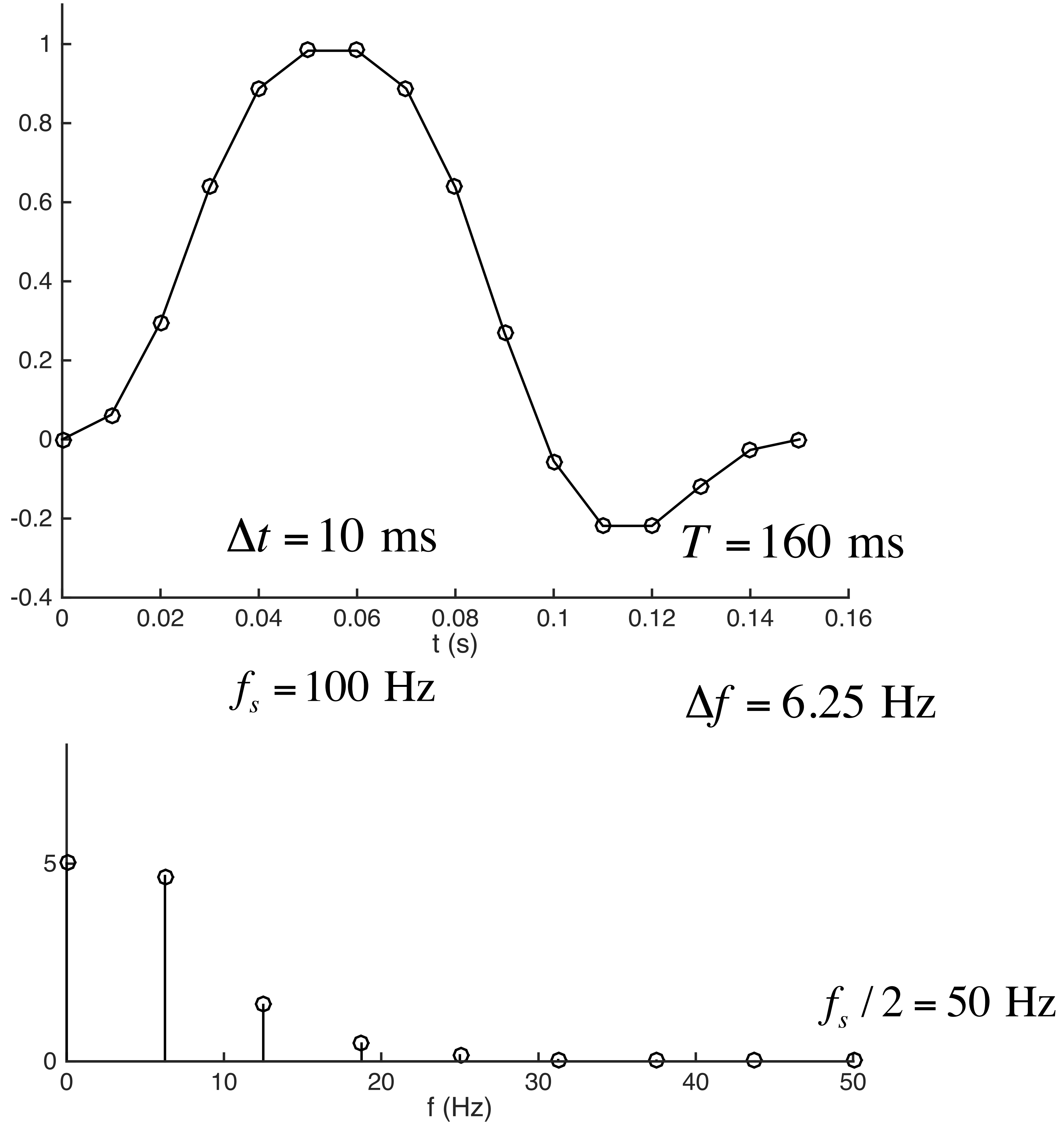

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$$x[t] = \frac{1}{N}\sum_{k=0}^{N-1} X[f_k]e^{i2\pi f_k t} \quad \text{where:}$$

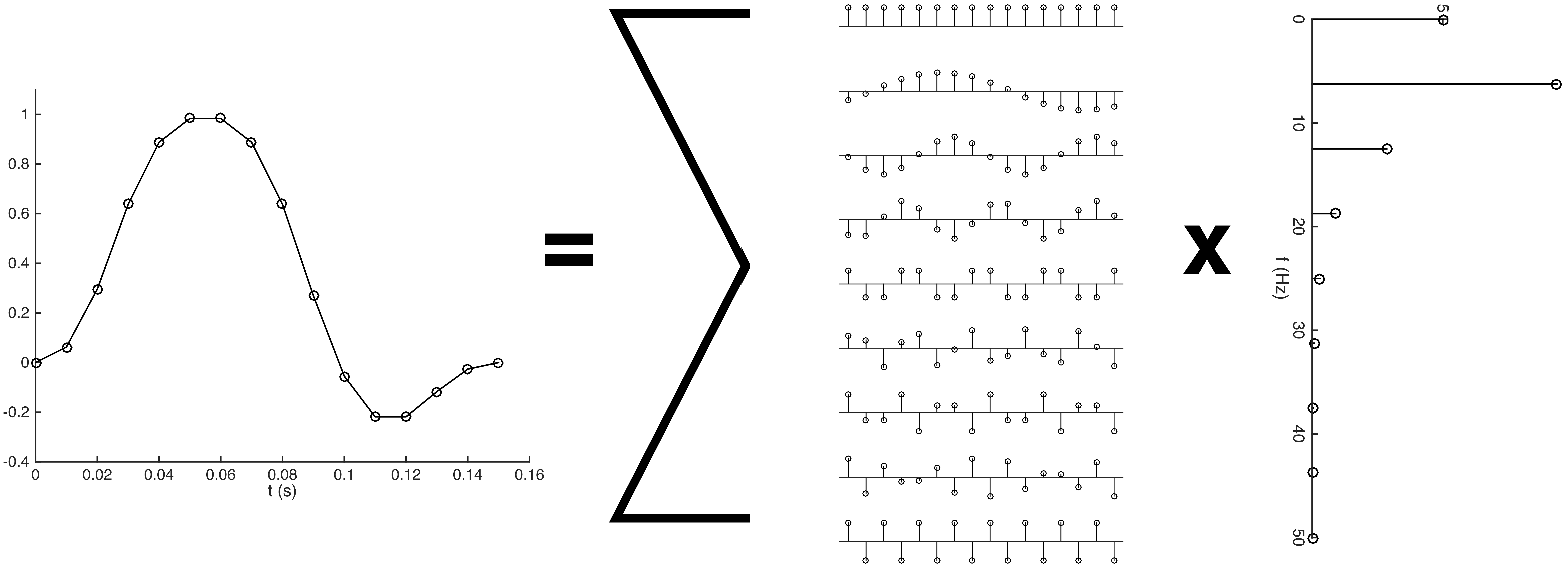
$$t = \underbrace{0, \Delta t, 2\Delta t, \ \ldots \ , T - \Delta t}_{N}$$

$$f_k = \underbrace{0, \Delta f, 2\Delta f, \ \ldots \ , f_s - \Delta f}_{N}$$

$$f_s = \textit{sampling frequency} = \frac{1}{\Delta t}$$

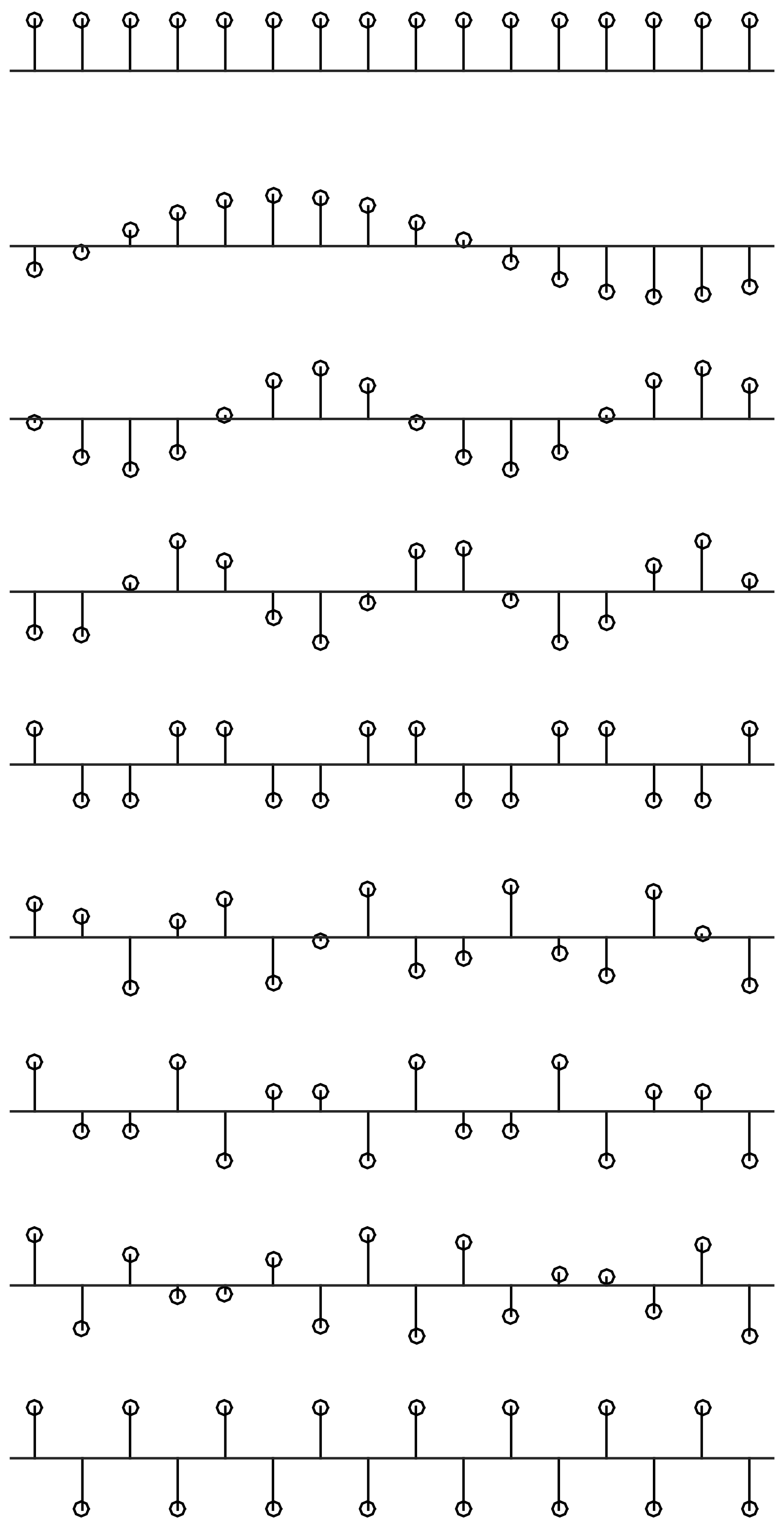
$$T = \textit{signal duration} = \frac{1}{\Delta f}$$

The Fourier Transform



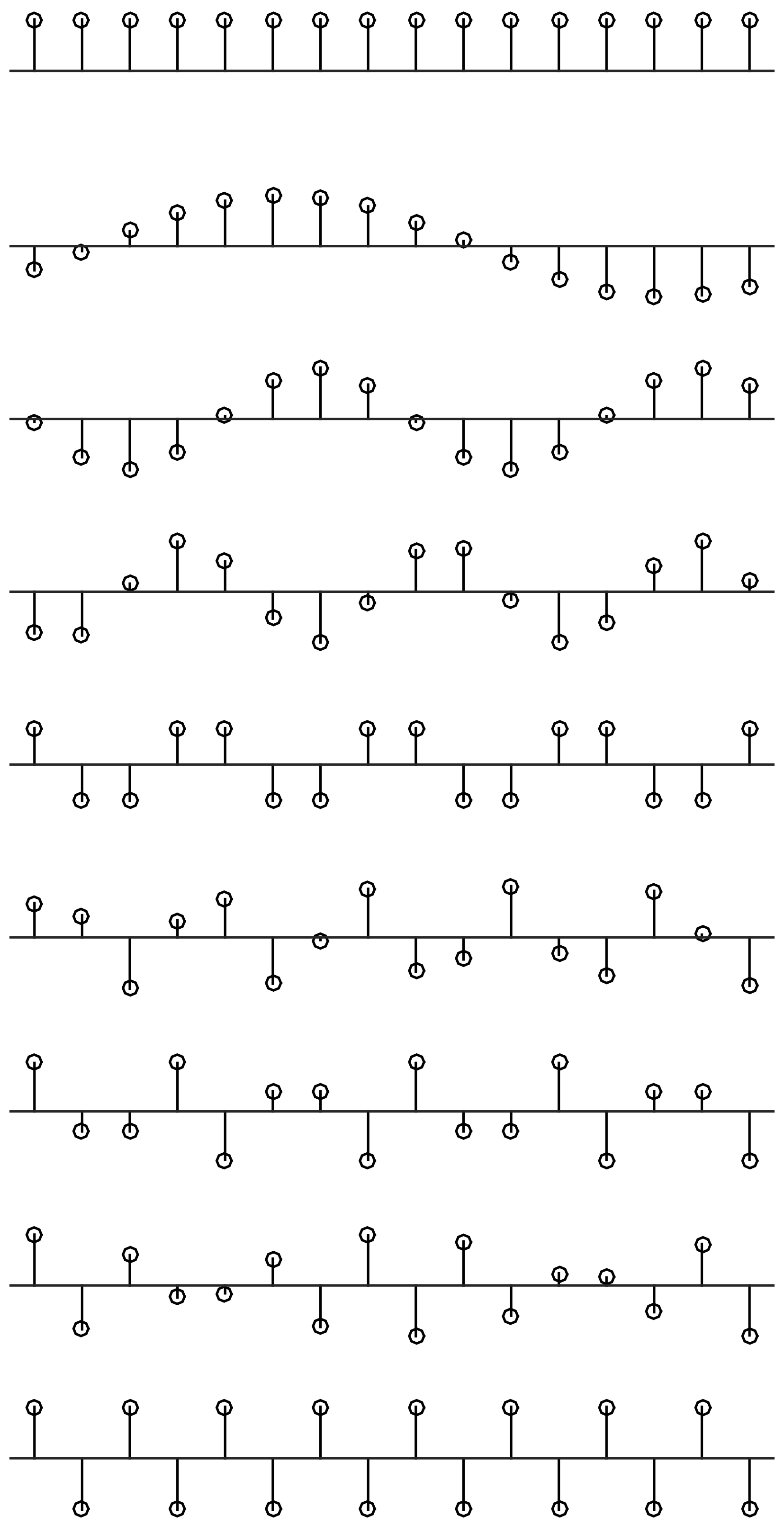
The Fourier Transform

Every Signal $=$ $\sum$



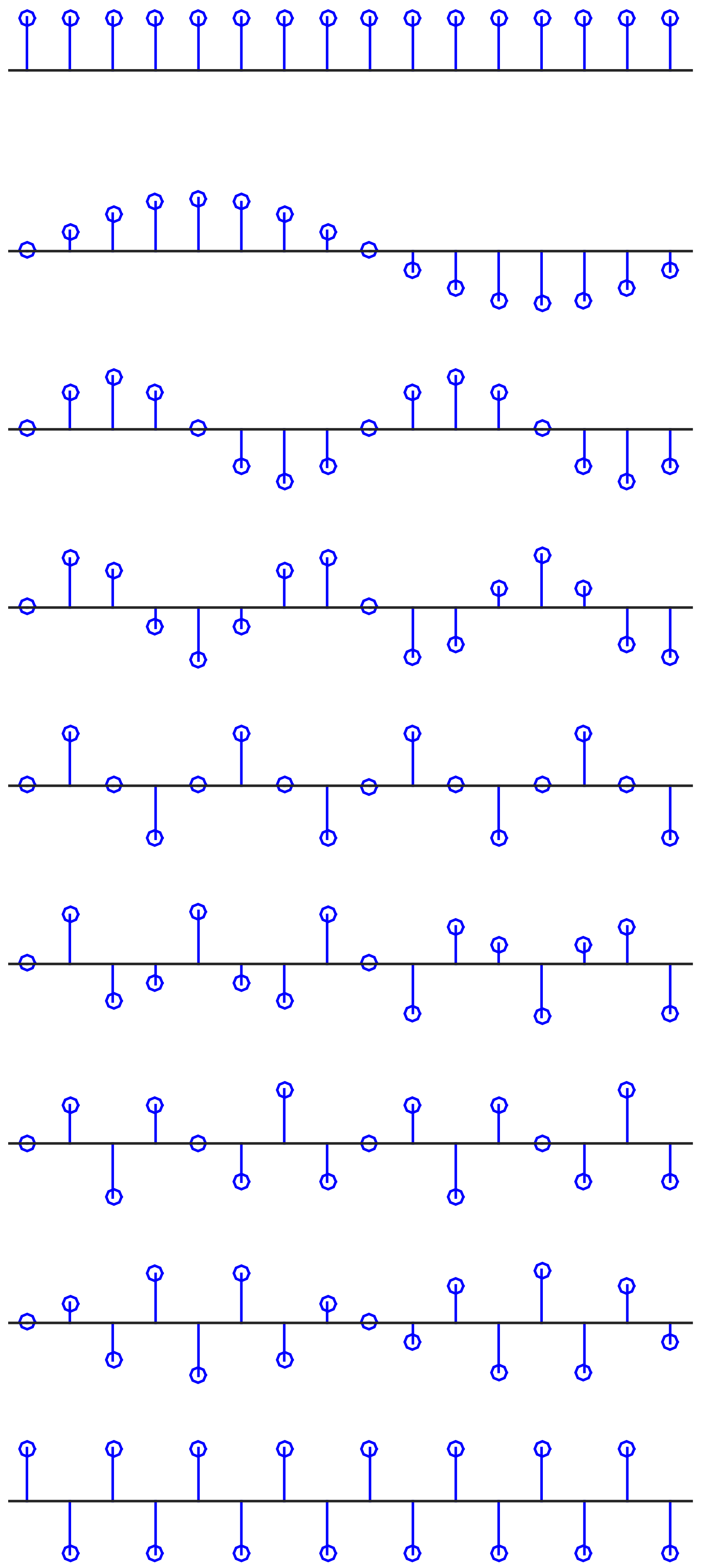
x Fourier Coefficients

Filters and the Fourier Transform

Filter(signal) **=** $\sum$ Filter() **x** Fourier Coefficients

Filters and the Fourier Transform

So it's really important what the filter does to these:

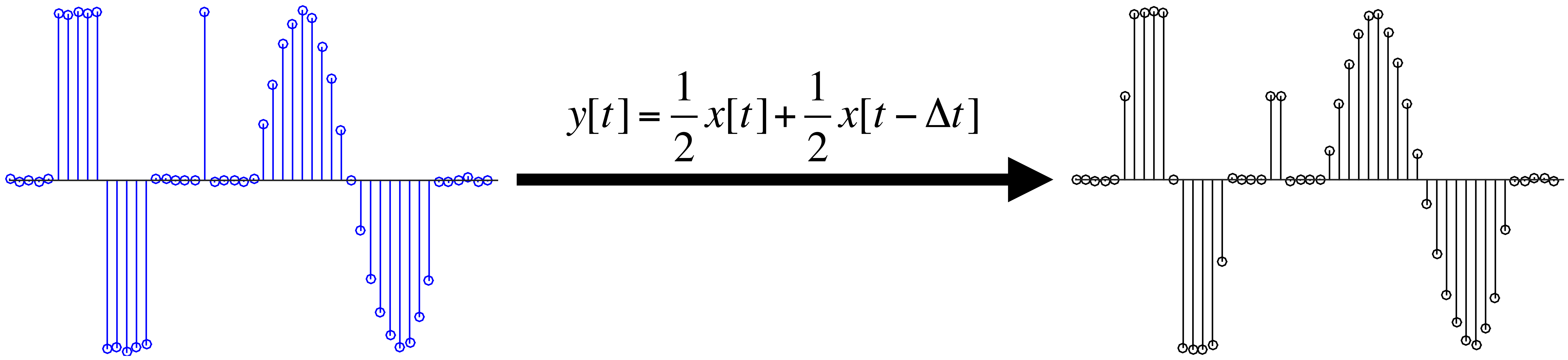


$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t-\Delta t]$$

→ **?**

Filters and the Fourier Transform

Recall:



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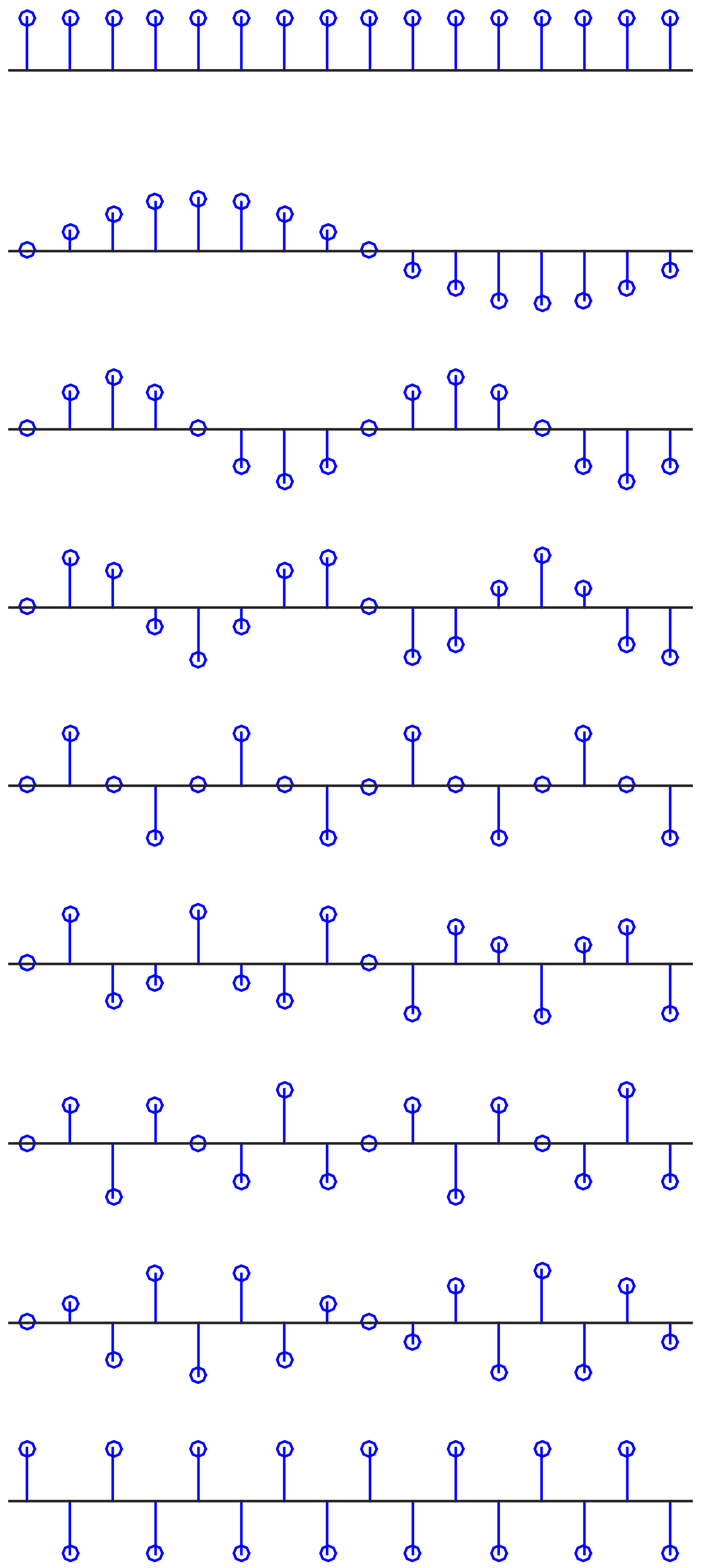

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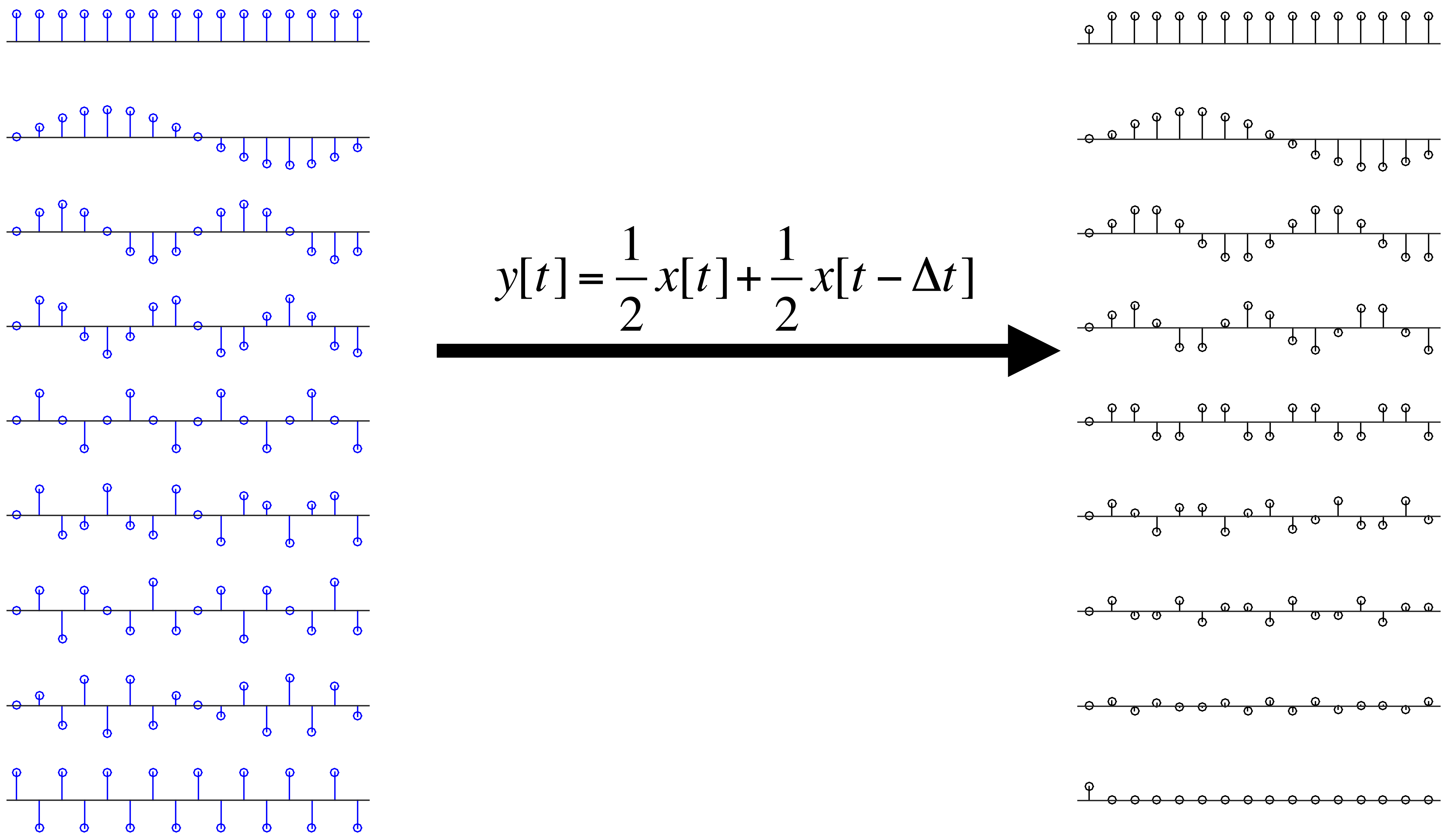

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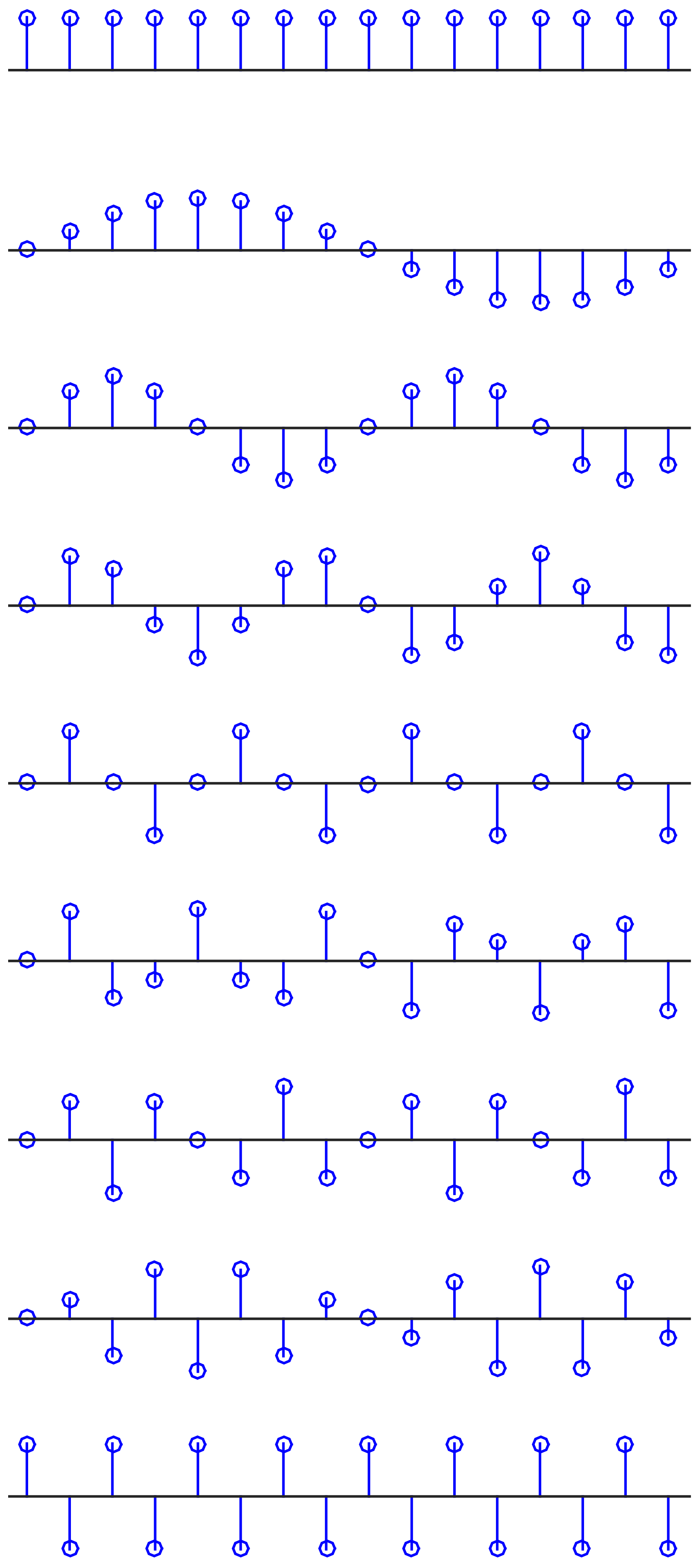

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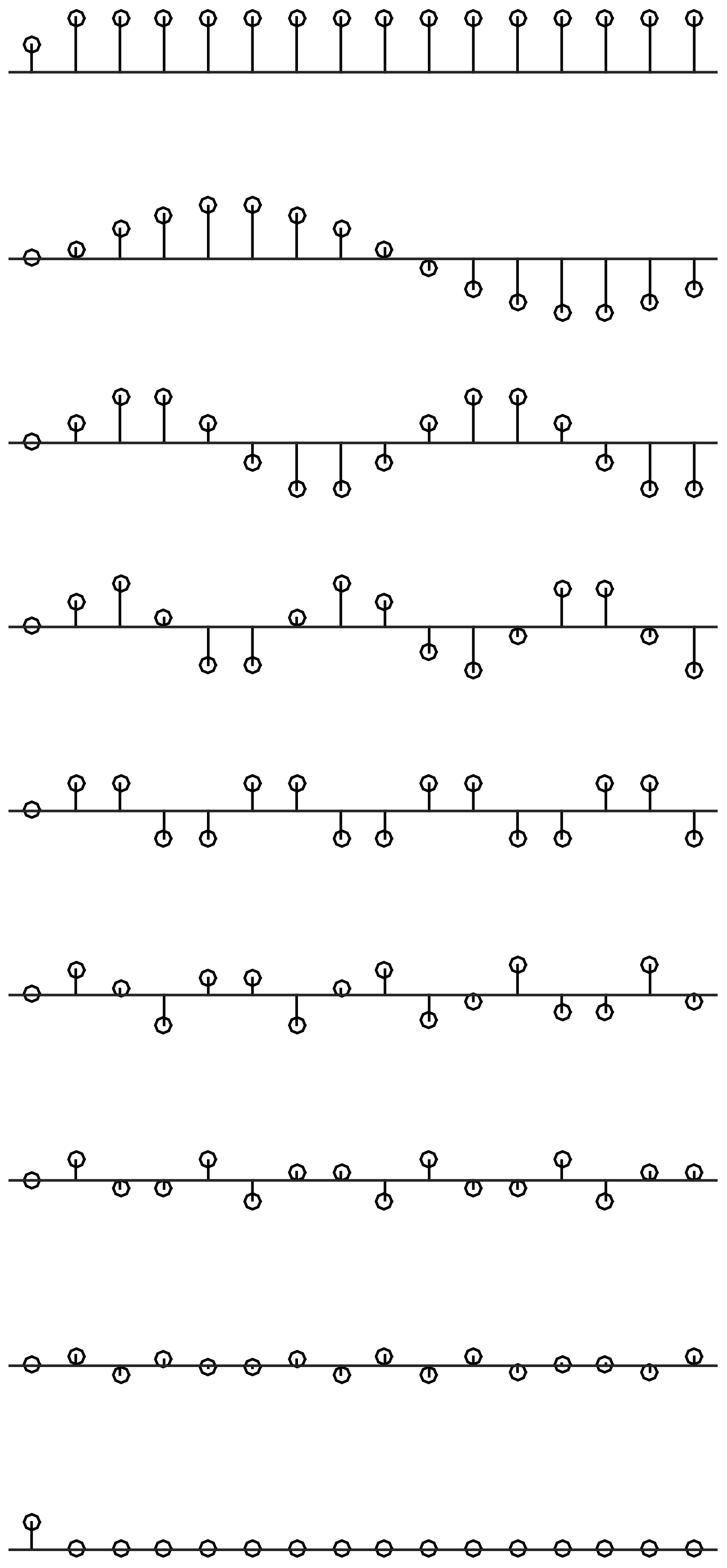

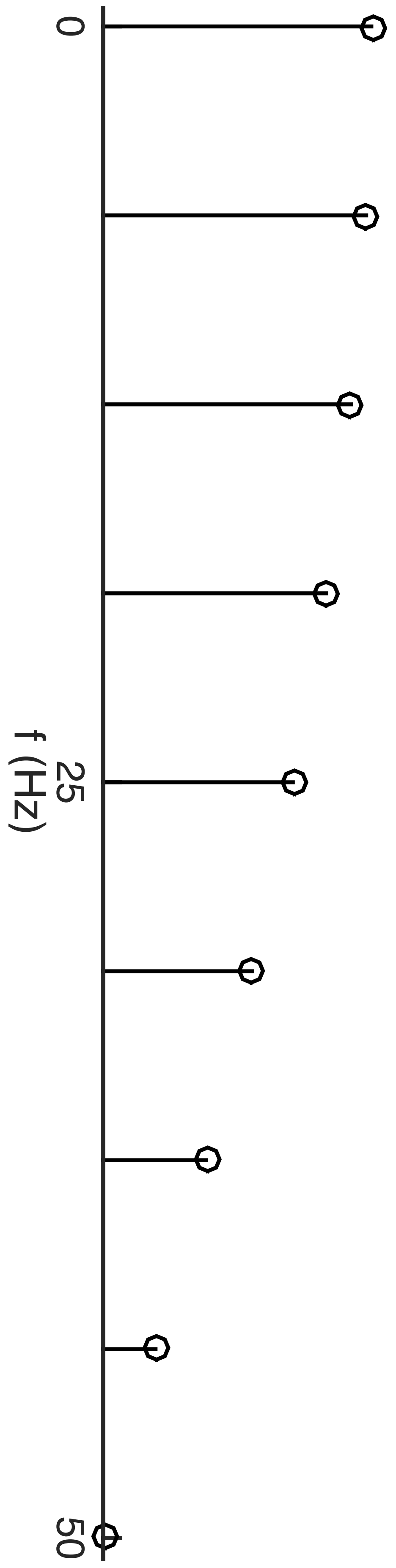

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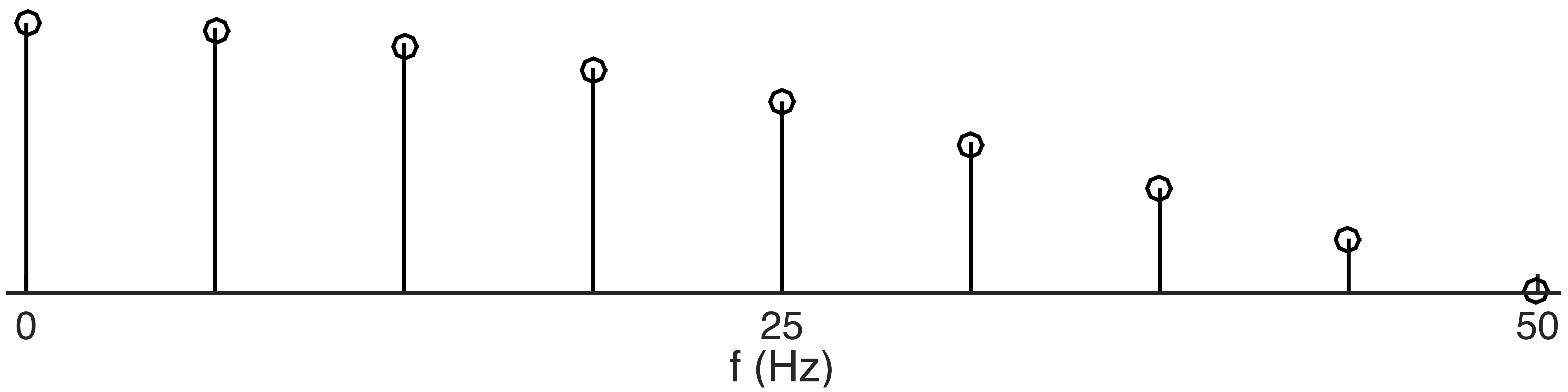


Low Pass Filter

Example: Two-Point Moving Difference

$$y[t] = \frac{x[t] - x[t - \Delta t]}{2}$$

What to Expect:

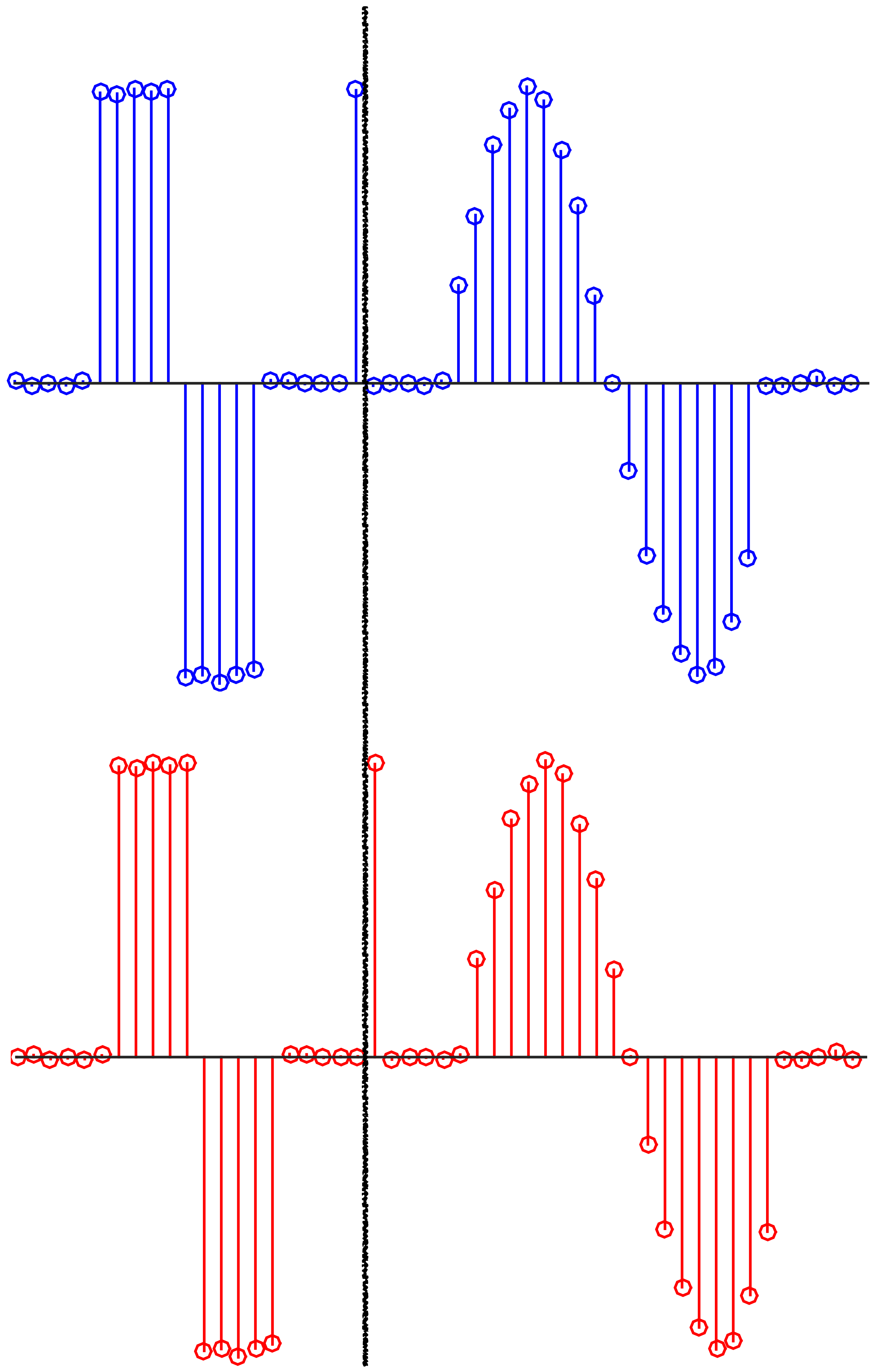
- Exaggerate differences
- Amplify quickly varying signals
- Attenuate slowly varying signals
- High Pass Filter?

Example: Two-Point Moving Difference

$$y[t] = \frac{1}{2}x[t] - \frac{1}{2}x[t - \Delta t]$$

$x[t]$

$x[t - \Delta t]$



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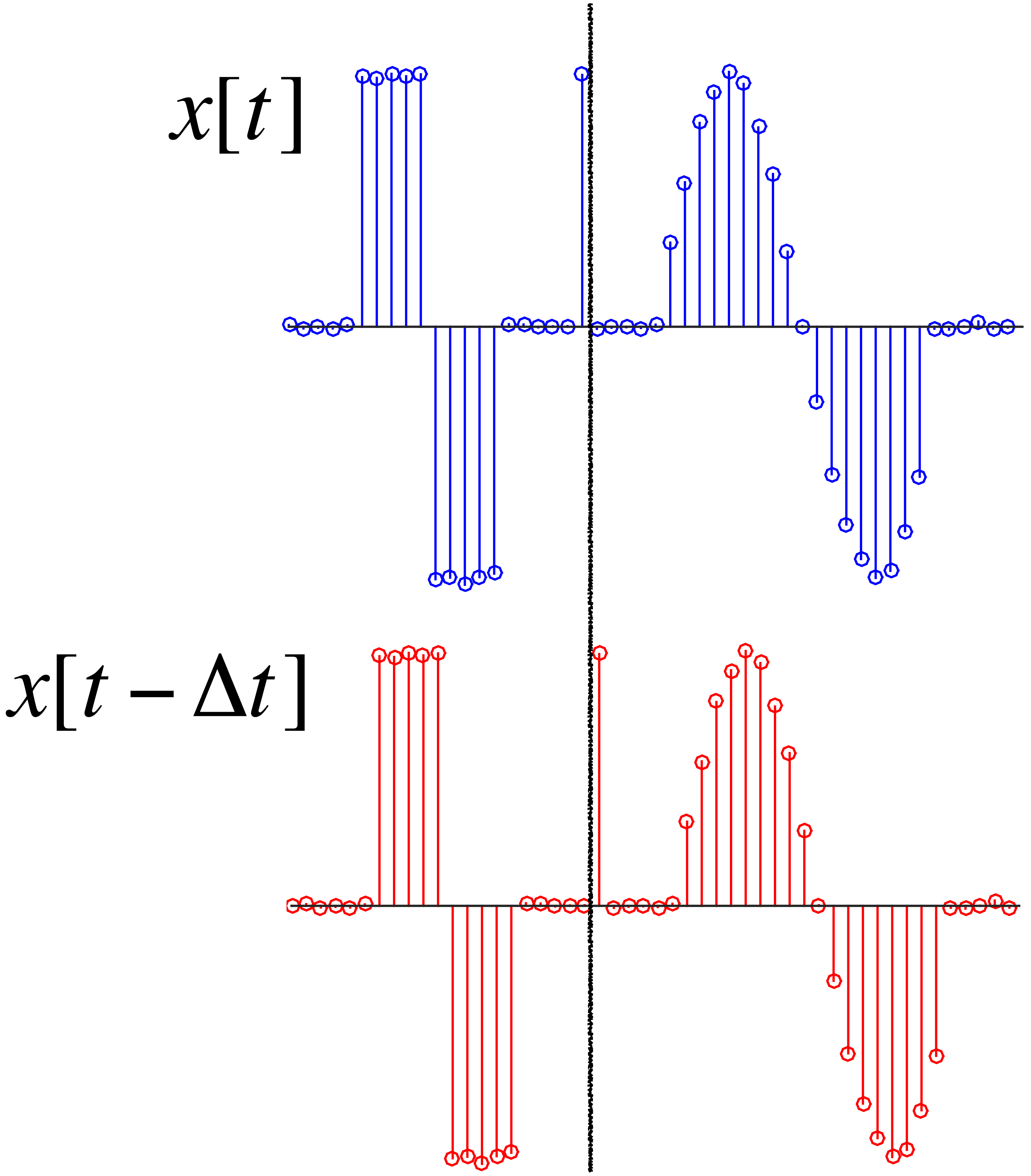

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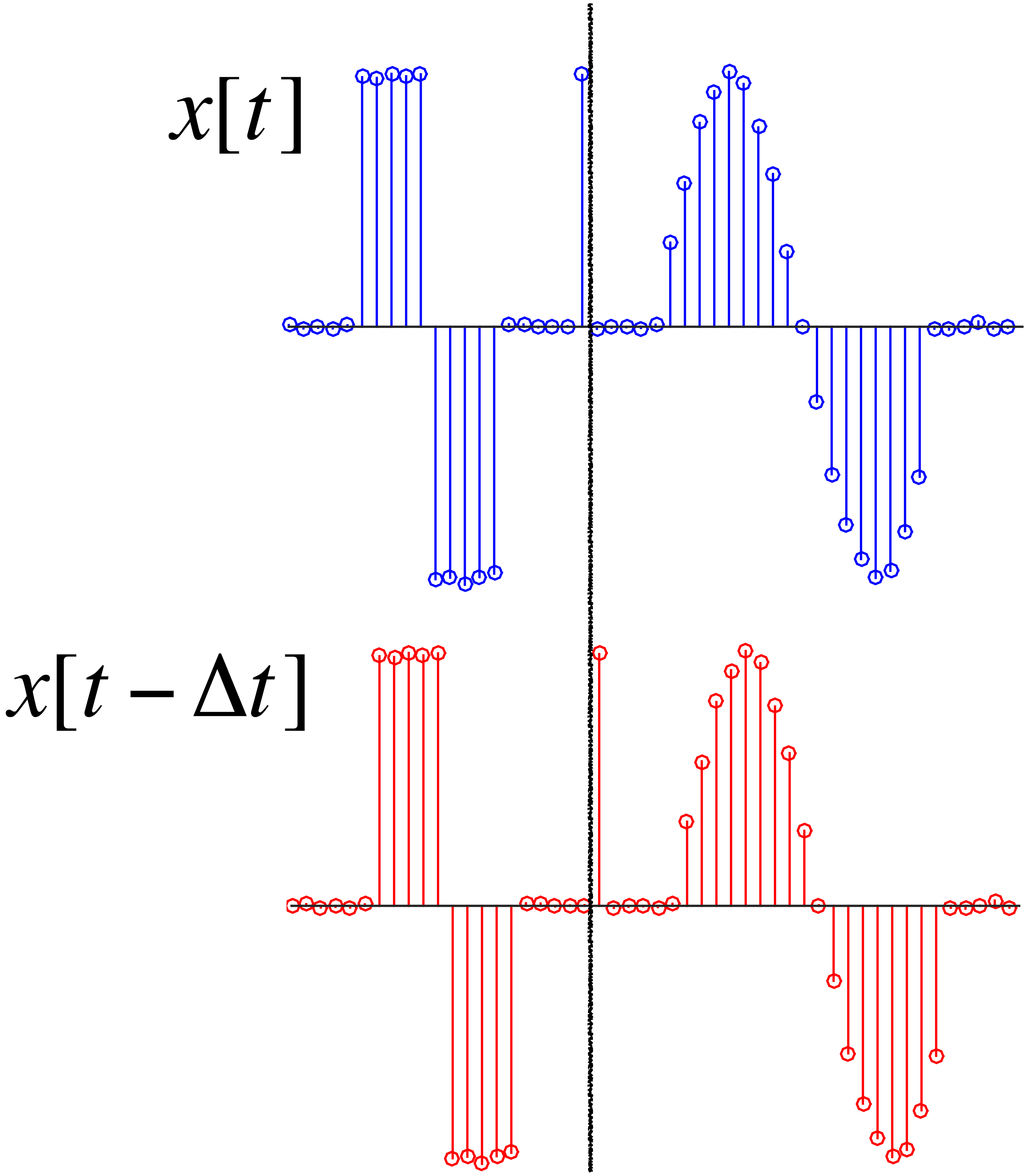
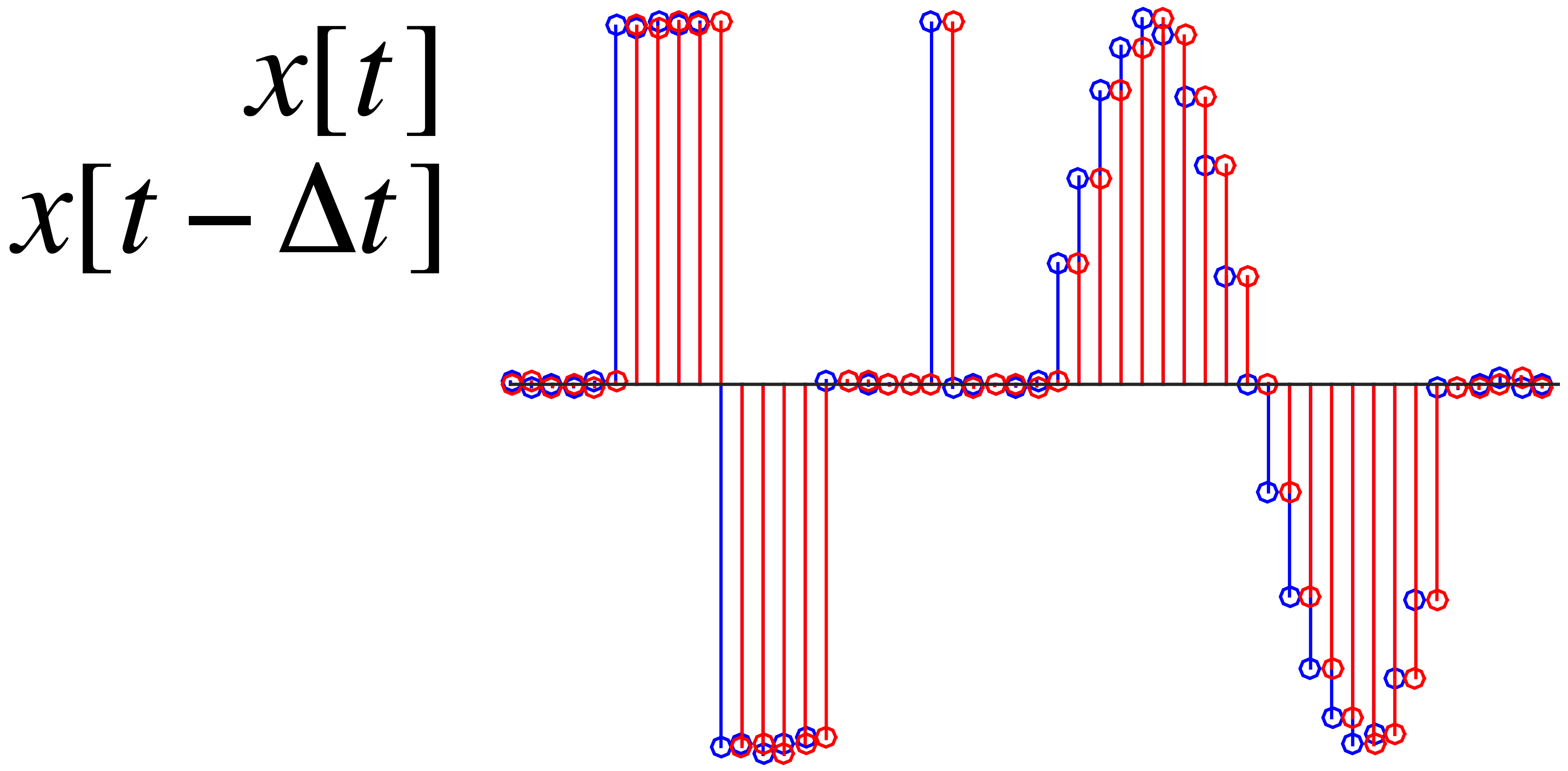
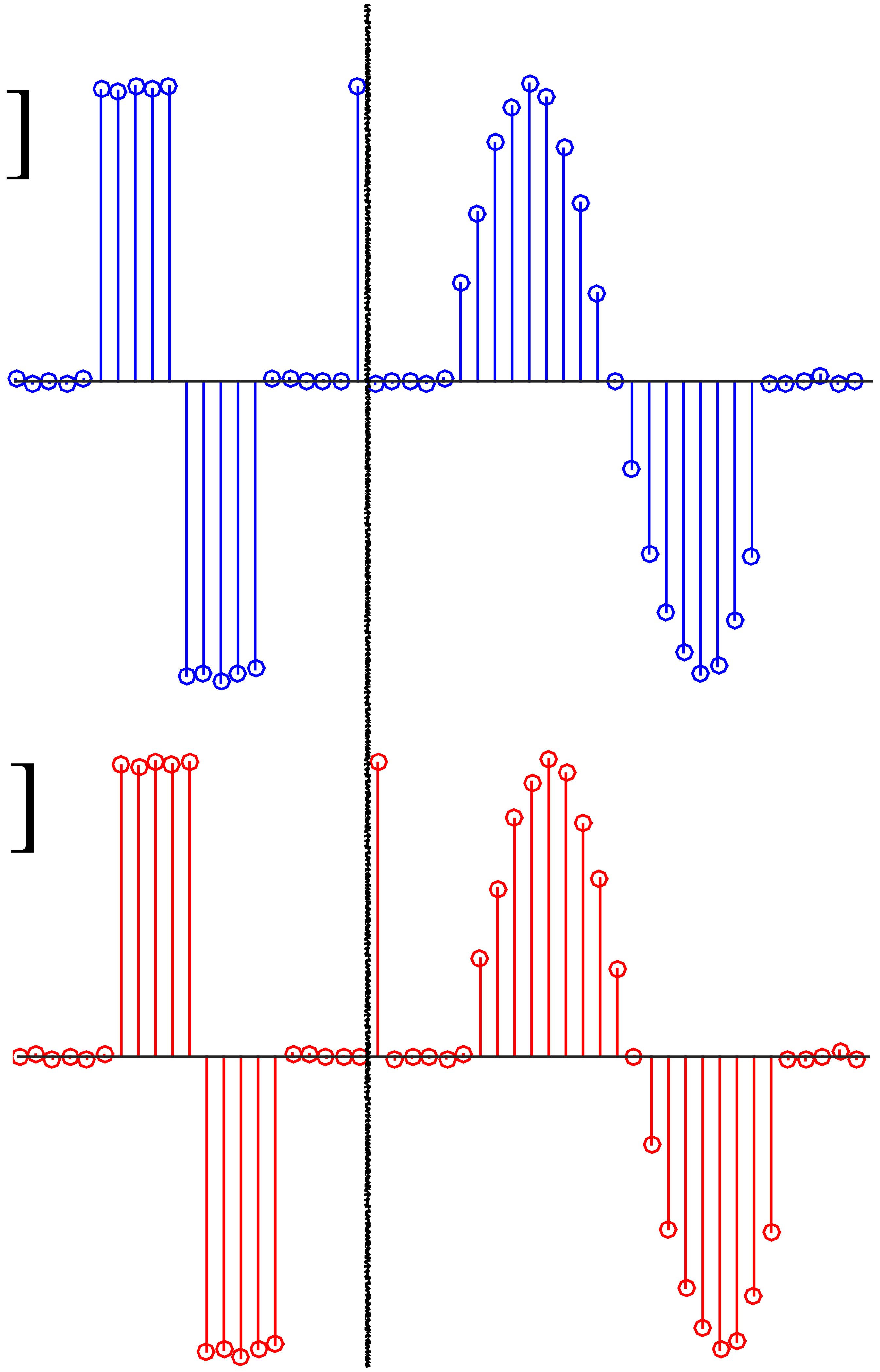




Example: Two-Point Moving Difference

$x[t]$

$x[t-\Delta t]$



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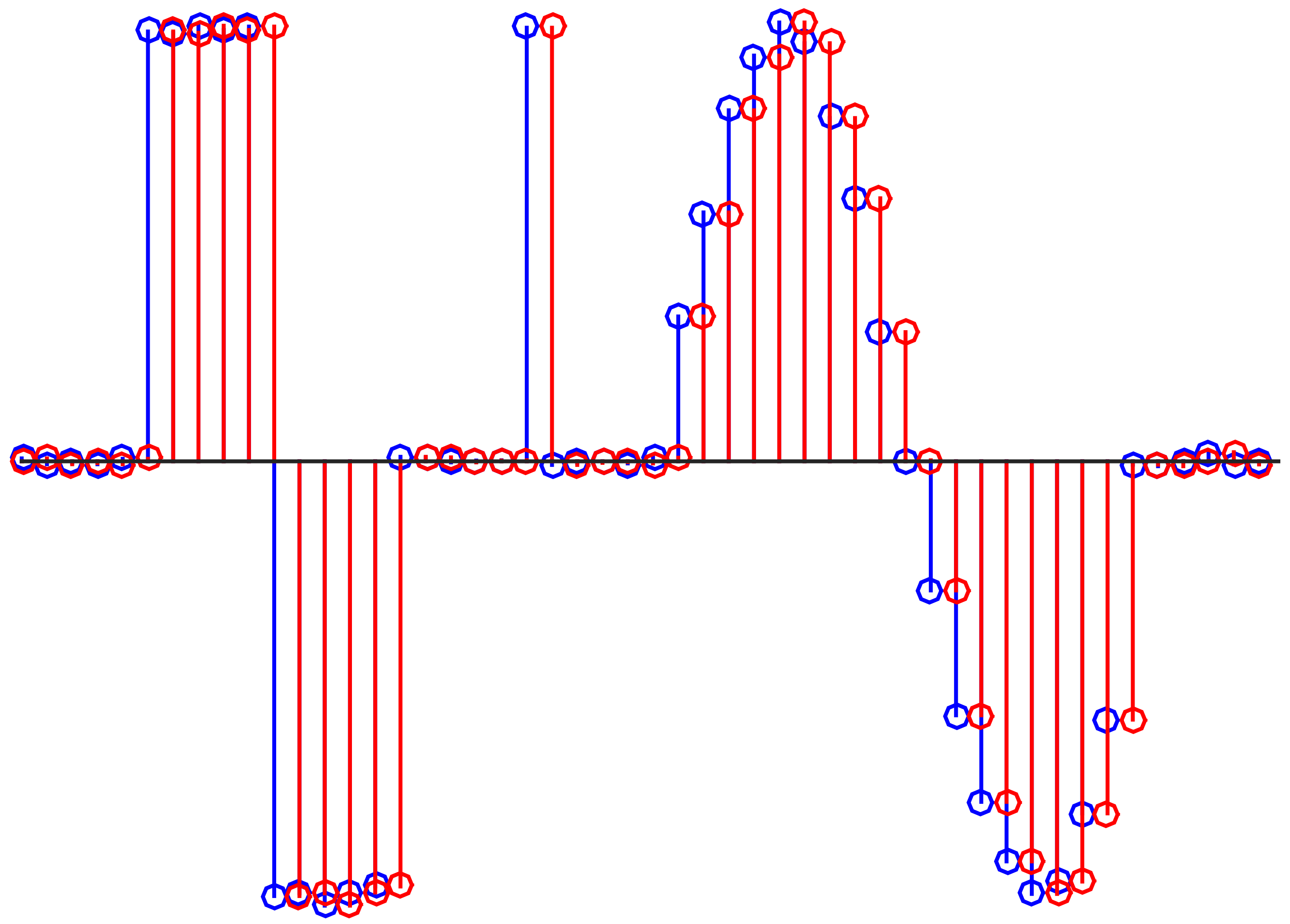

$$x[t]/2$$

$$\underline{-\,x[t-\Delta t]/2}$$

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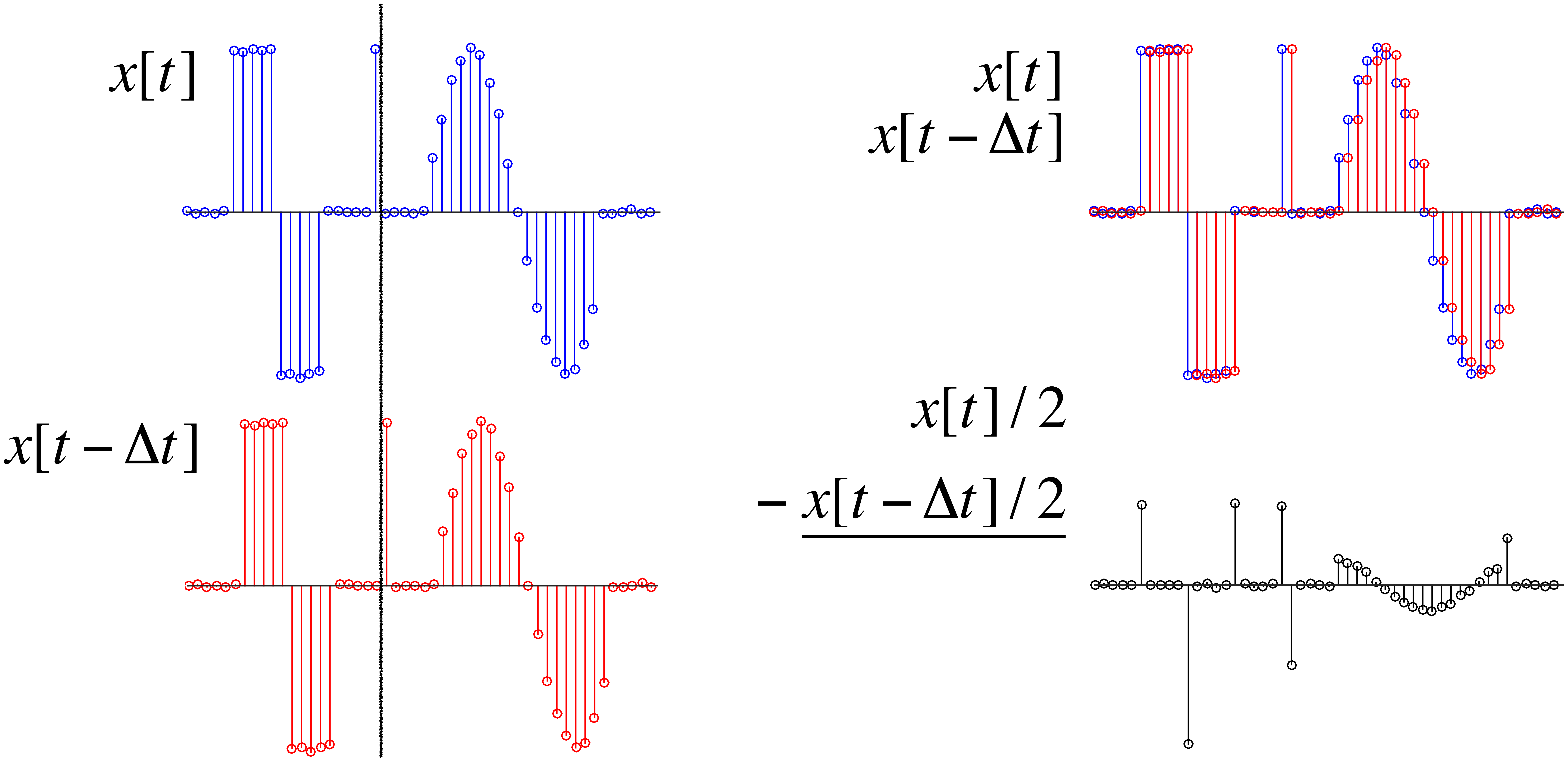

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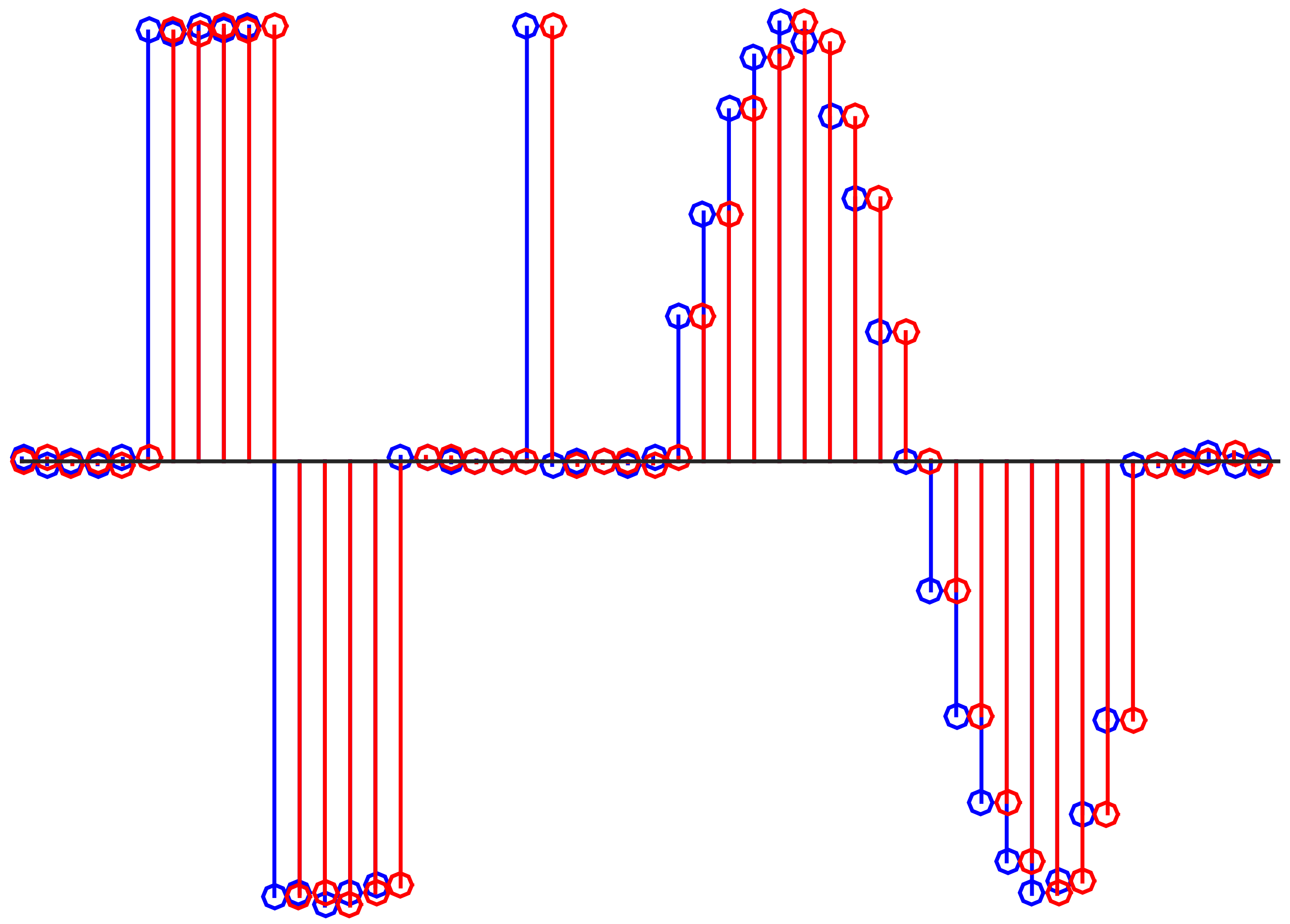

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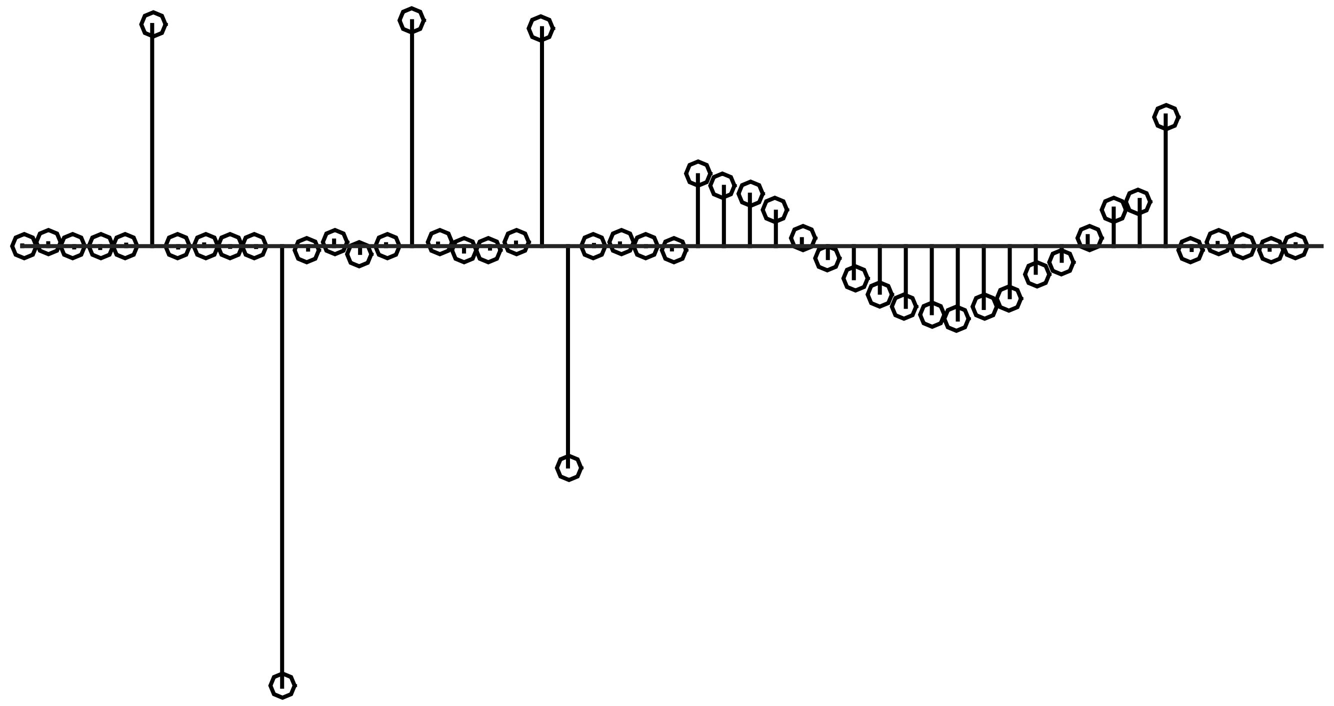

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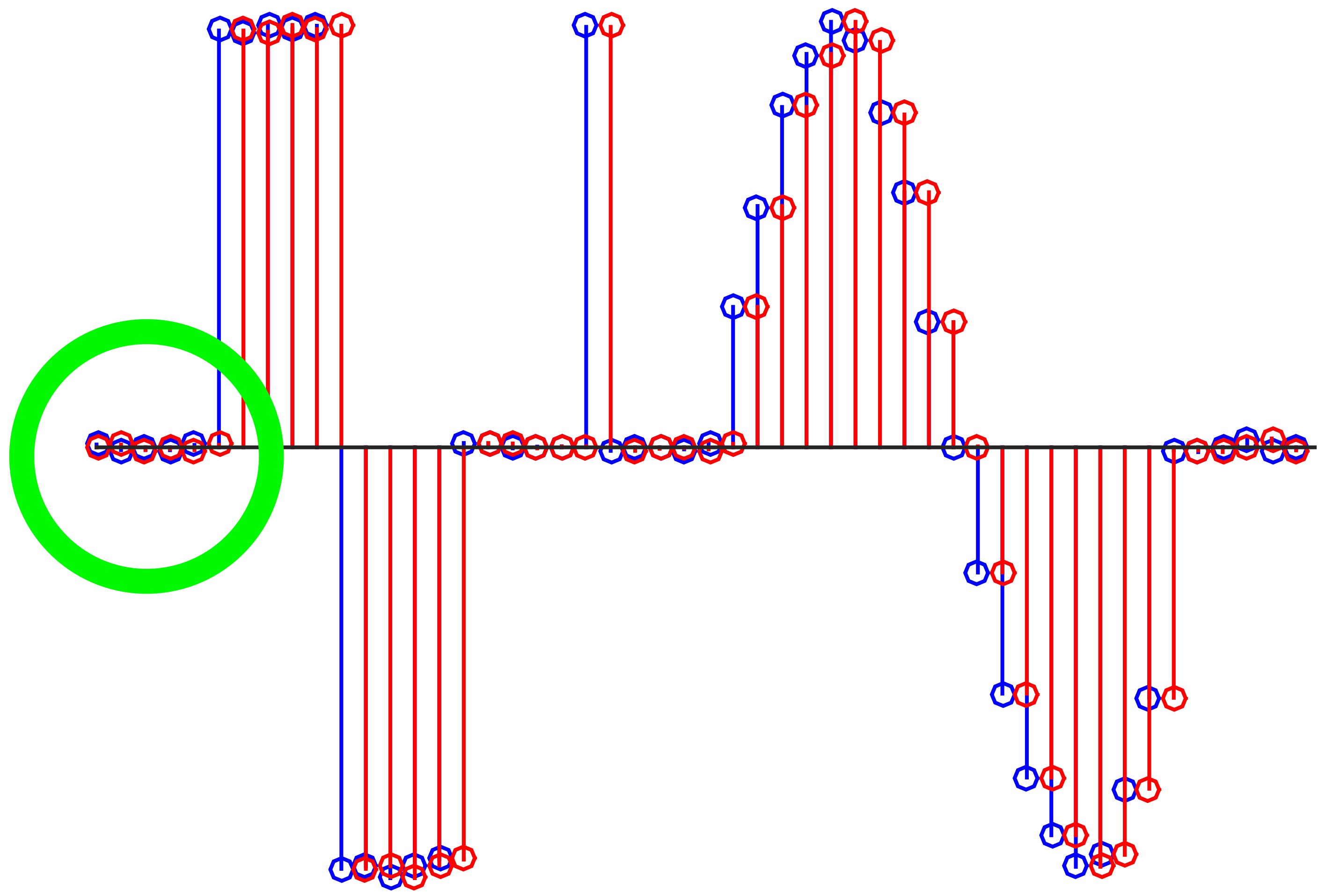

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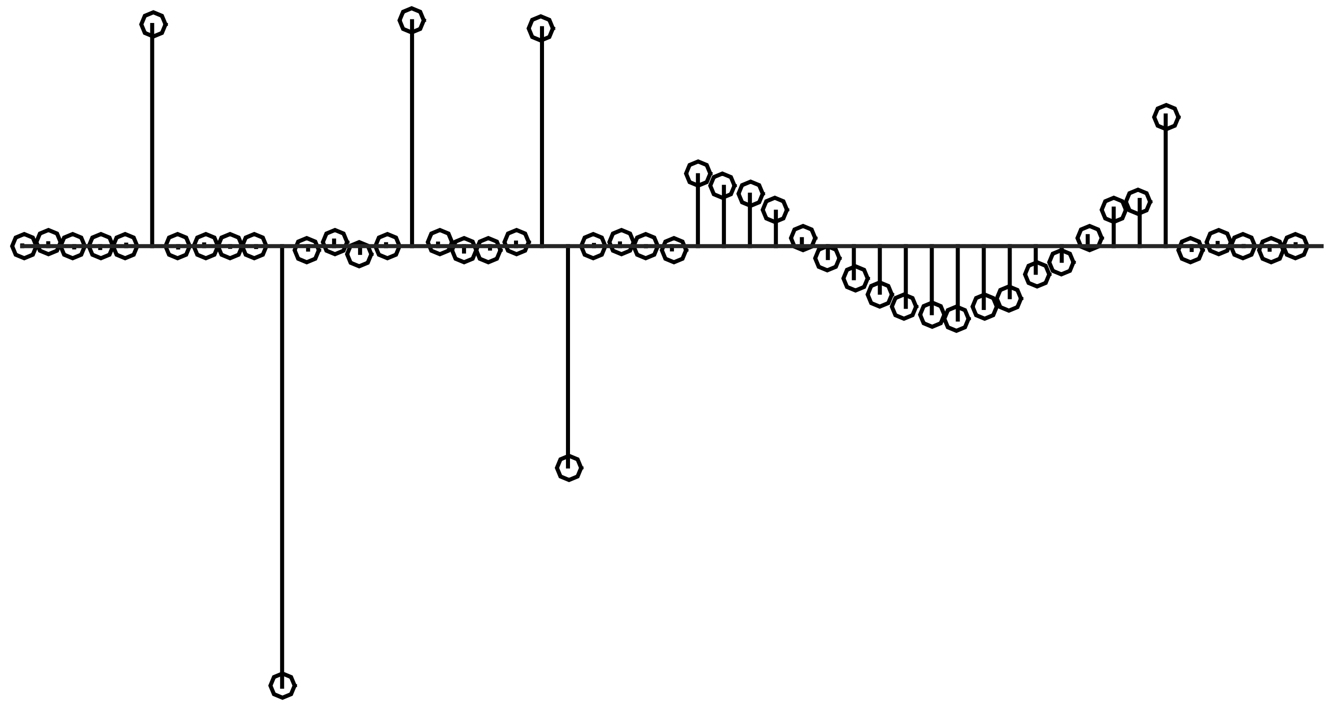

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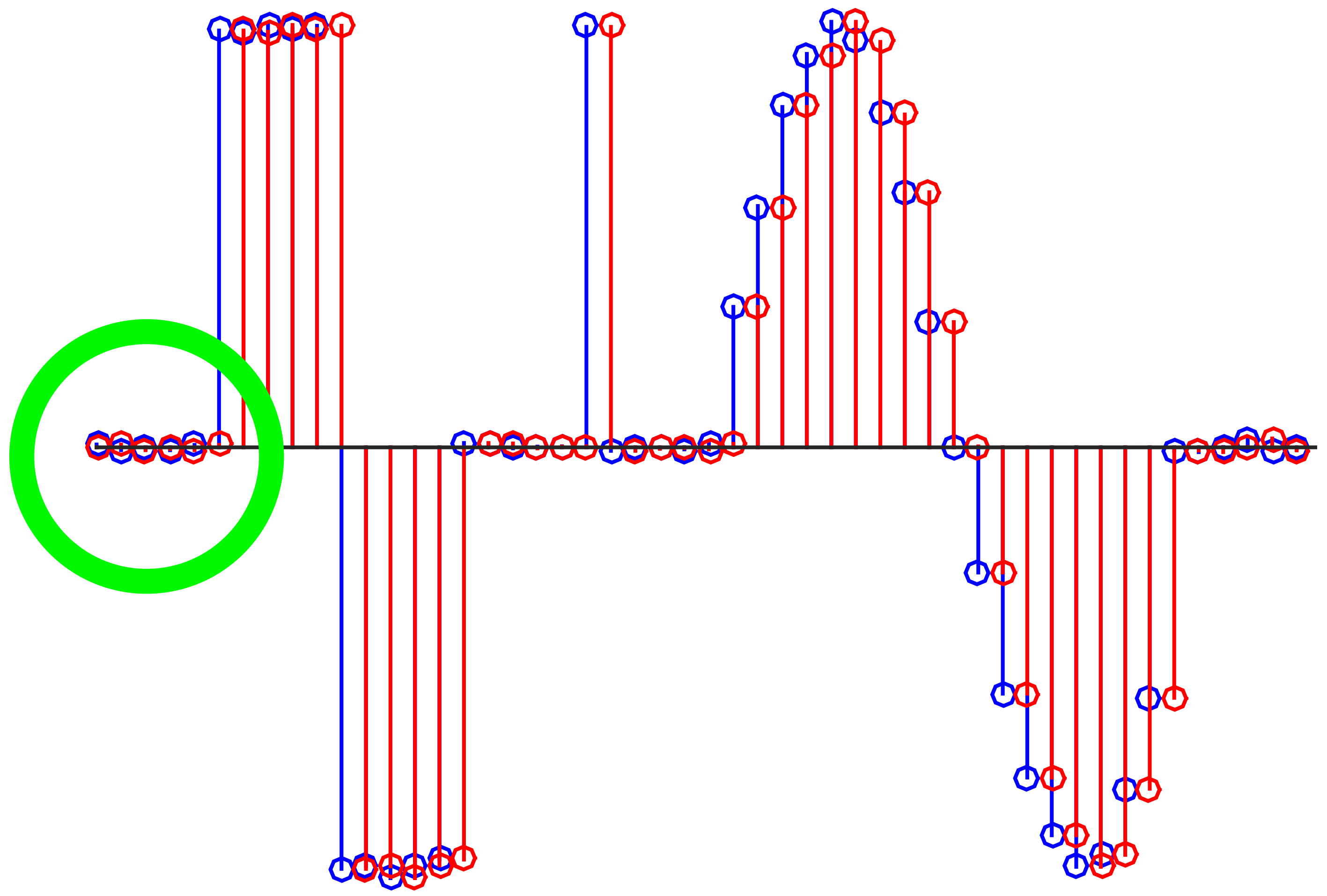

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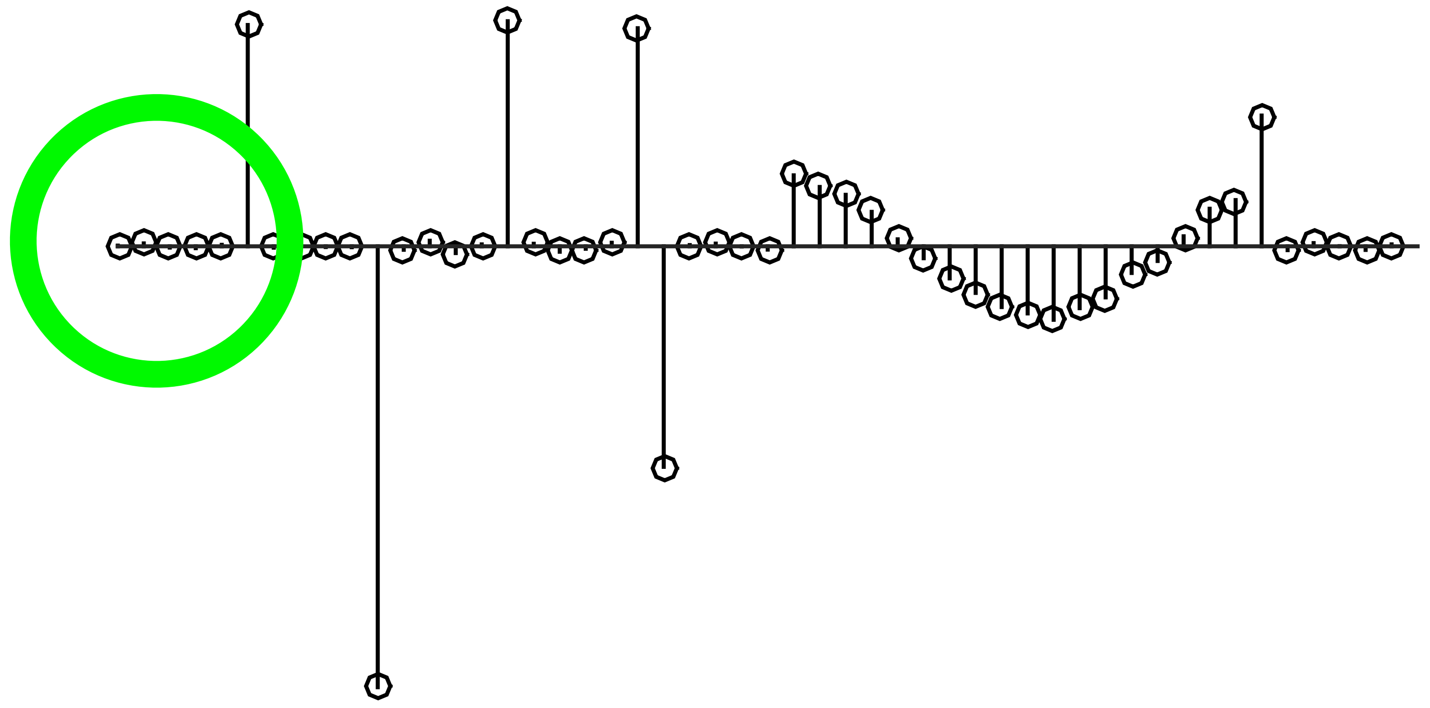

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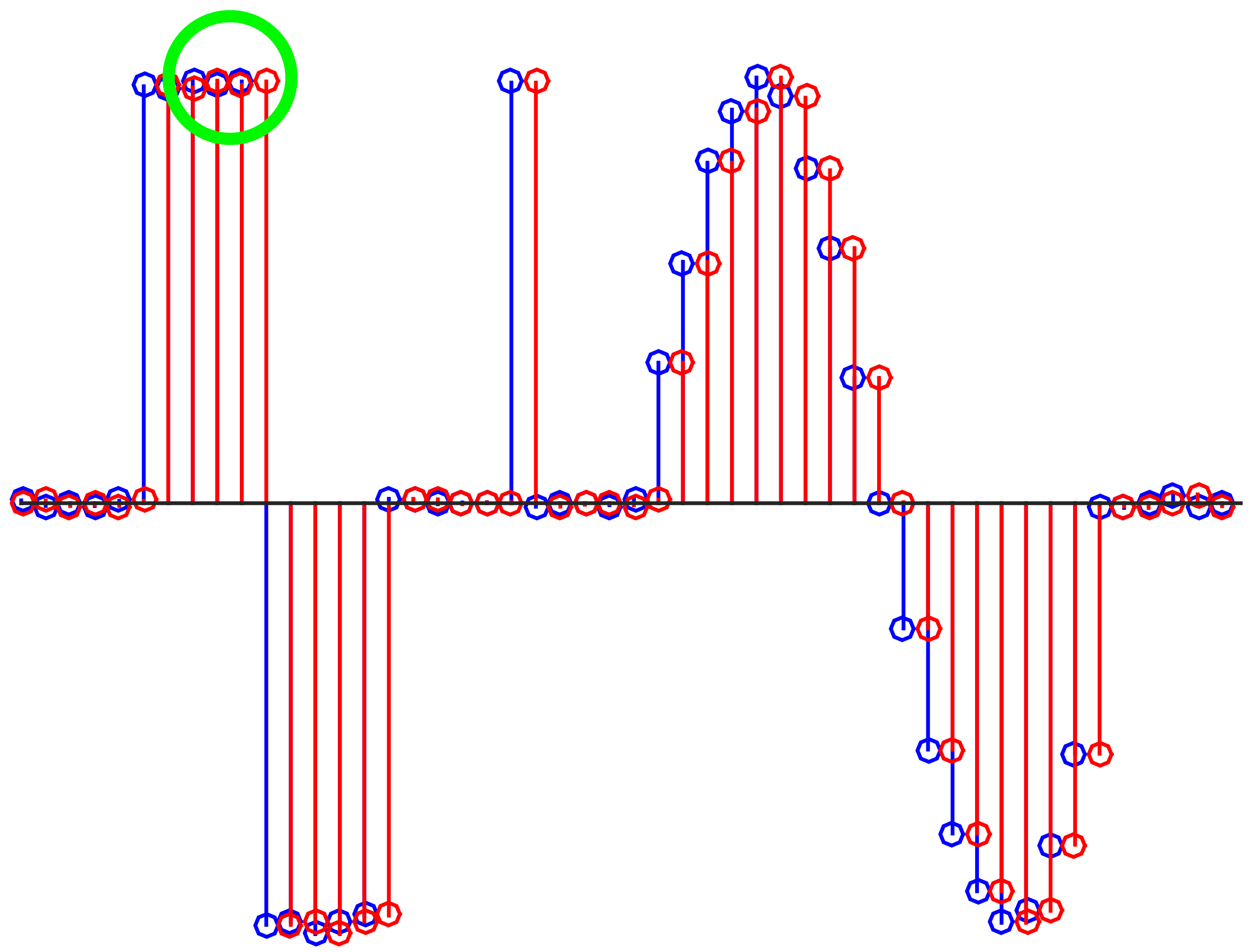

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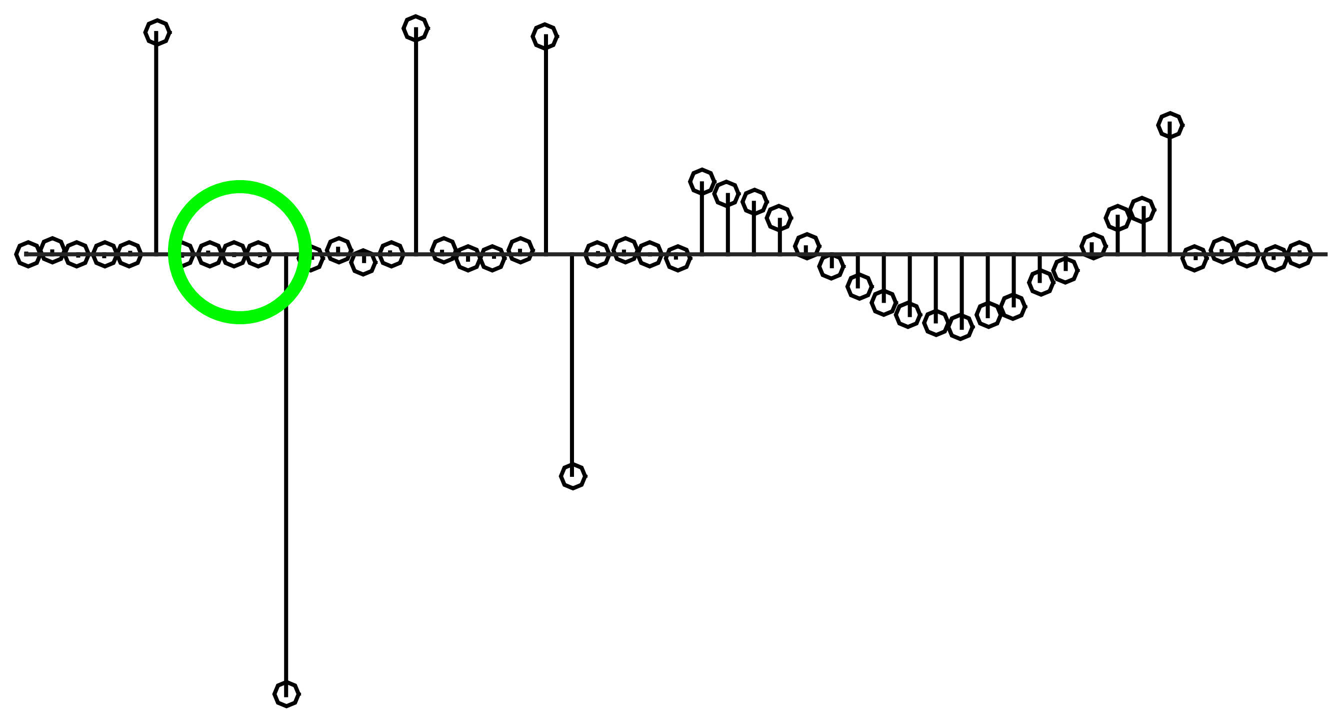

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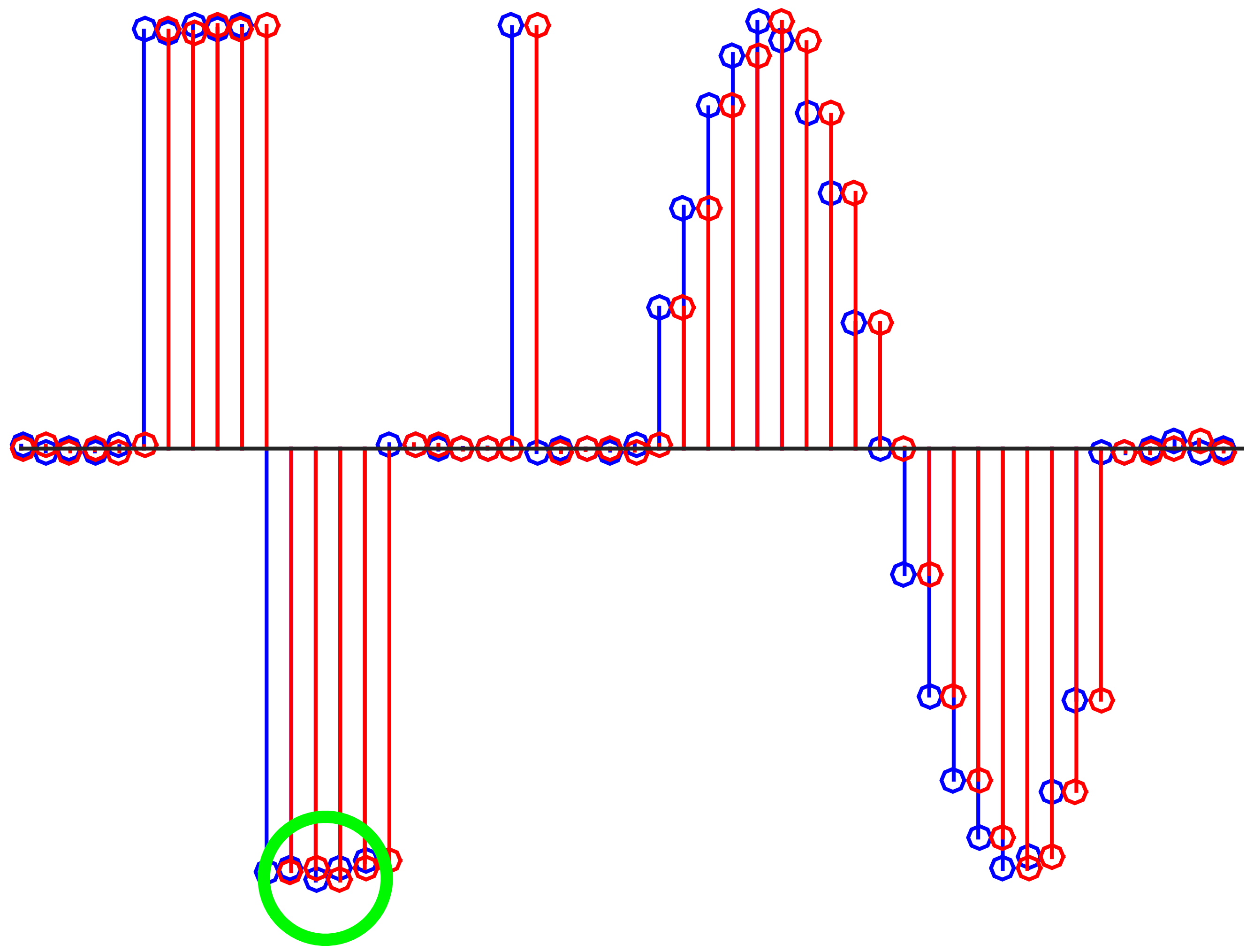

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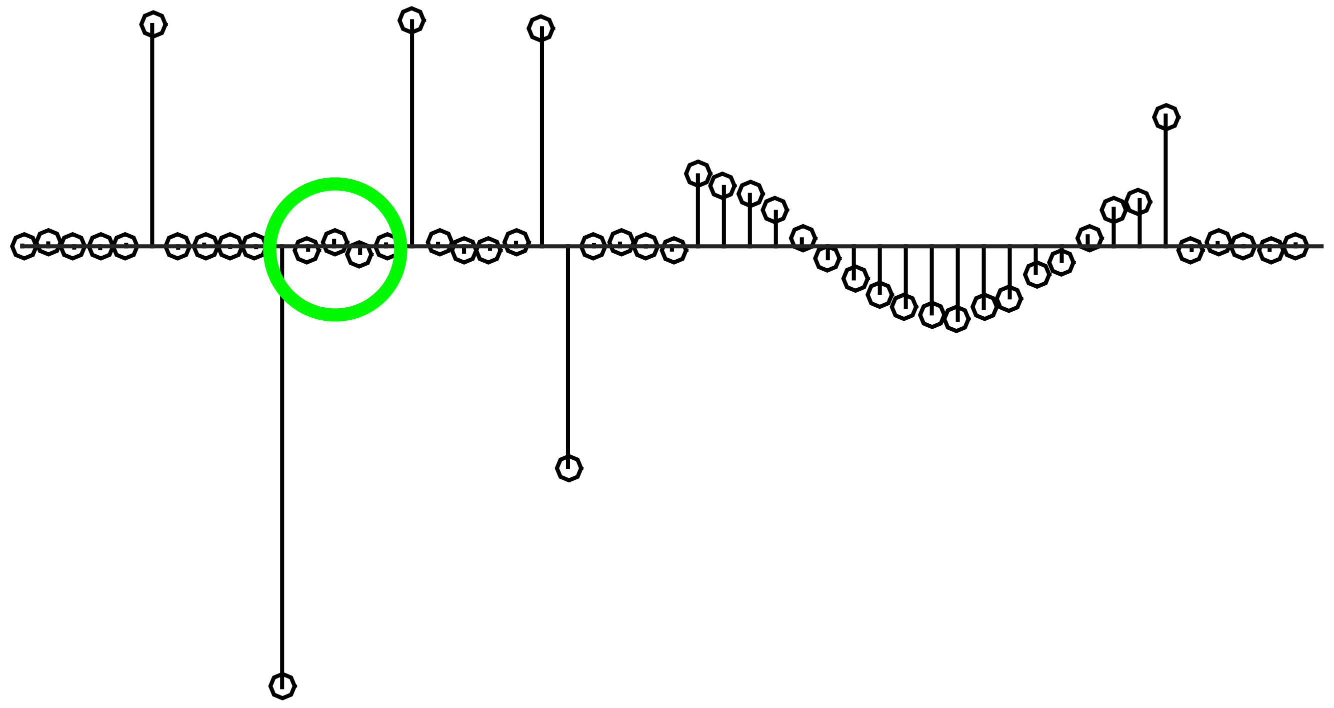

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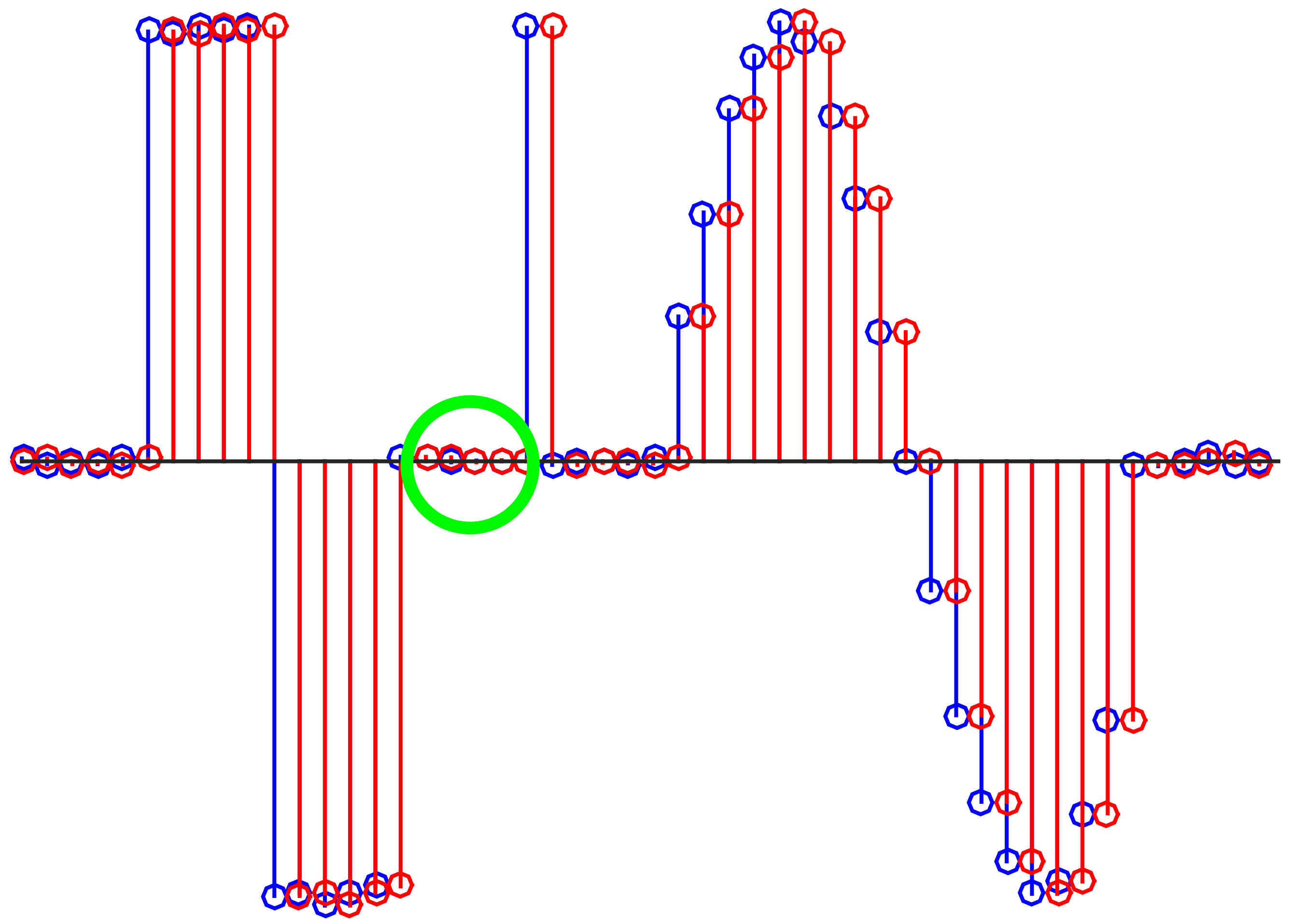

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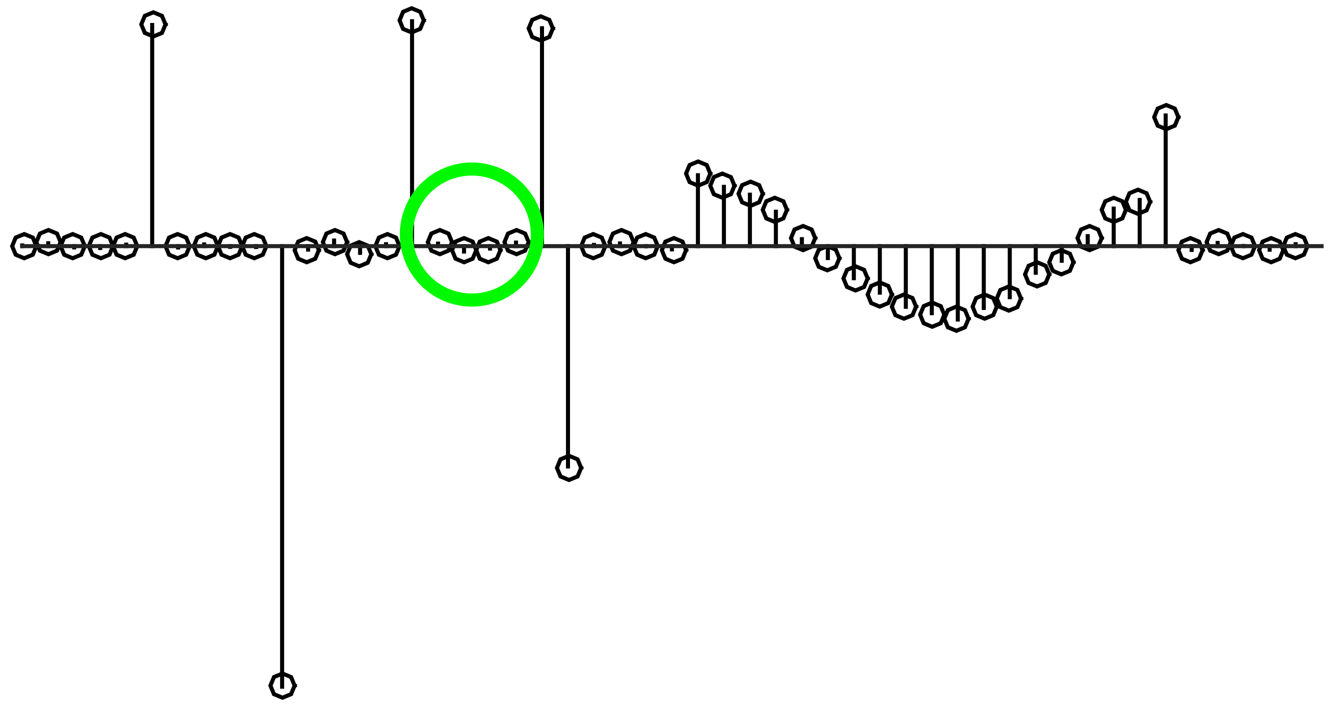

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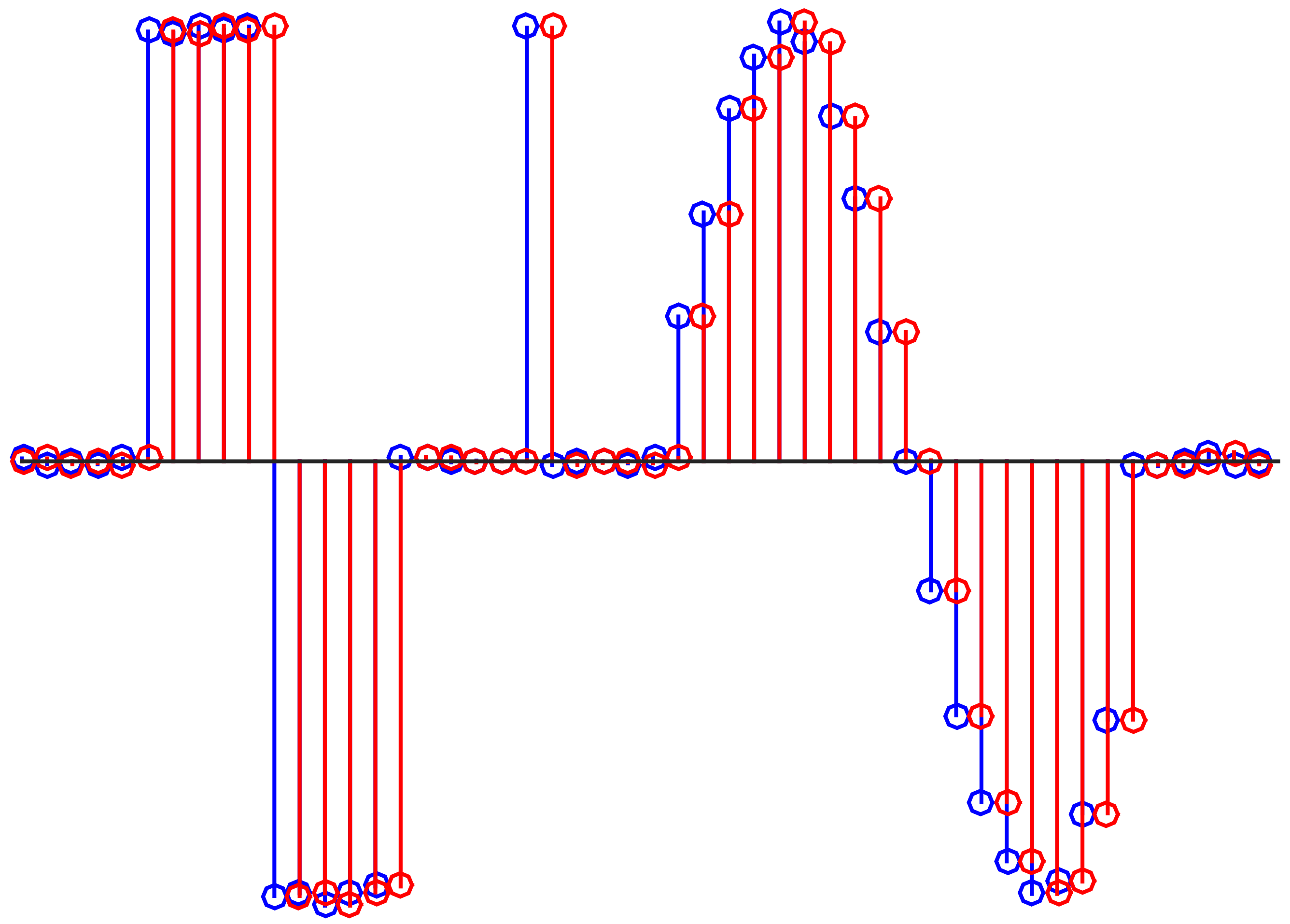

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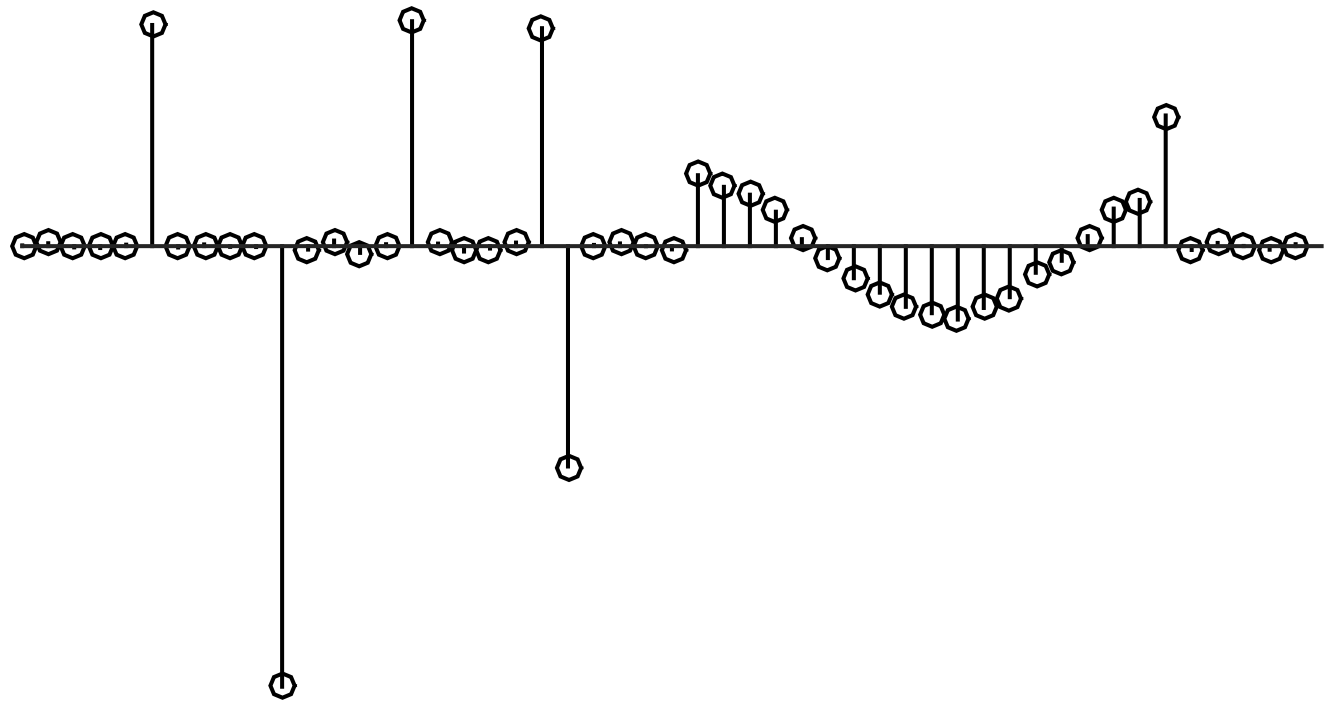

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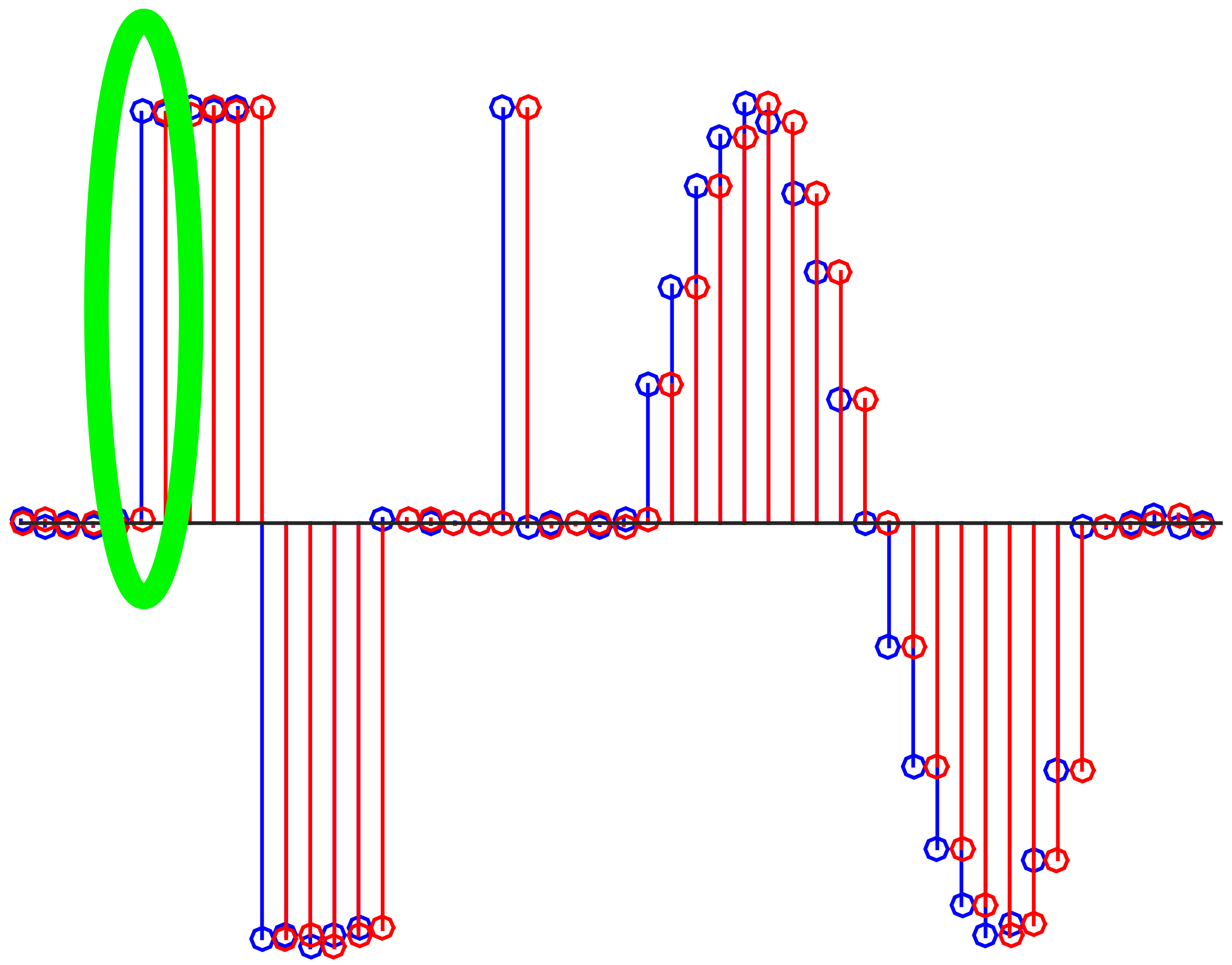

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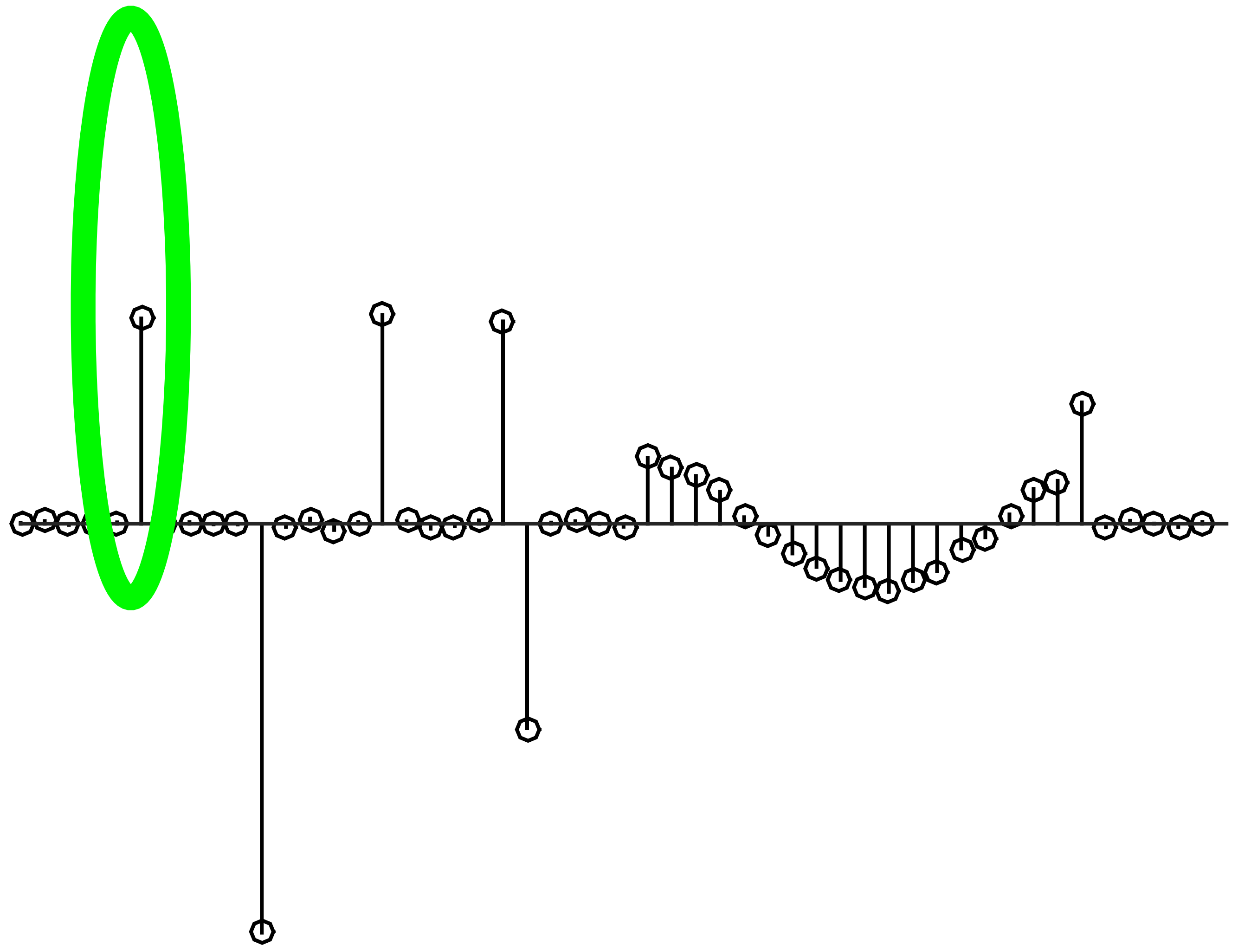

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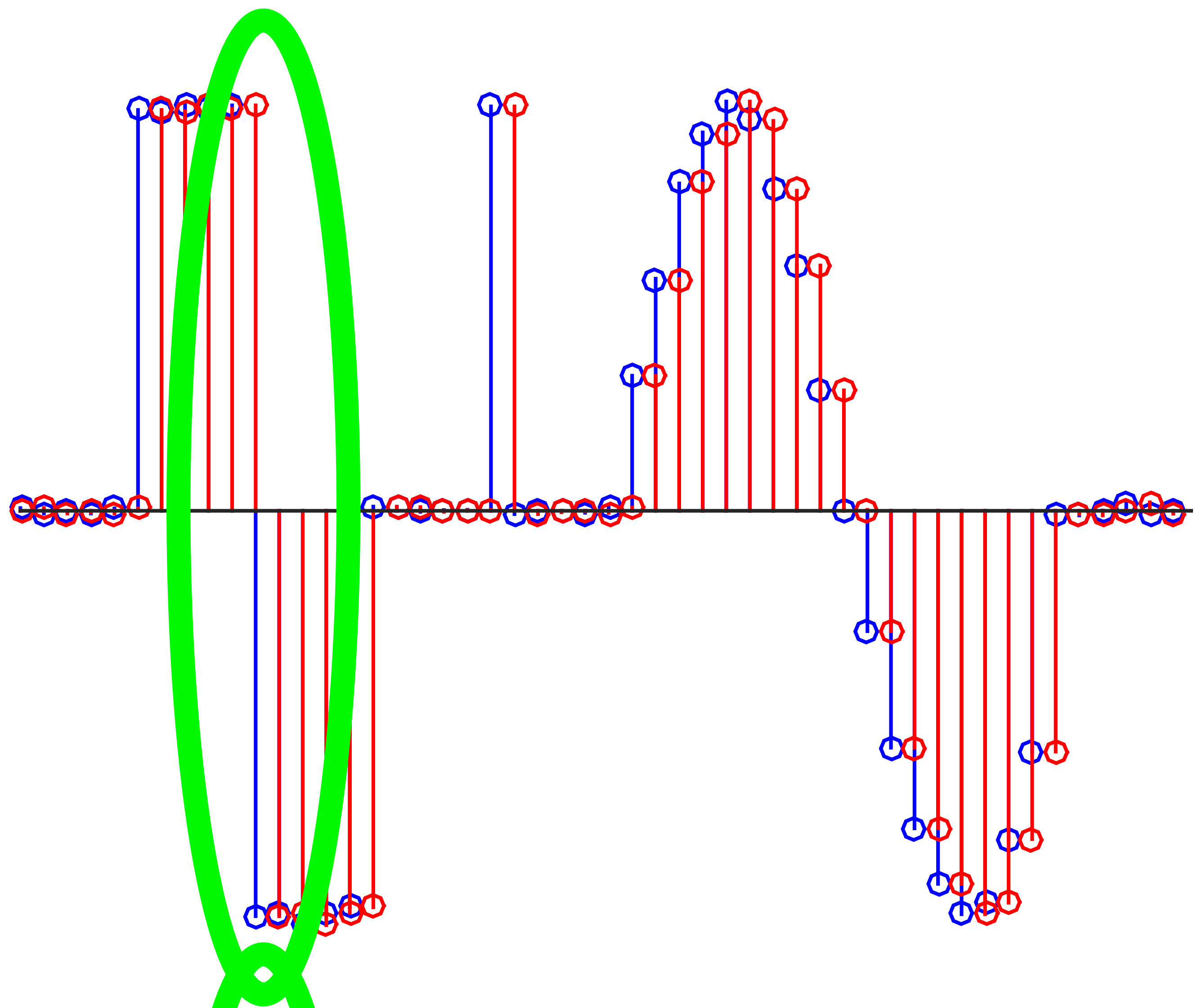

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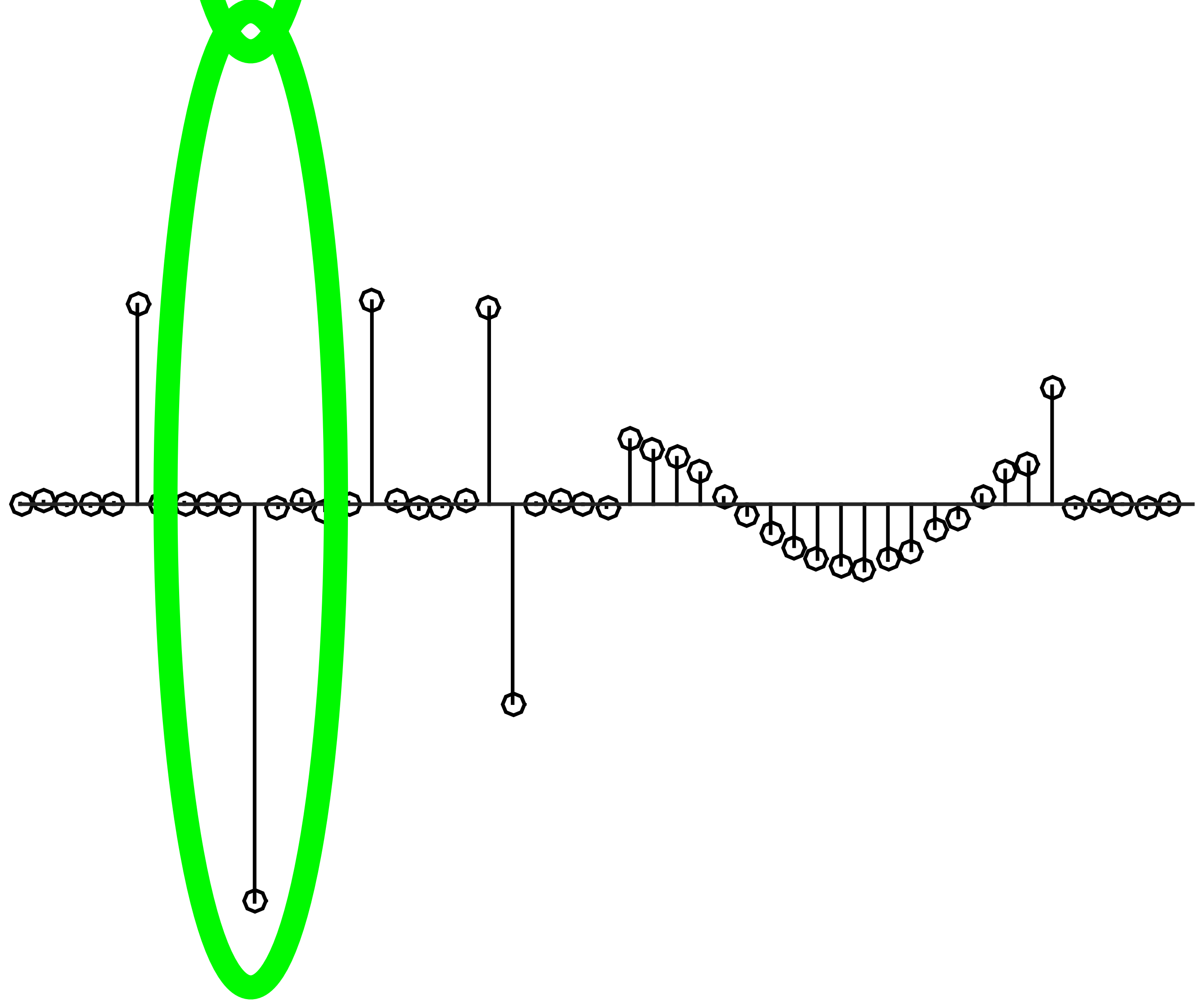

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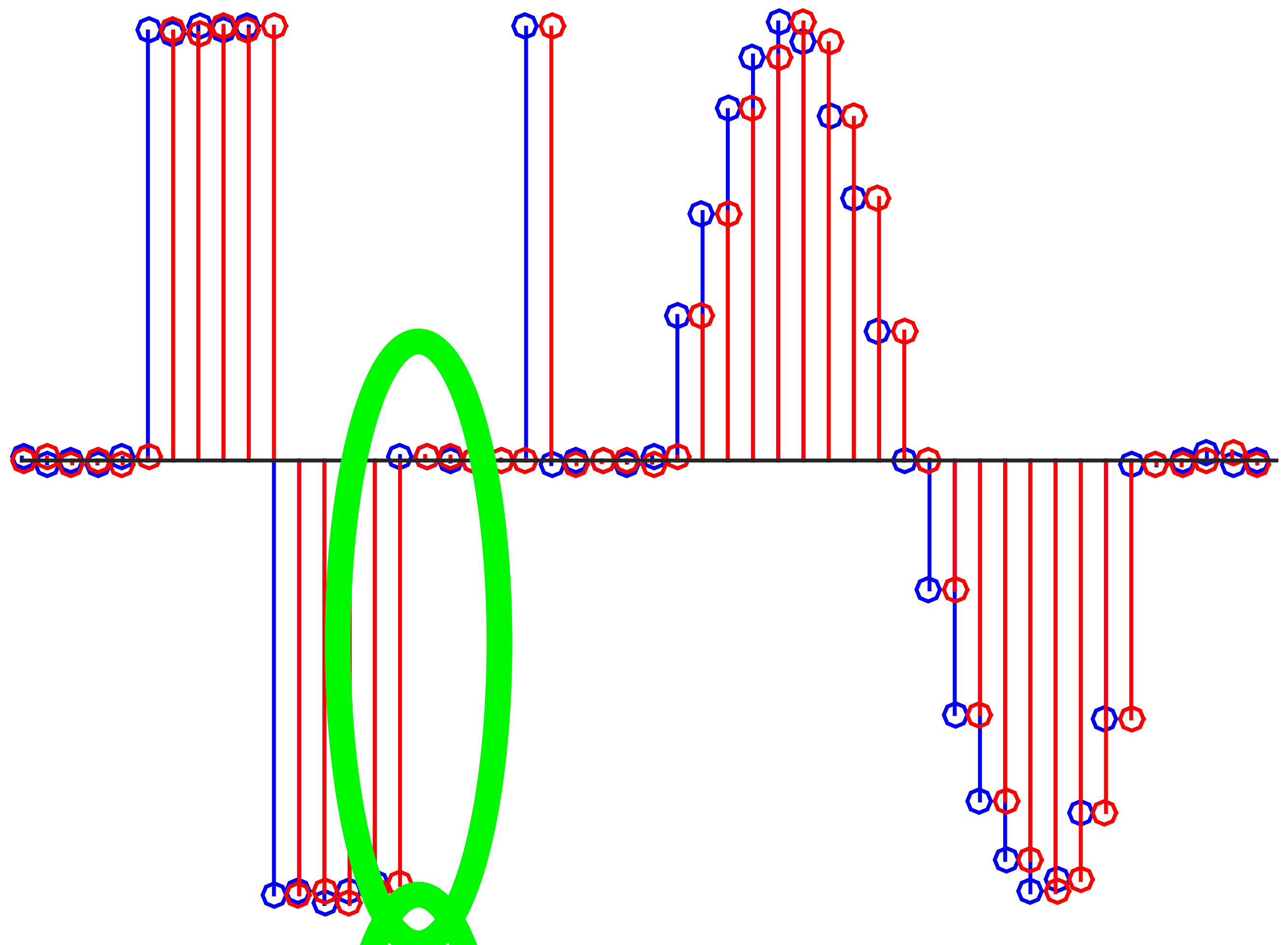

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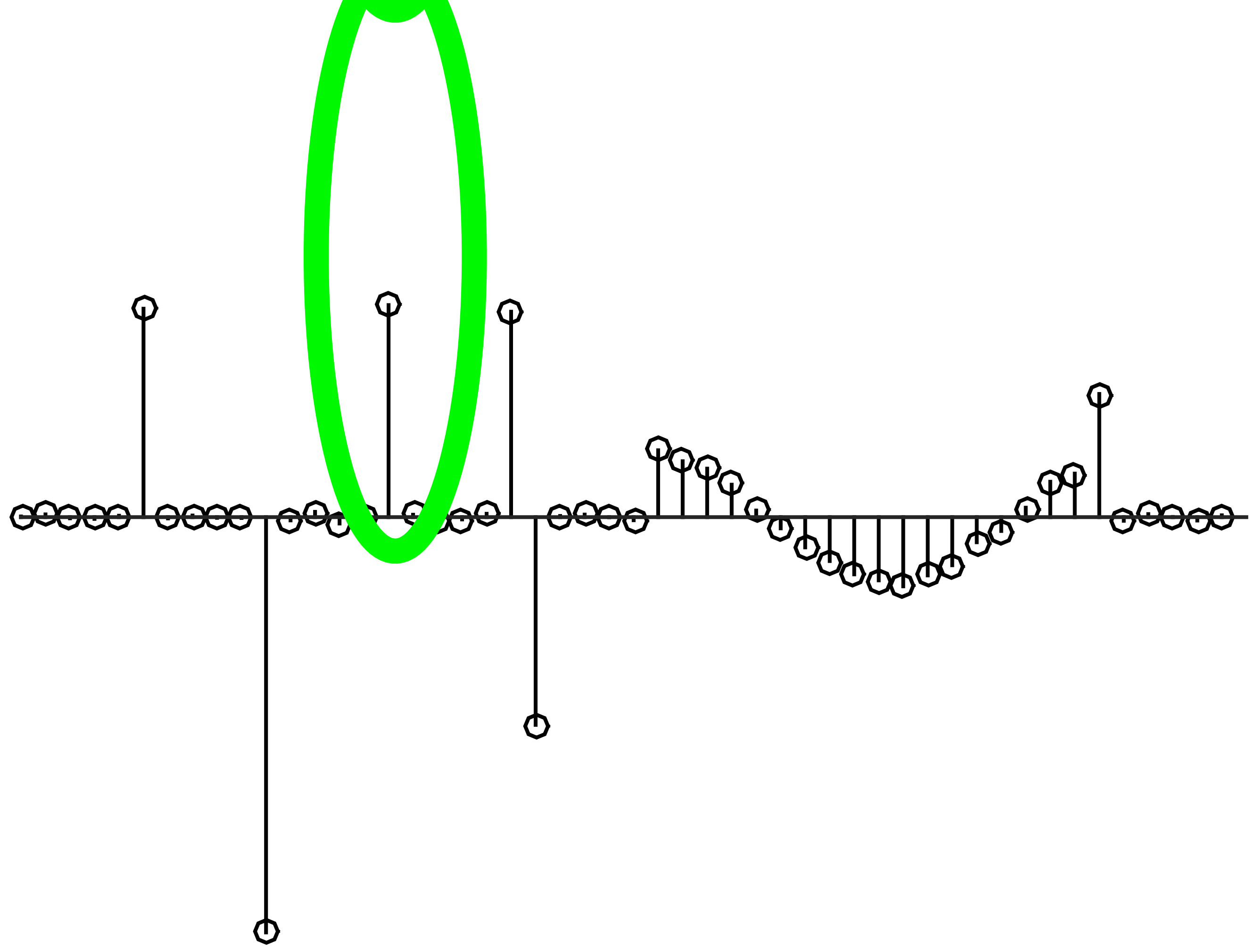

Example: Two-Point Moving Difference

$x[t]$
$x[t - \Delta t]$

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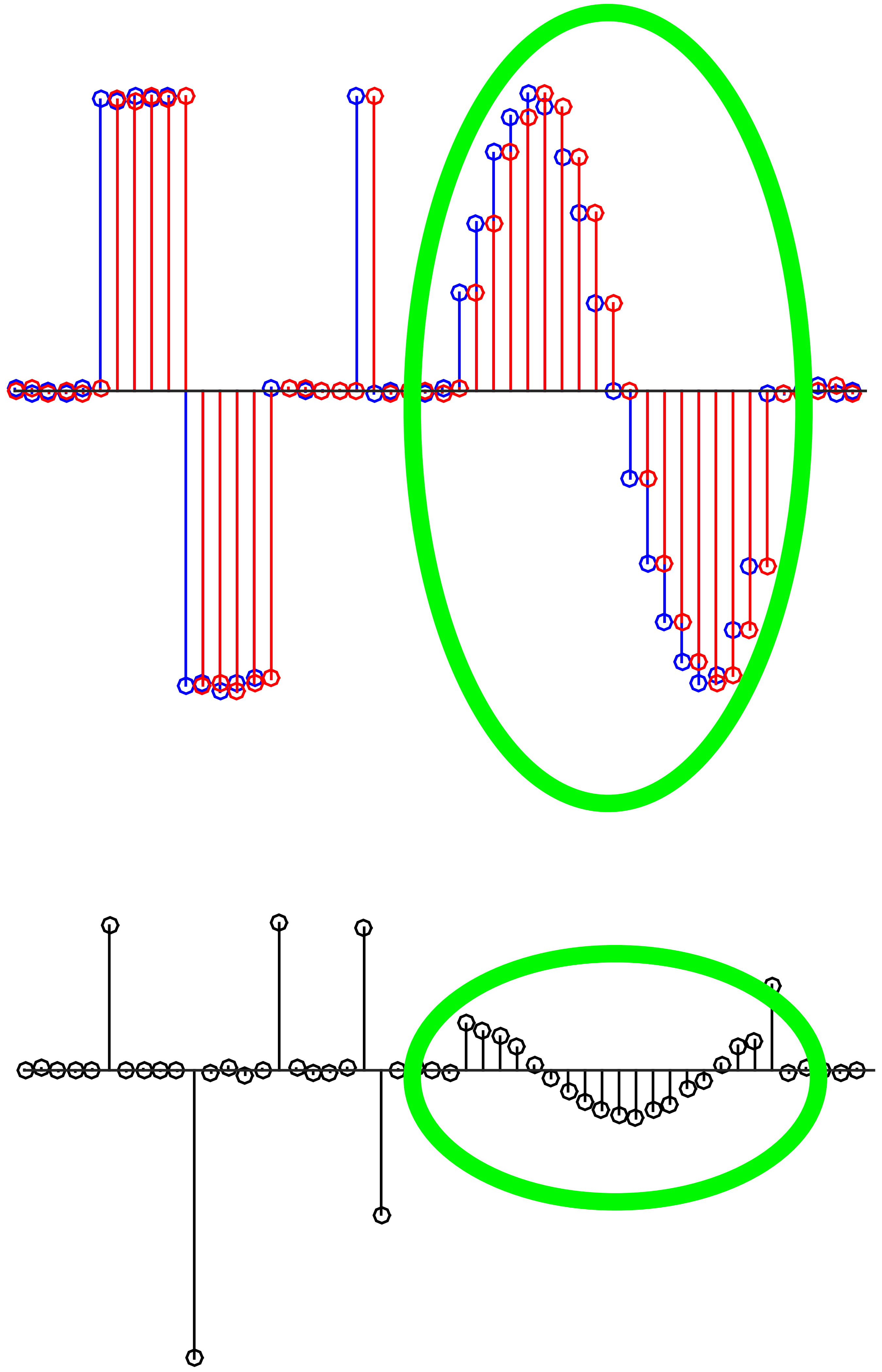

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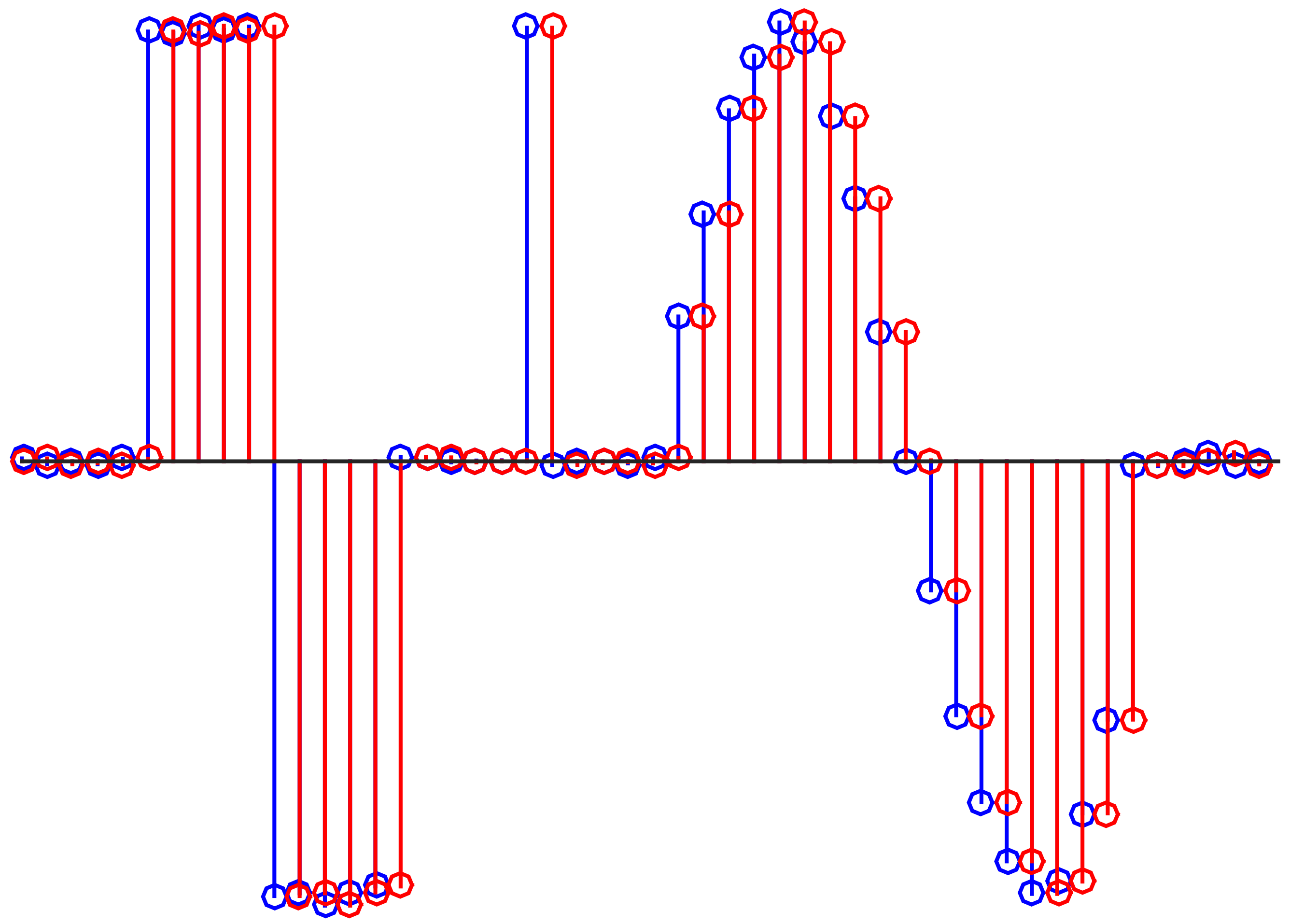

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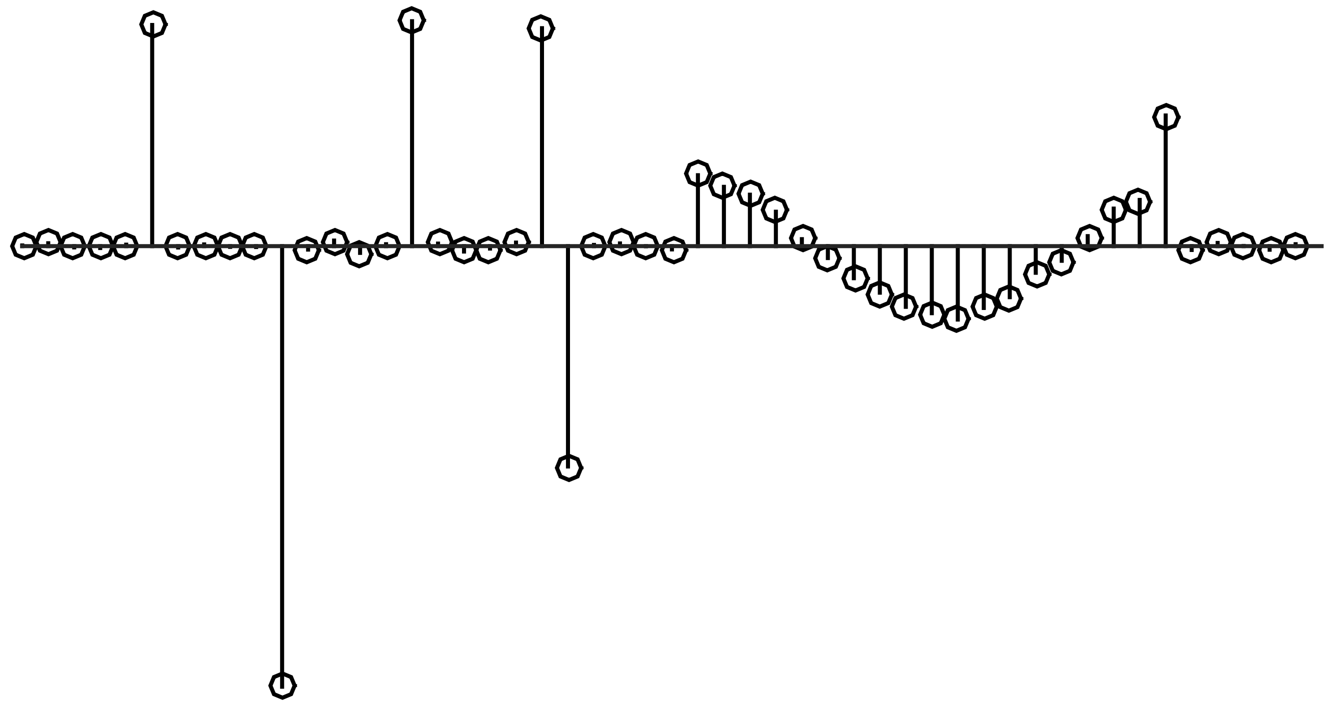

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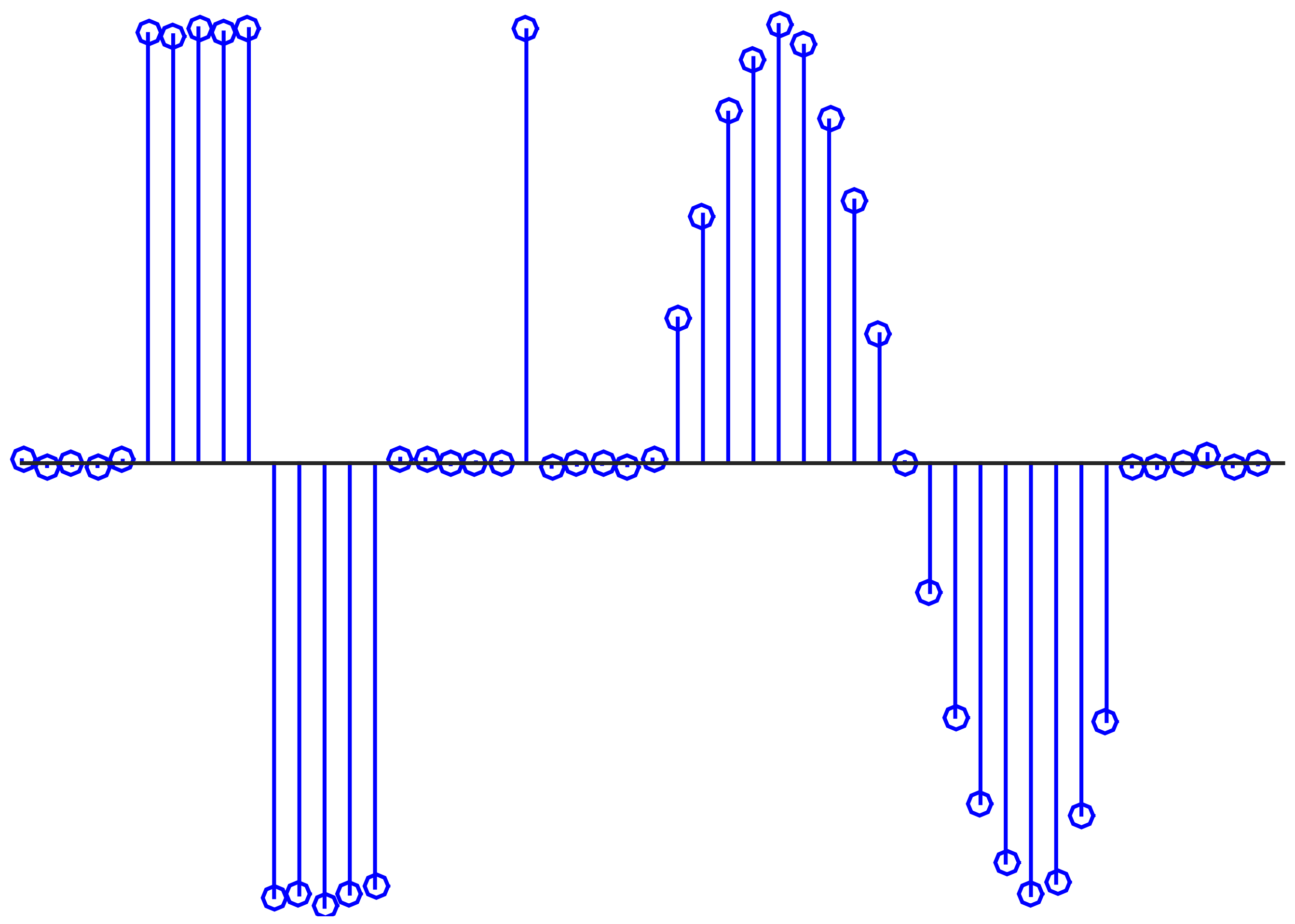

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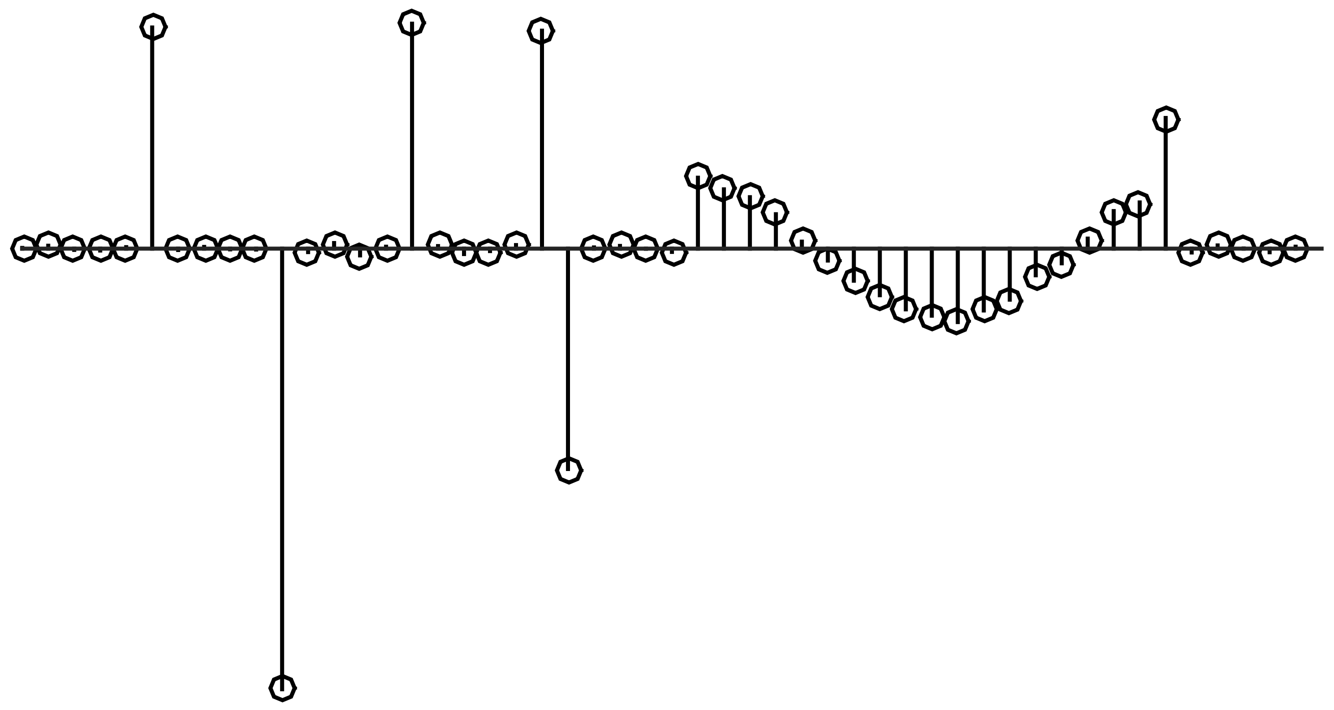
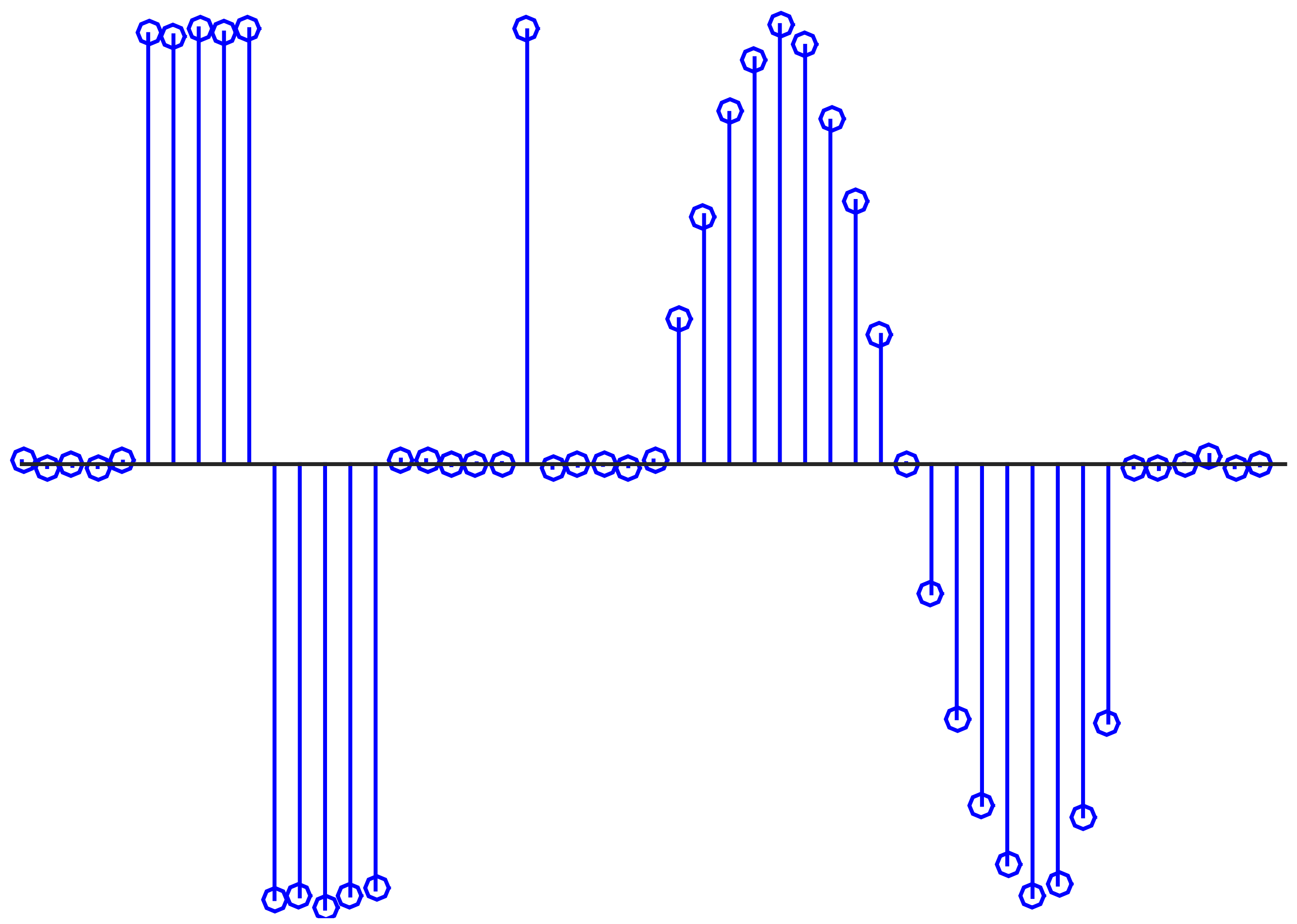


Results:

- Exaggerates differences
- Amplifies quickly varying signals
- Attenuates slowly varying signals
- High Pass Filter?

Example: Two-Point Moving Difference

$x[t]$



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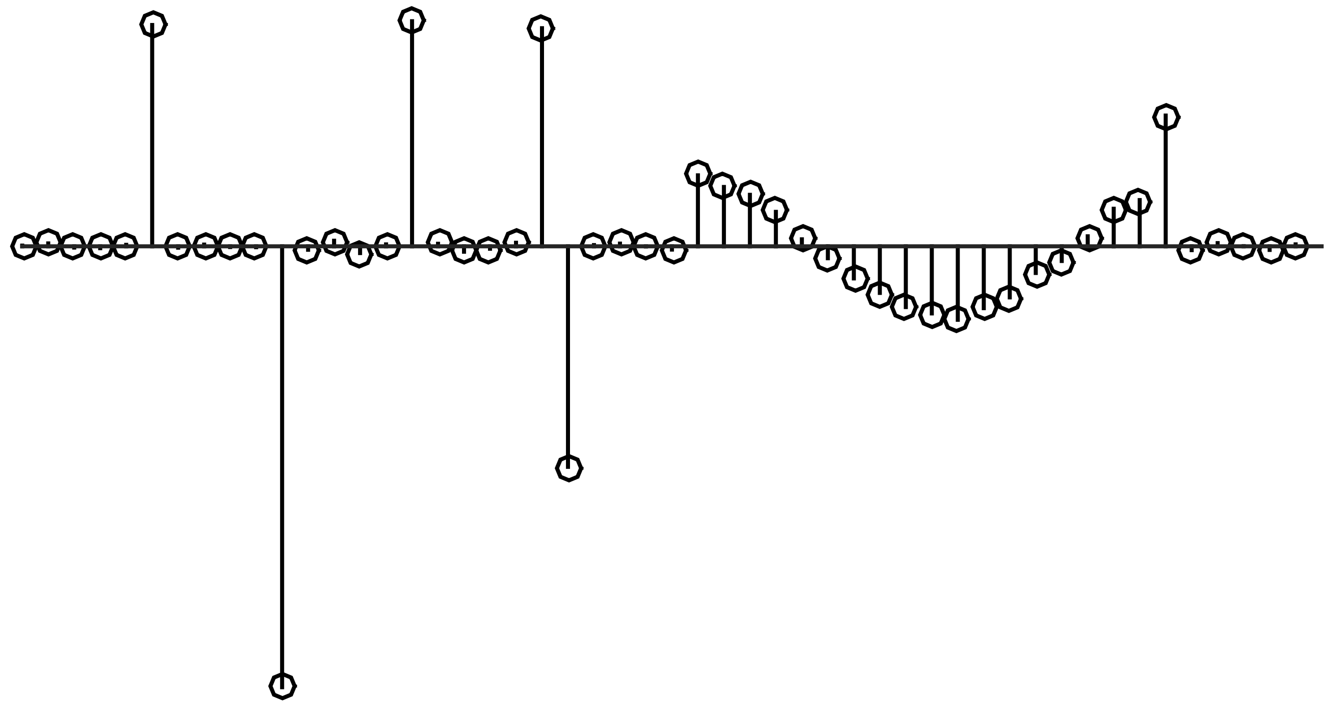

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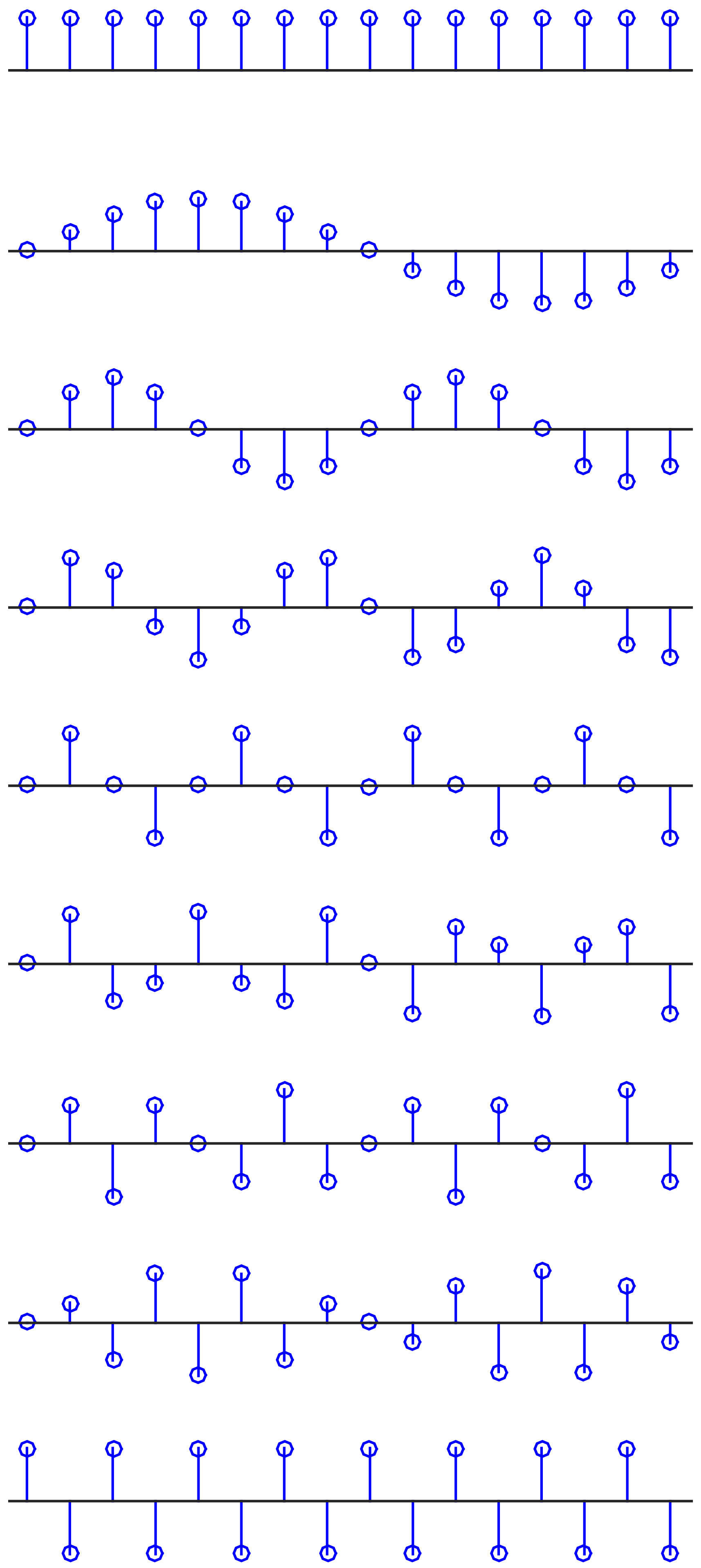
Filters and the Fourier Transform



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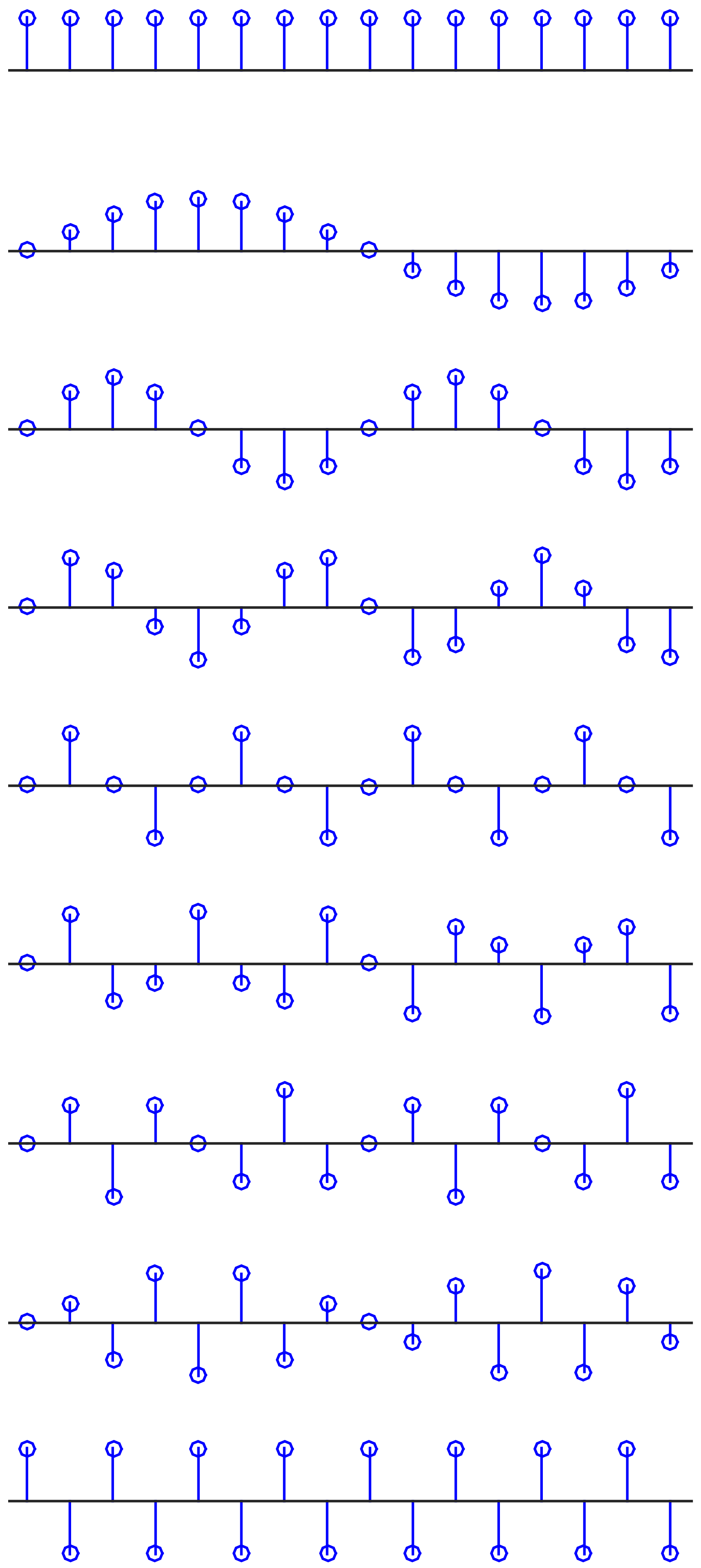
Filters and the Fourier Transform



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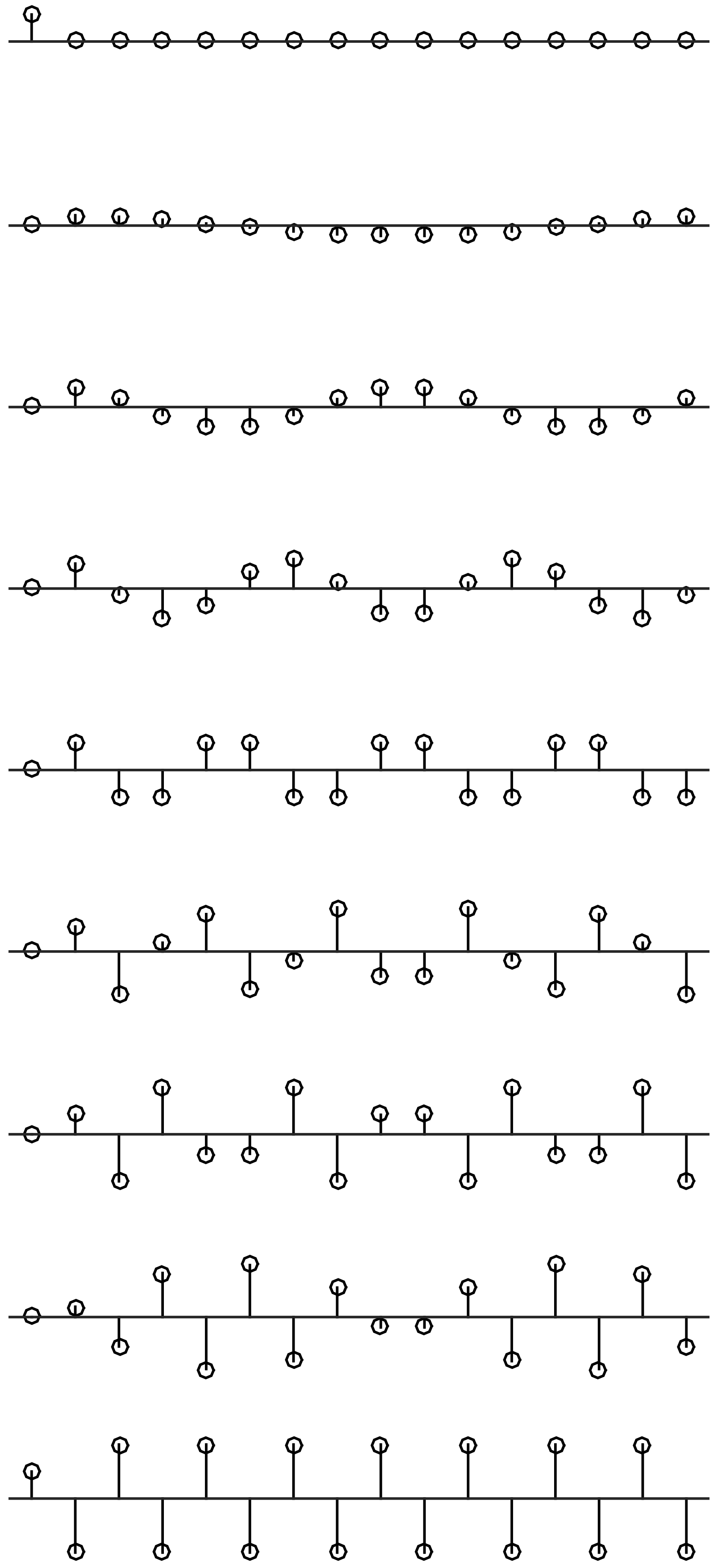
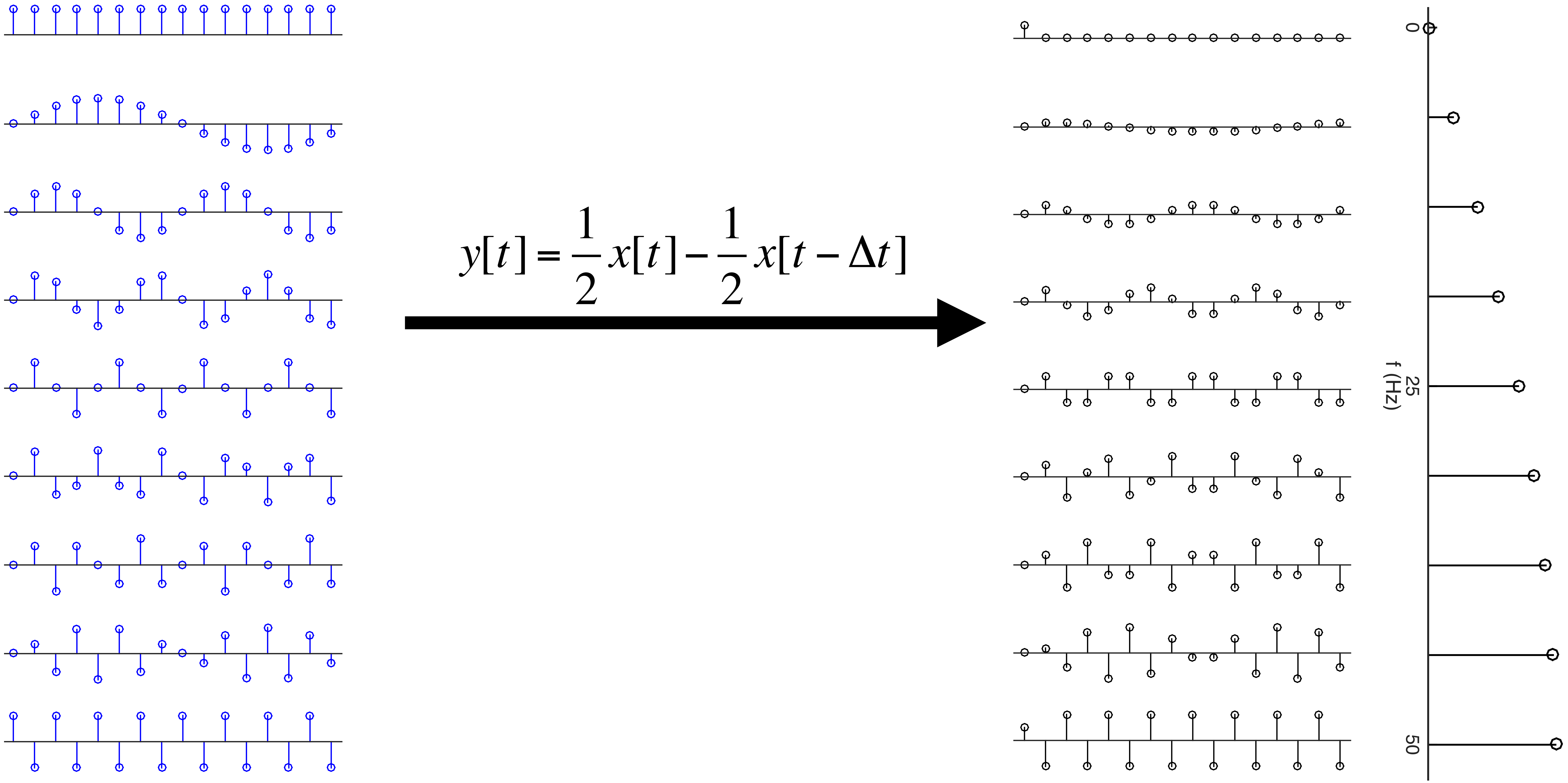


Filters and the Fourier Transform

$$y[t] = \frac{1}{2}x[t] - \frac{1}{2}x[t-\Delta t]$$



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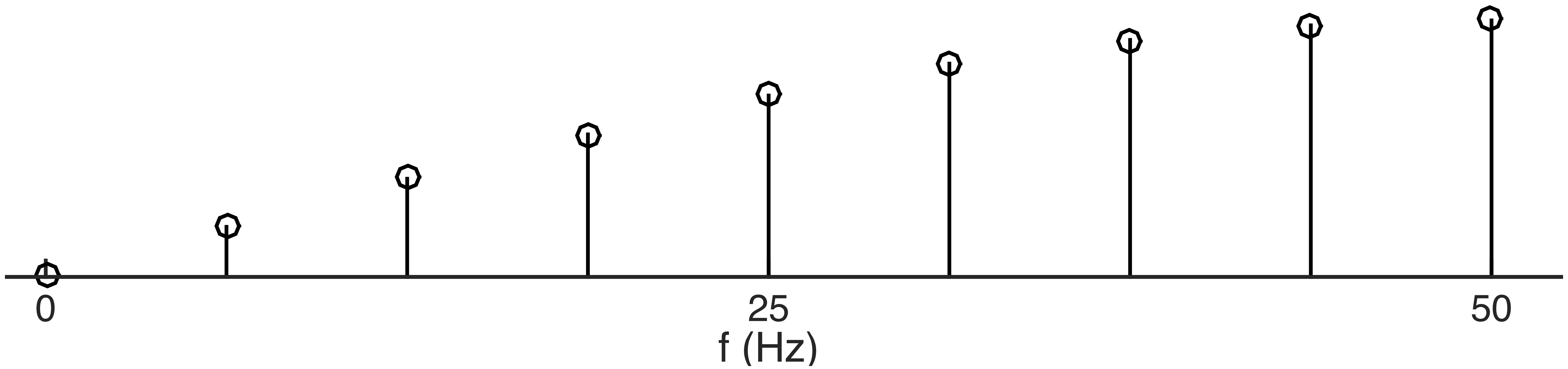
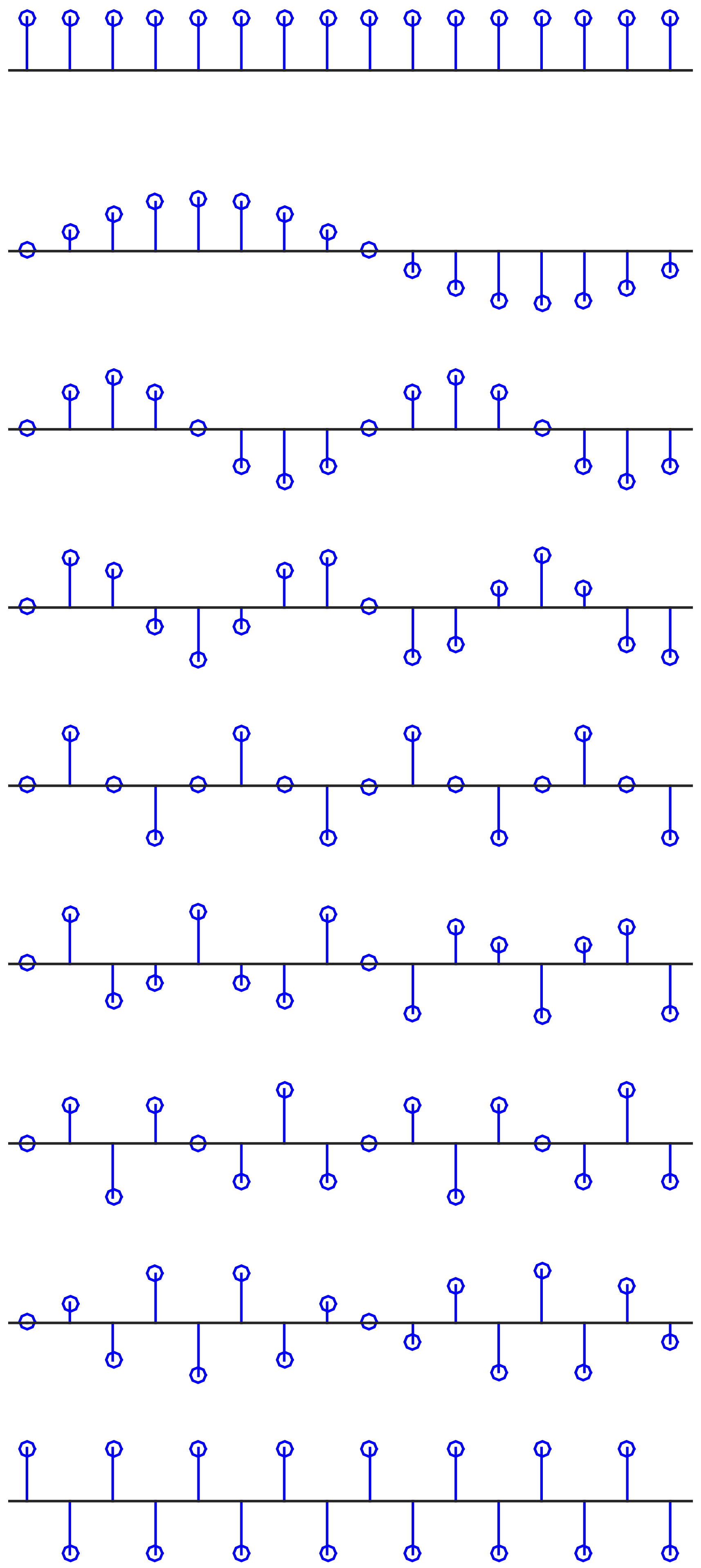


High Pass Filter

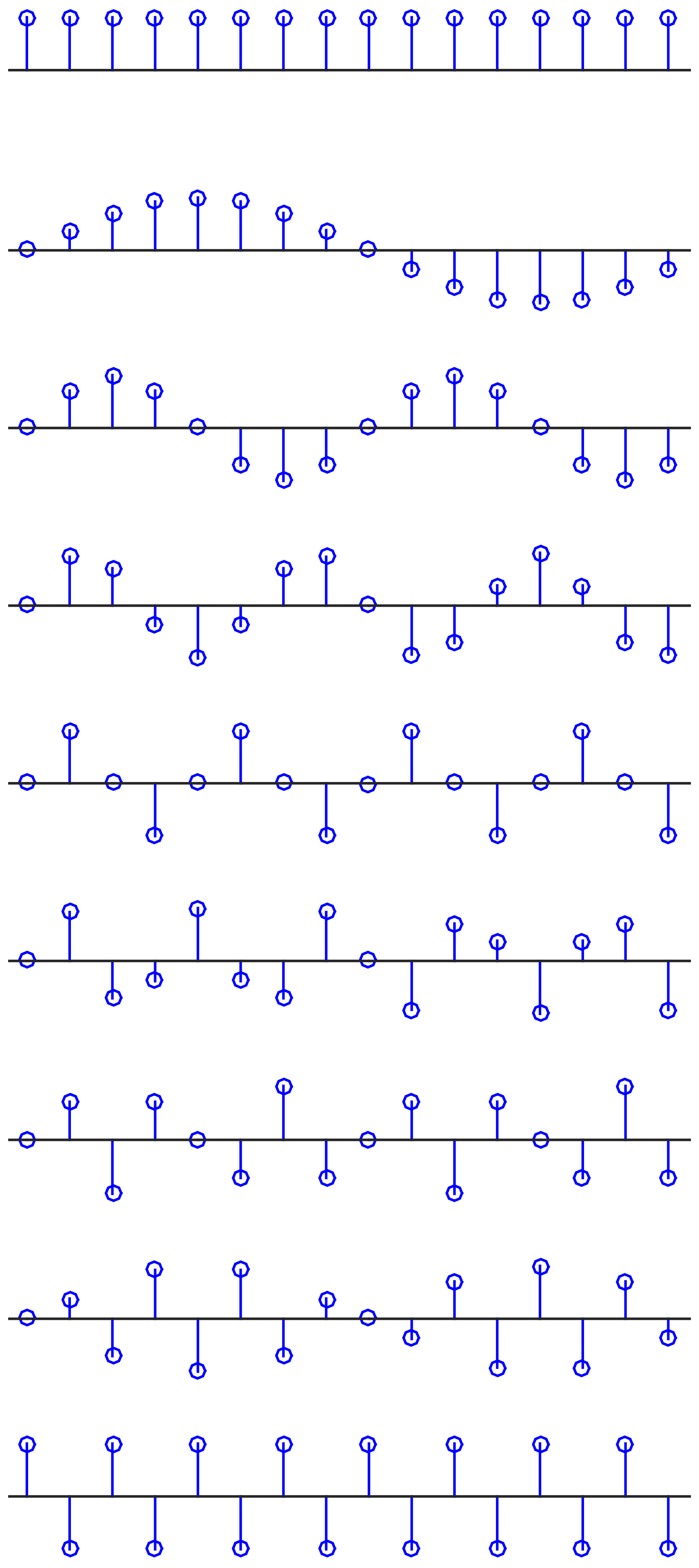
Filters and the Fourier Transform



$$y[t] = \frac{1}{10}x[t] - \frac{9}{10}y[t - \Delta t]$$

?

Filters and the Fourier Transform



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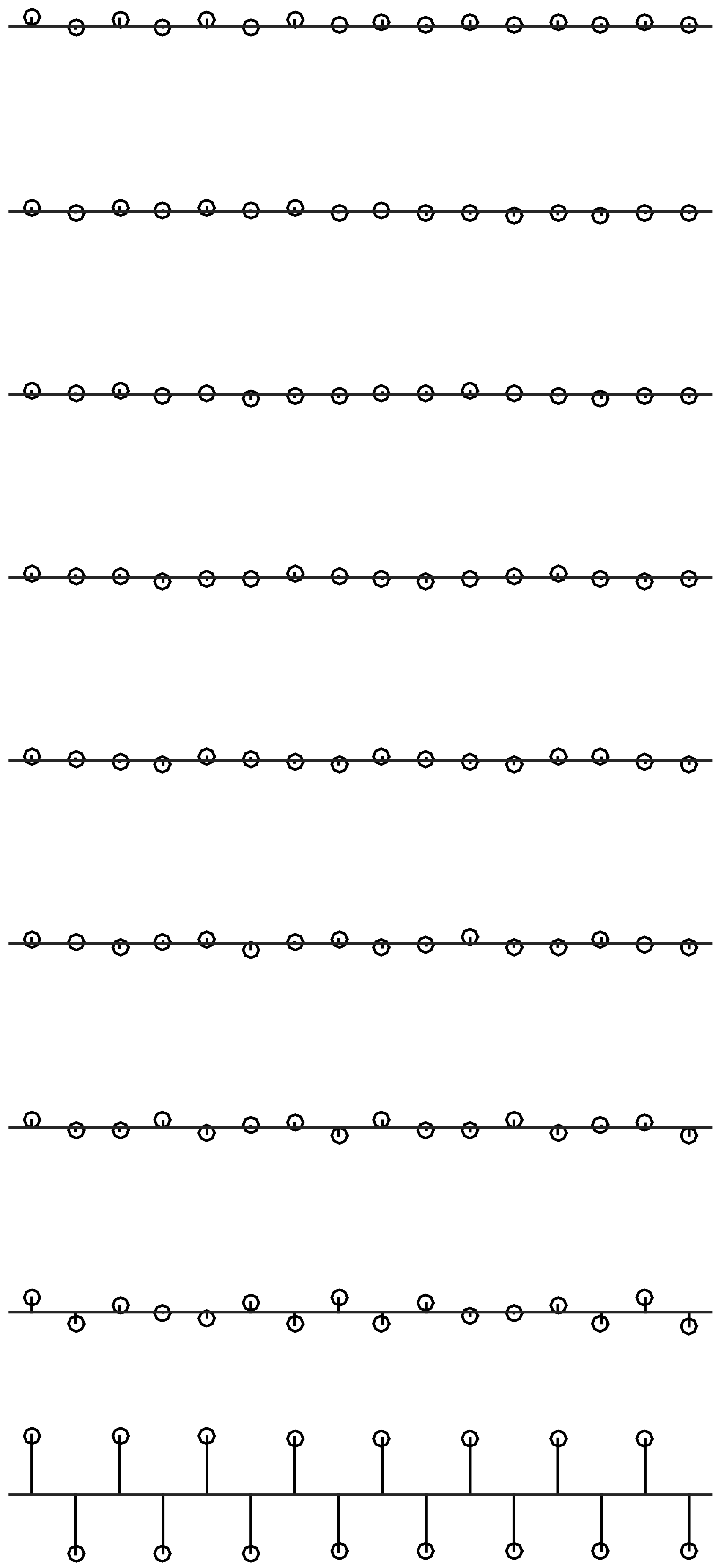

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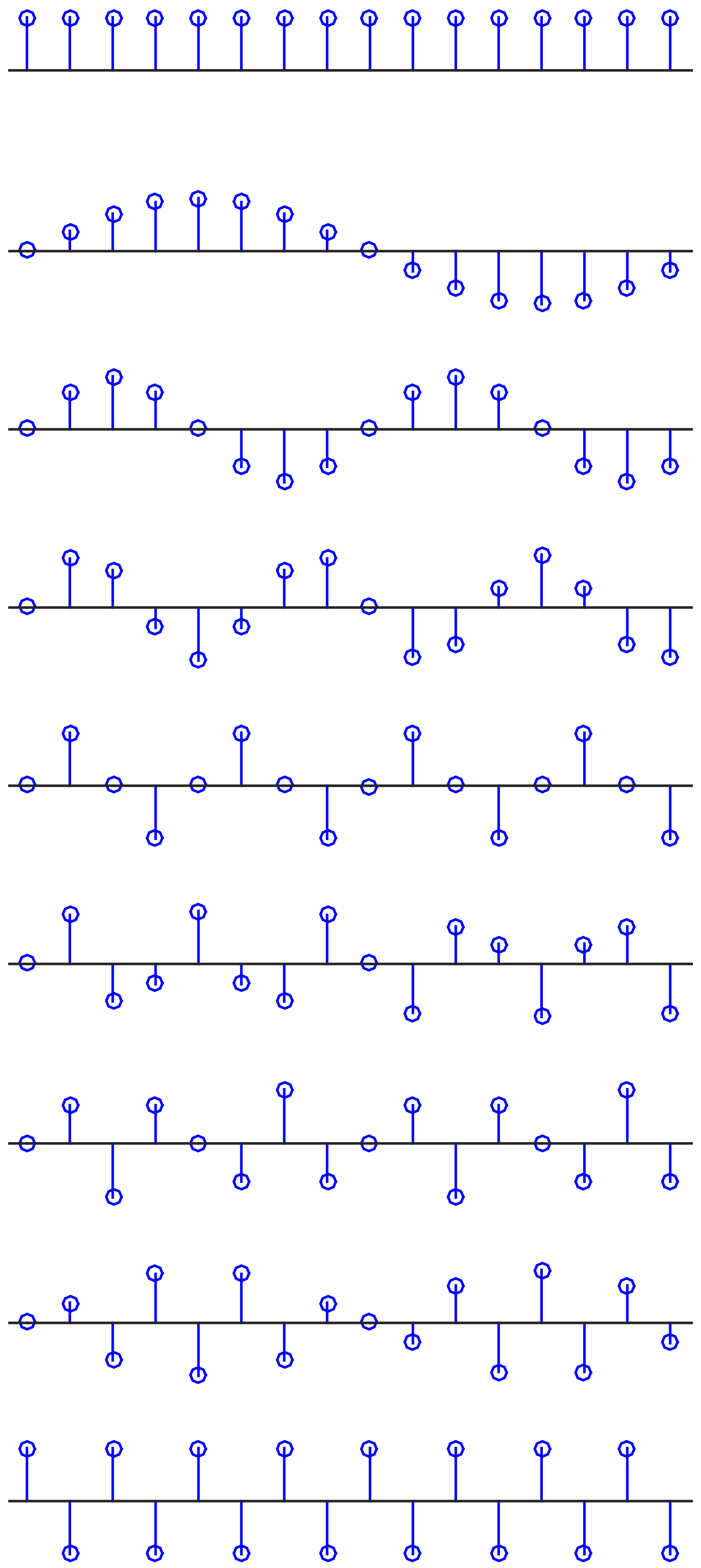

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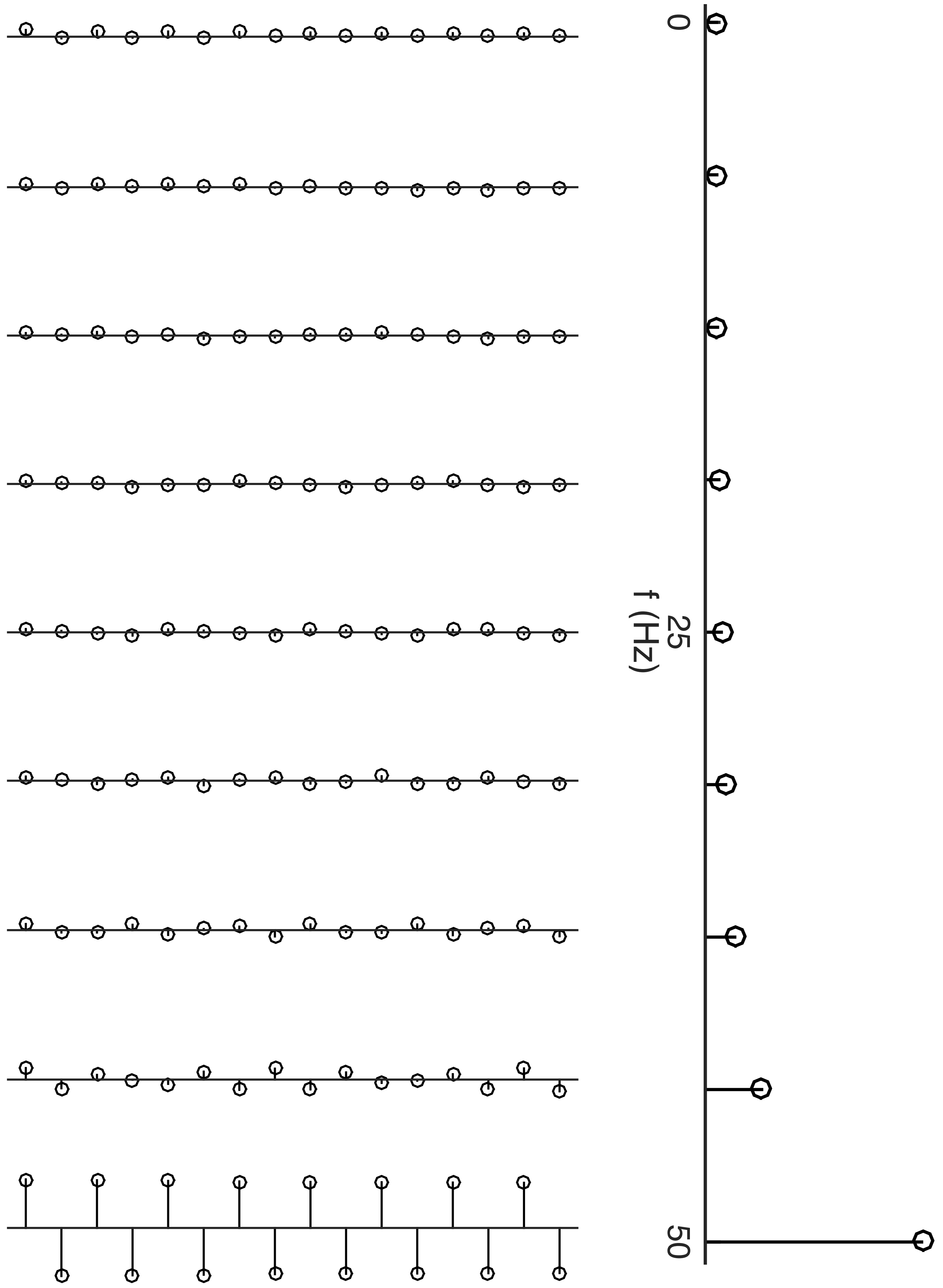

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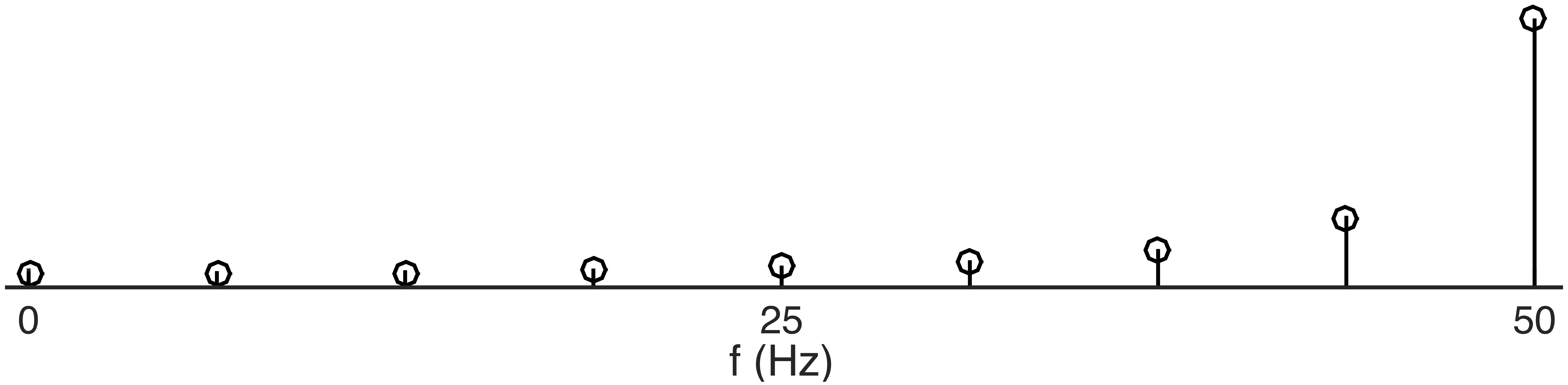

High Pass Filter

Outline

- Fourier Transform: *Why It's Useful*, and *What it Can/Cannot Do For You*
- Filters: *What They Do*, and *How They Do It*
- Filters: *Why So Many Different Kinds? Which Should I Use and When*?
- Grab Bag:
    - *Use Causal Filters; Windowing is Good*; *Low-Pass your Envelopes*

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“Which Filter Should I Use?”

–Every student I’ve ever worked with

Many Filter Decisions

- Frequency Selectivity: Sharp vs. Soft Frequency Transition
- Feedforward Only/Feedback: FIR vs. IIR
- Filter Order: Low order vs. High Order
- Causality: Causal vs. non-Causal (e.g. “zero-phase” filters)
- and more (e.g., FIR: moving average vs. Parks-McClellan, IIR: Butterworth vs. elliptic)

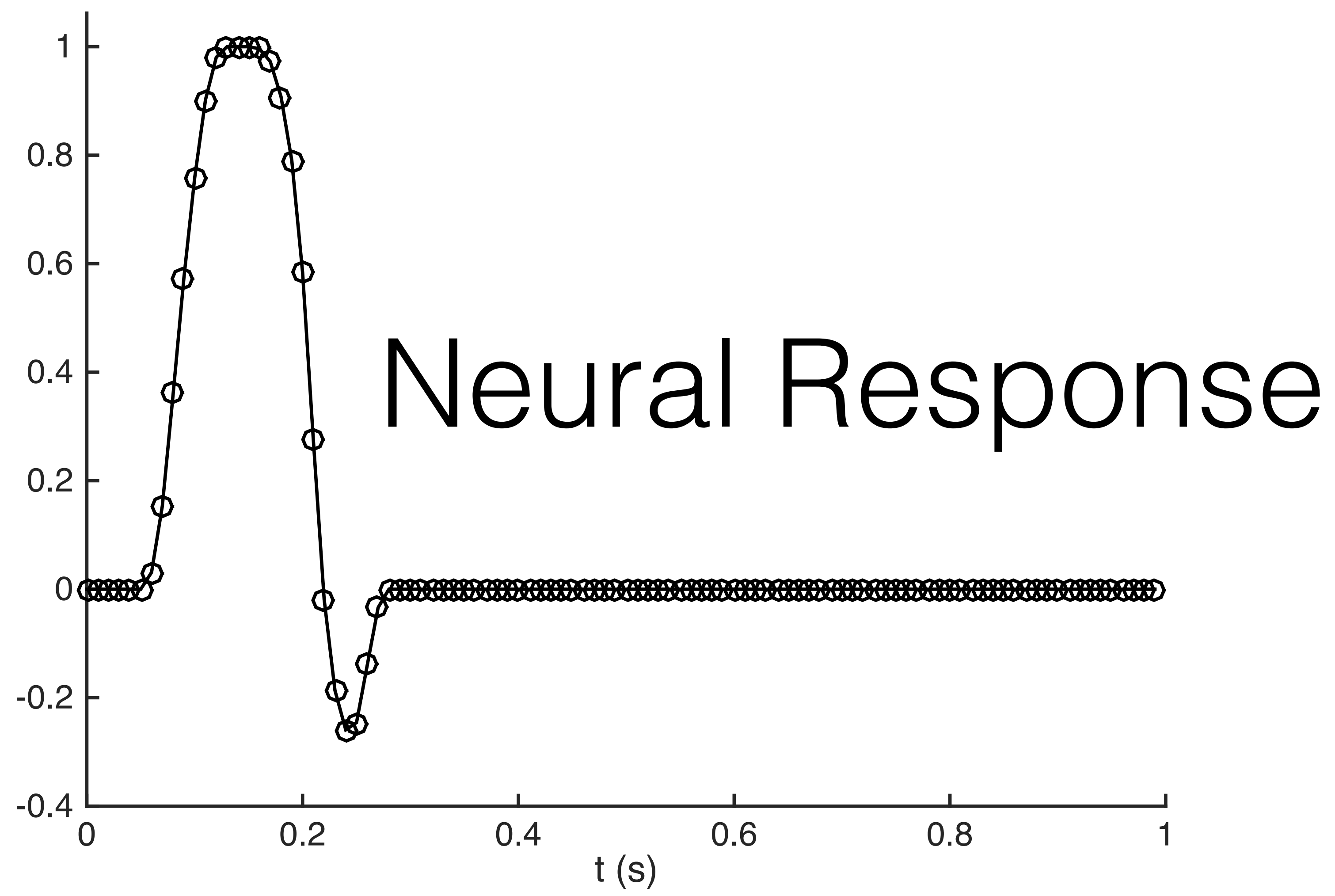
Ideas to Keep in Mind

- Filters modify signals, *by design.*
- There is no such thing as a filter that leaves signals (or signal components) unaltered
- Most filter decisions involve considering a valid tradeoff
  - Don't go overboard one way or the other
- Some filter decisions allow one to avoid artifacts without any tradeoff

Frequency Selectivity/Transitions

- Time and Frequency are inextricably linked.
  - Changing the frequency content of a signal will change the temporal content of the signal.
    - Low-Pass Filters will lengthen fast temporal changes
    - High-Pass Filters will remove slow transitions from one baseline to another
  - Sharp frequency transitions produce artificial temporal elongation: "ringing".

Ringing Artifacts



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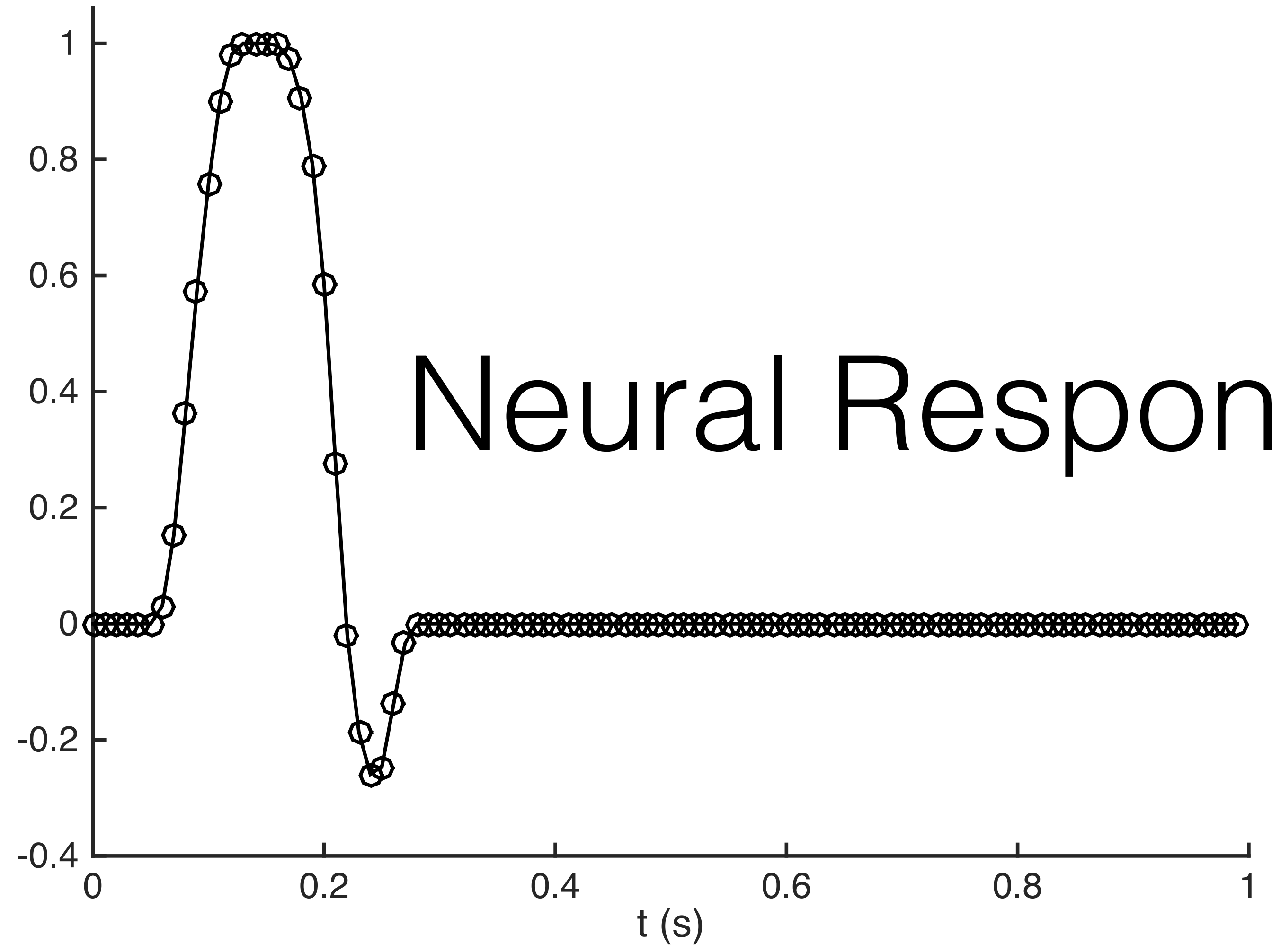
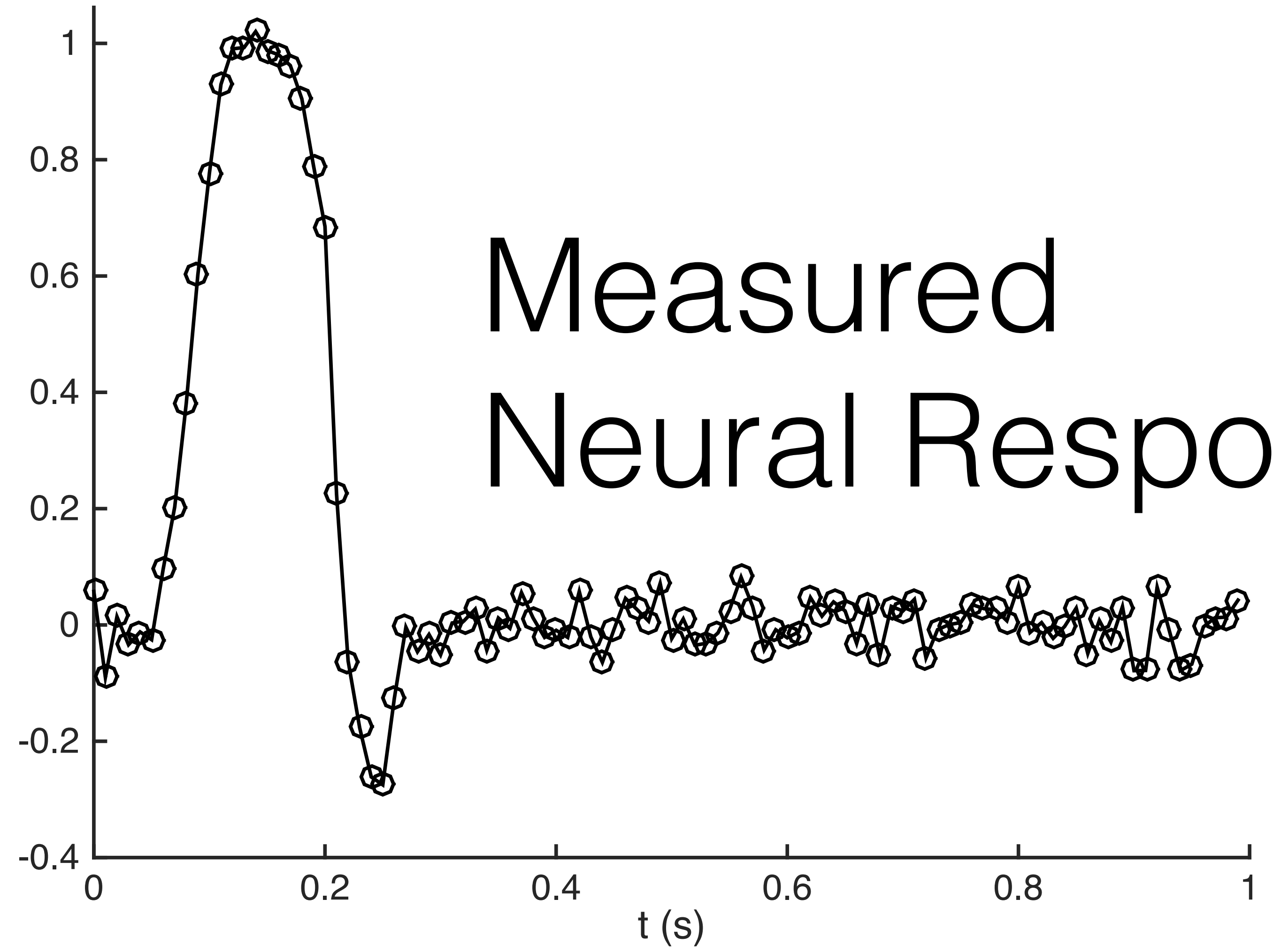
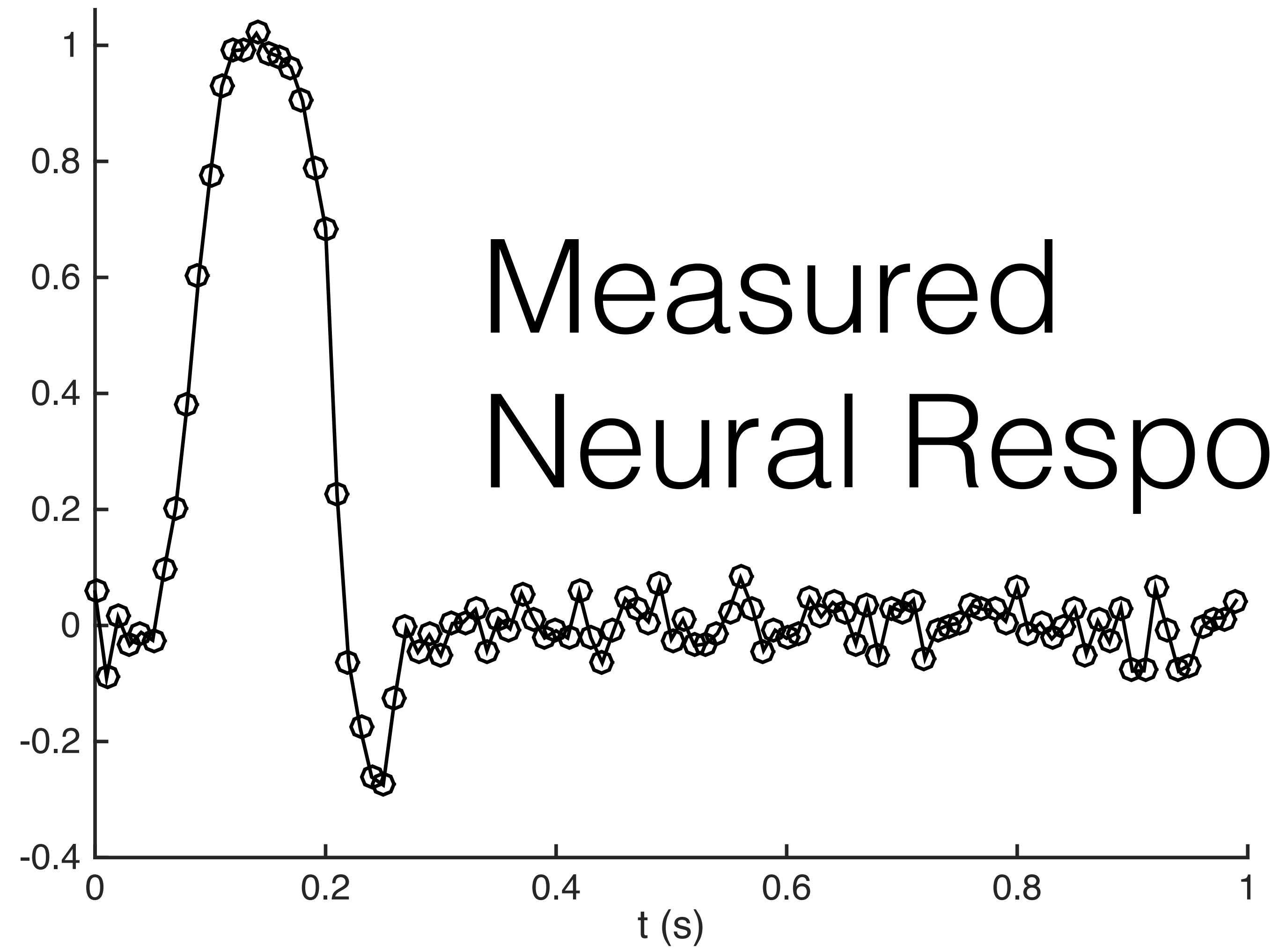


Neural Response



Measured
Neural Response

Ringing Artifacts



Measured
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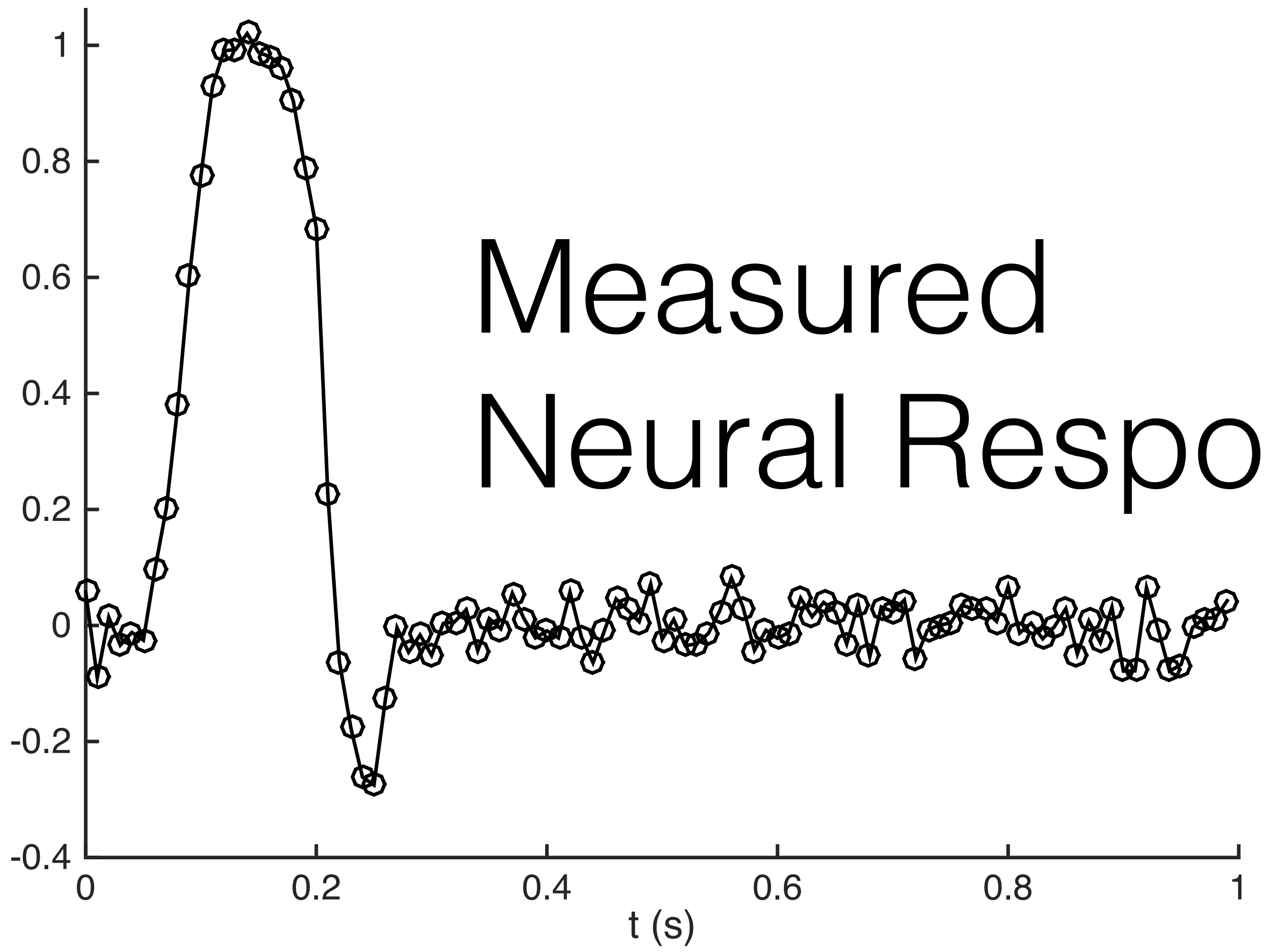

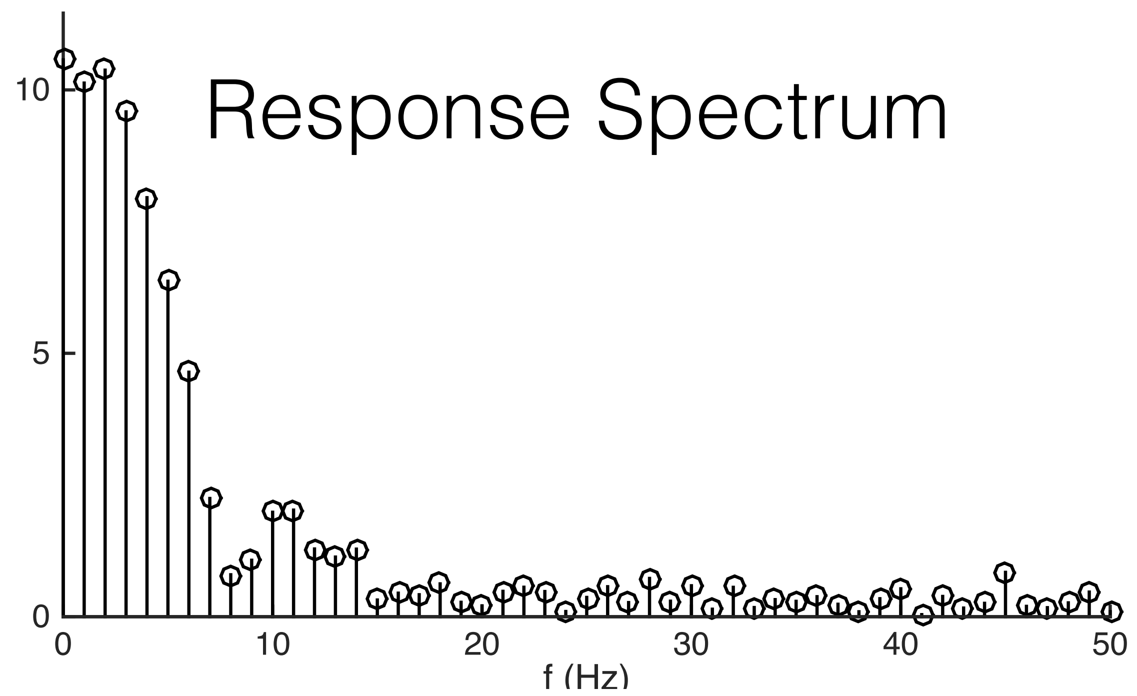

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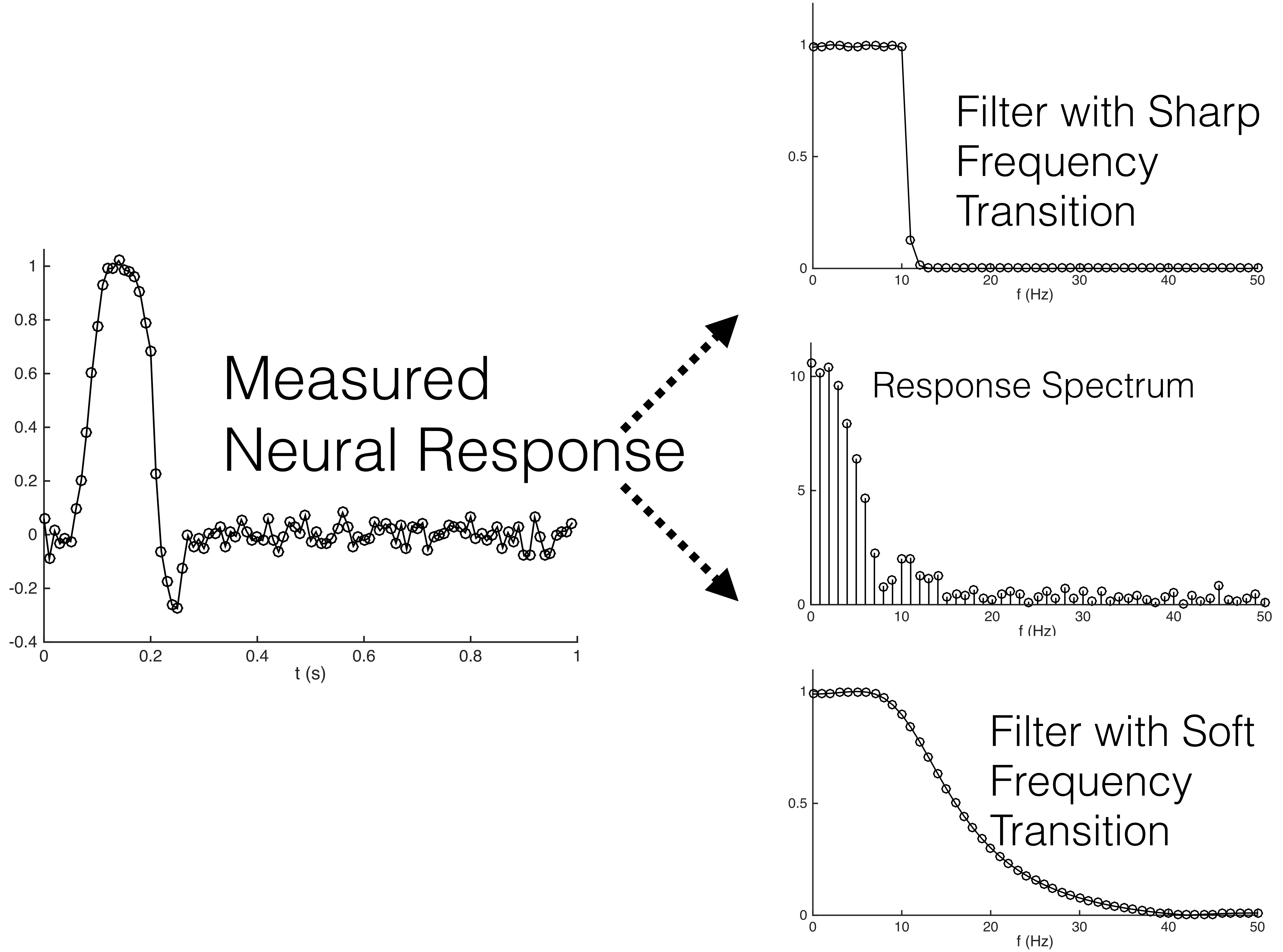

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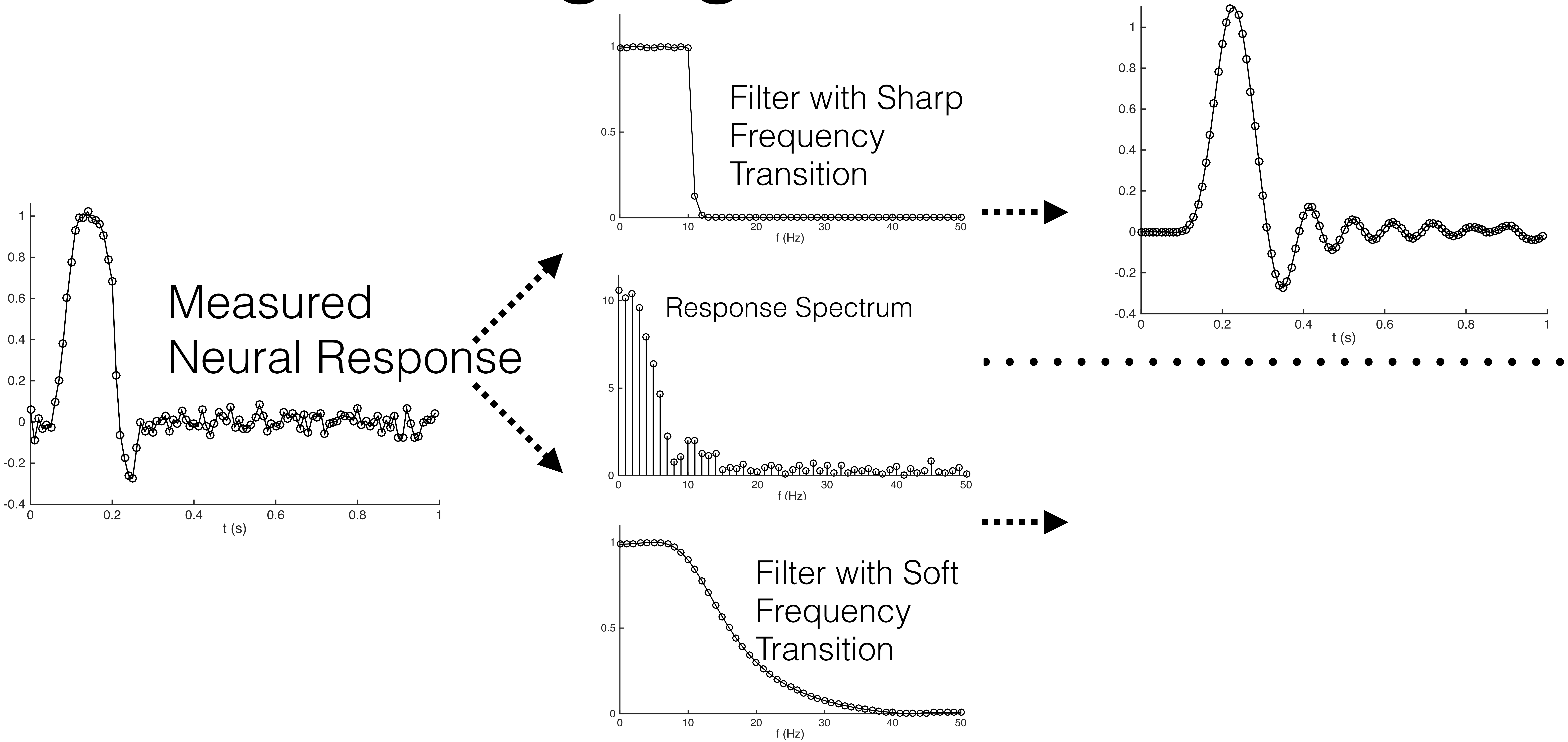

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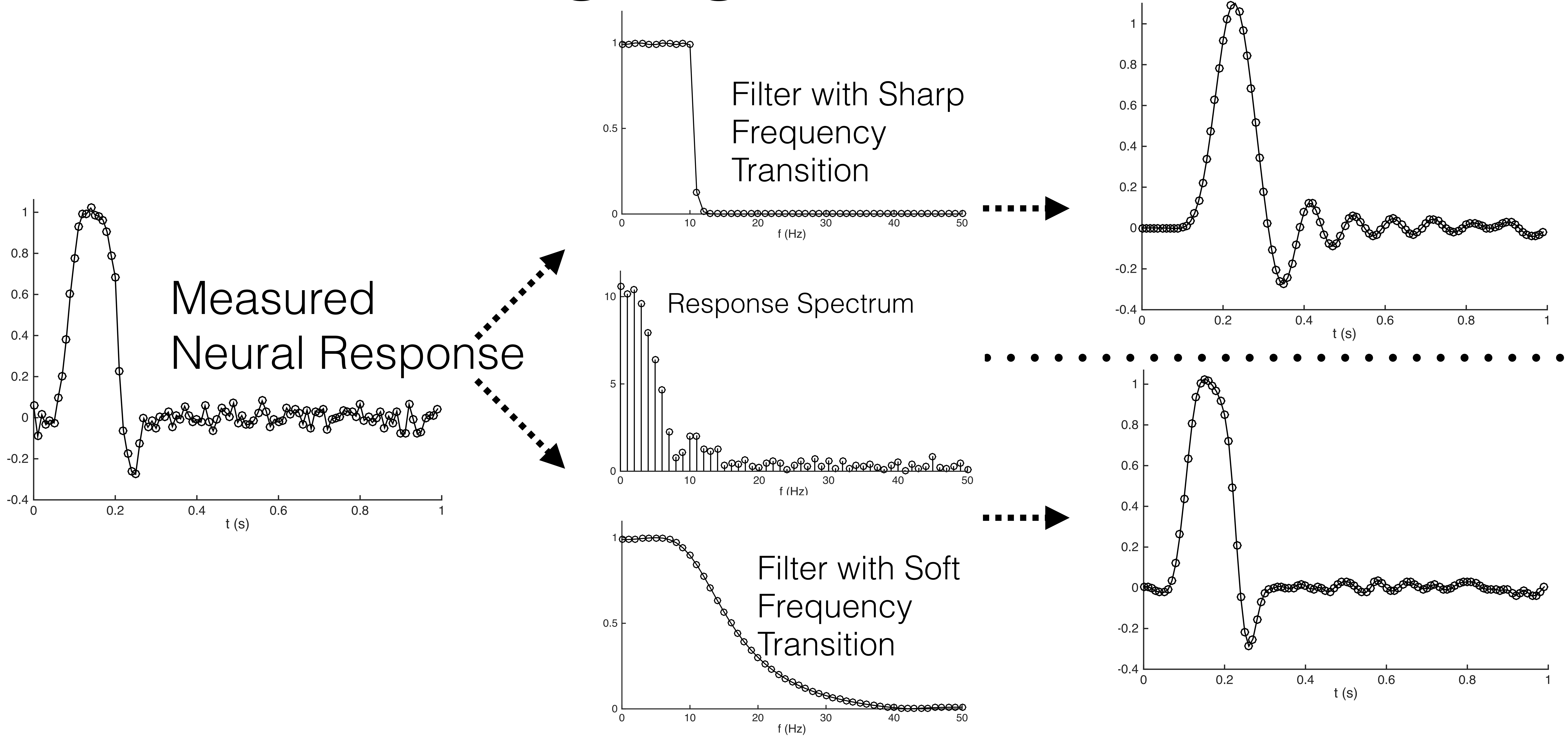
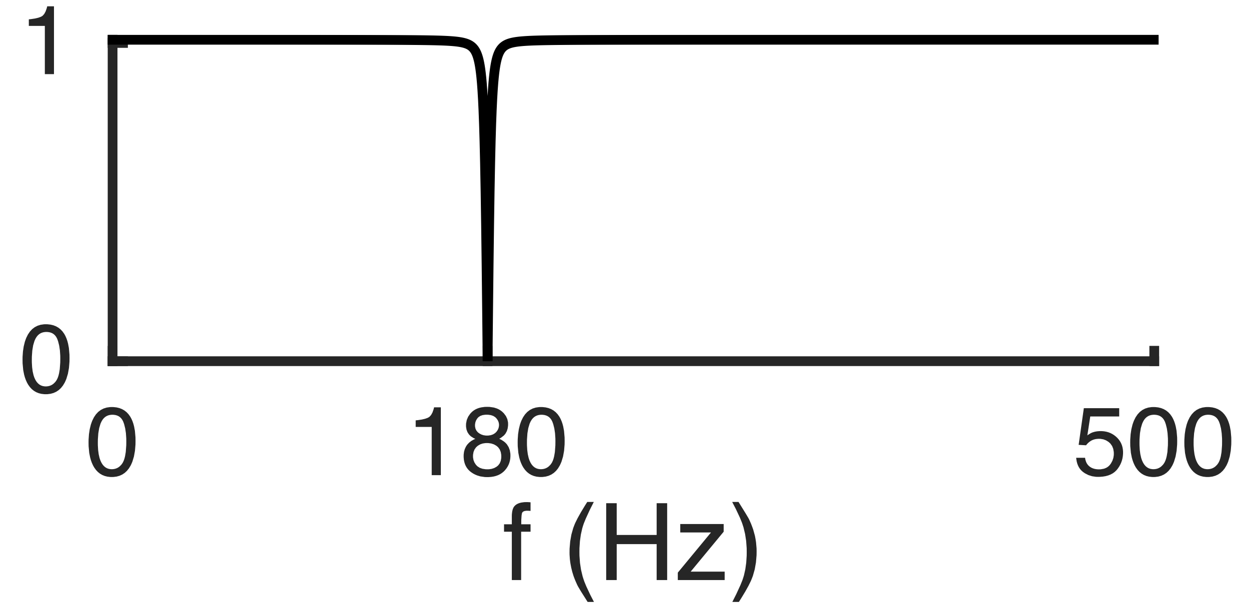


Ringing Artifacts

- Sharp Frequency Transitions are sometimes Necessary
  - e.g., Notch filters (and related filters, such as Comb filters)
- In these cases there will be unavoidable ringing

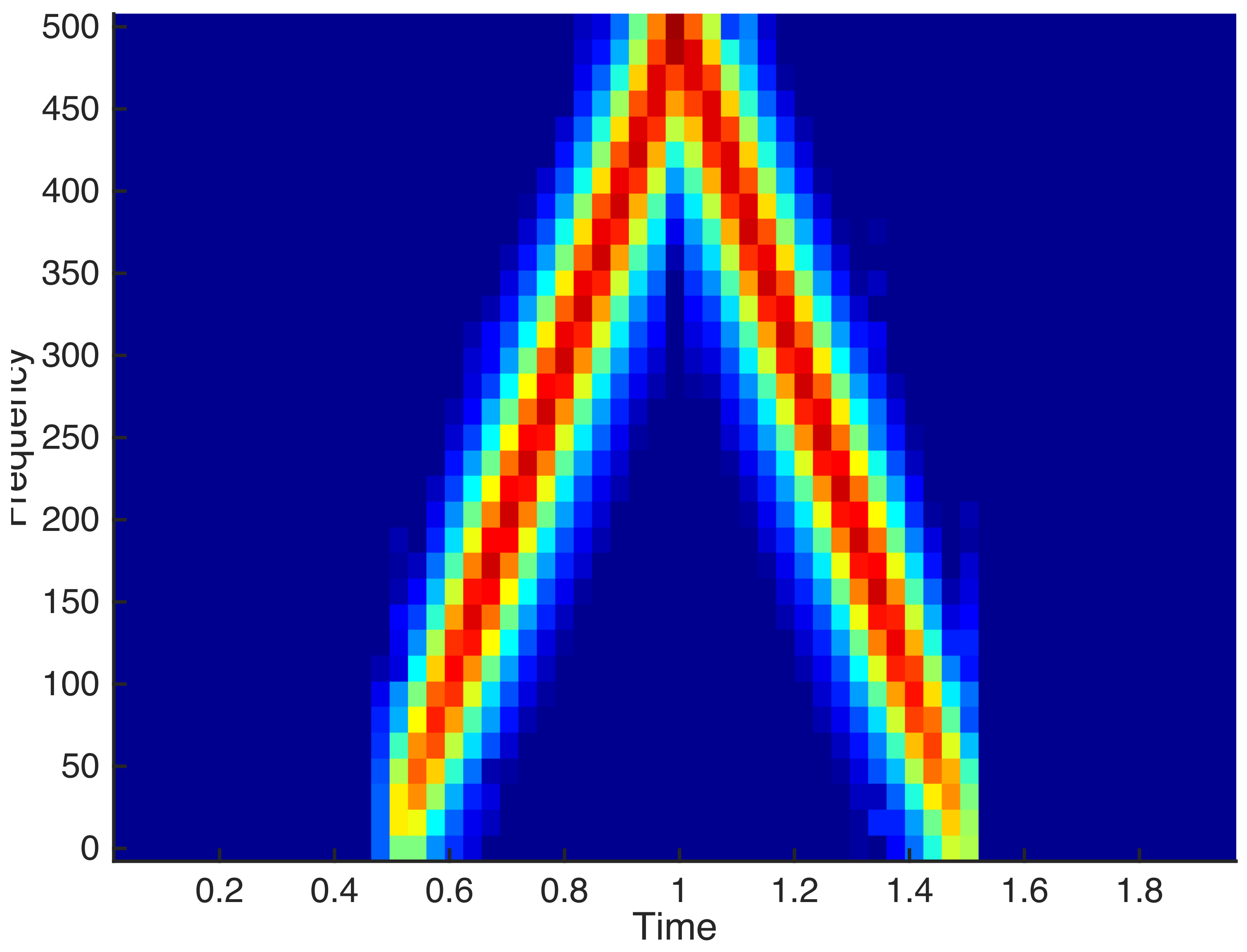
Ringing Artifacts

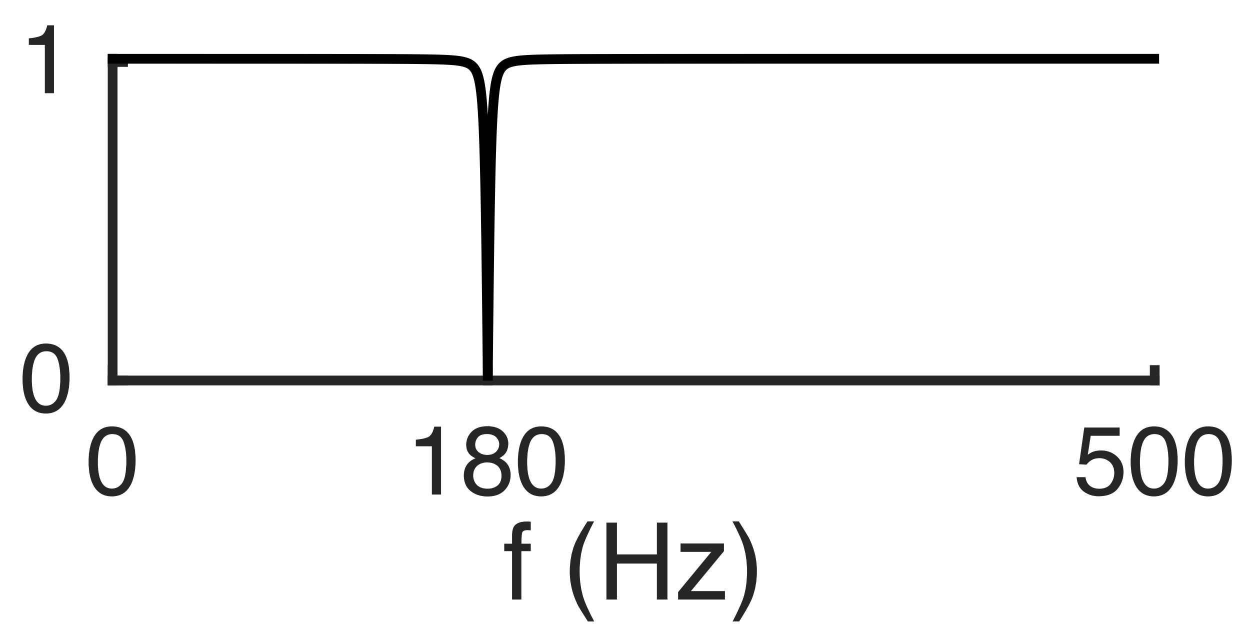


Notch Filter
(Sharp Frequency Transition)

Ringing Artifacts

FM Sweep
(Spectrogram)





Notch Filter
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FM Sweep
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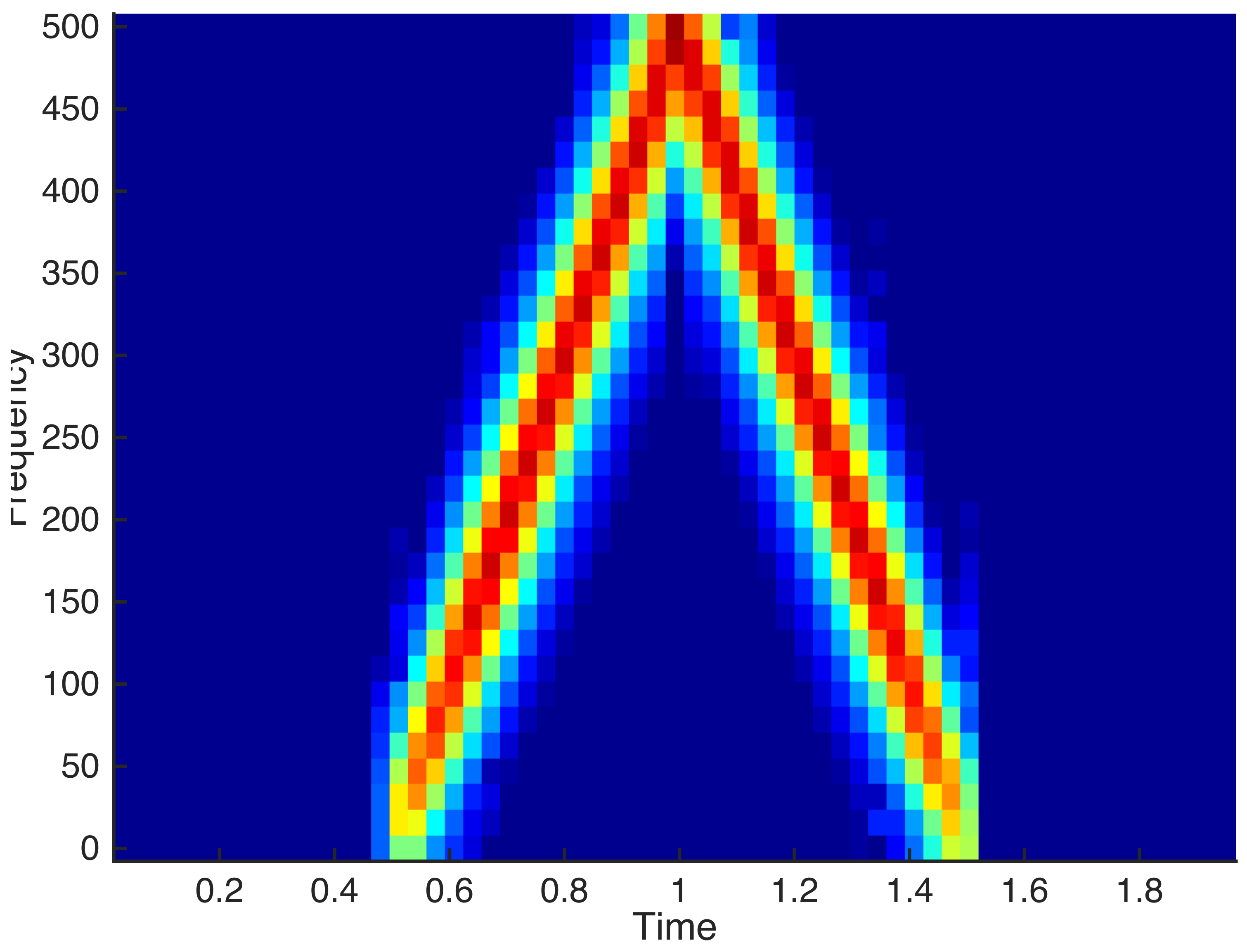
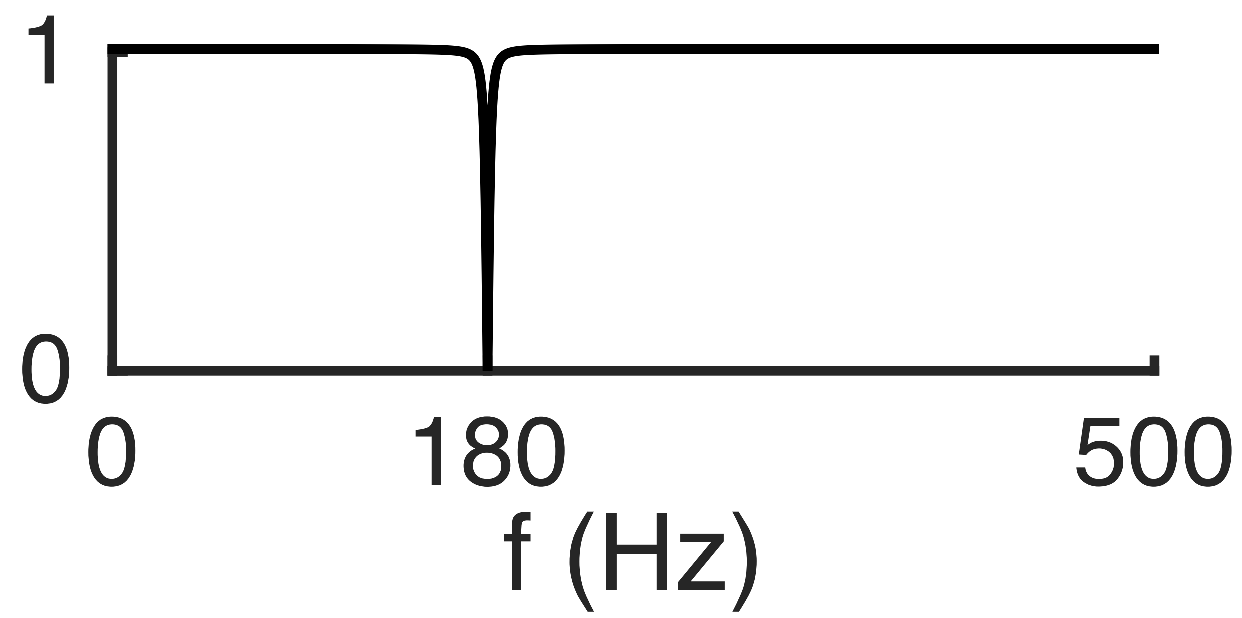
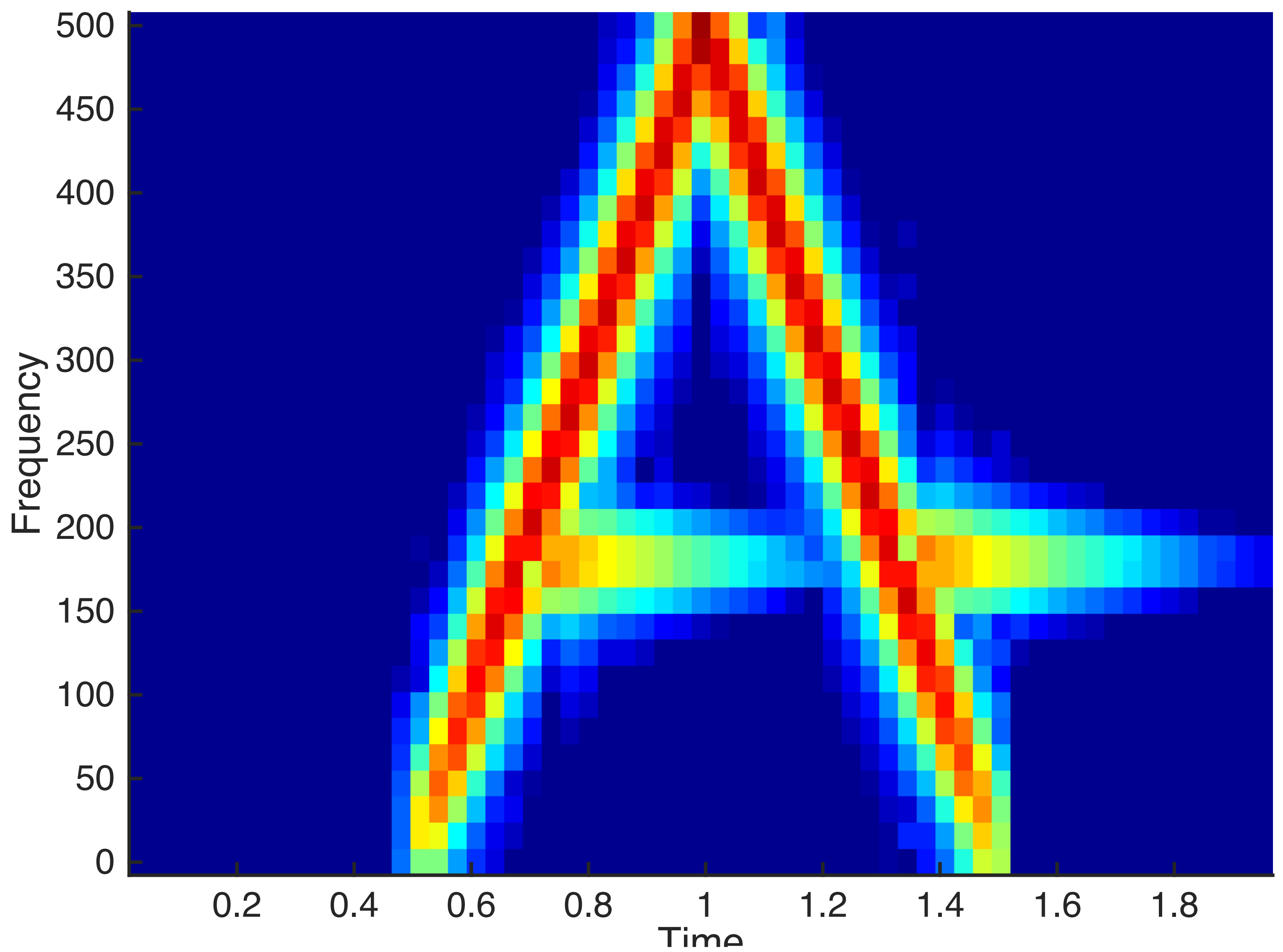




Notch Filter
(Sharp Frequency Transition)

Notched FM Sweep
(Spectrogram)

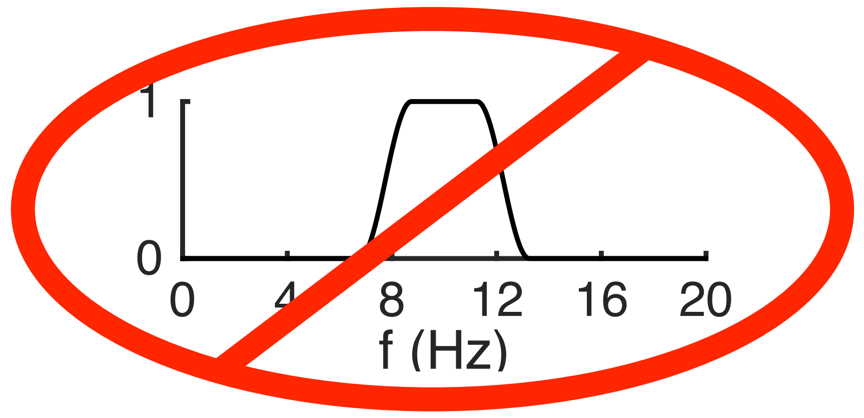


Notch too brief to see
But ringing clear:

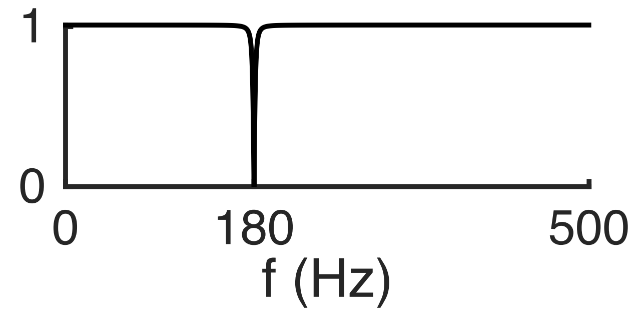
- narrowband
- extended in time

Take care, but don’t overreact

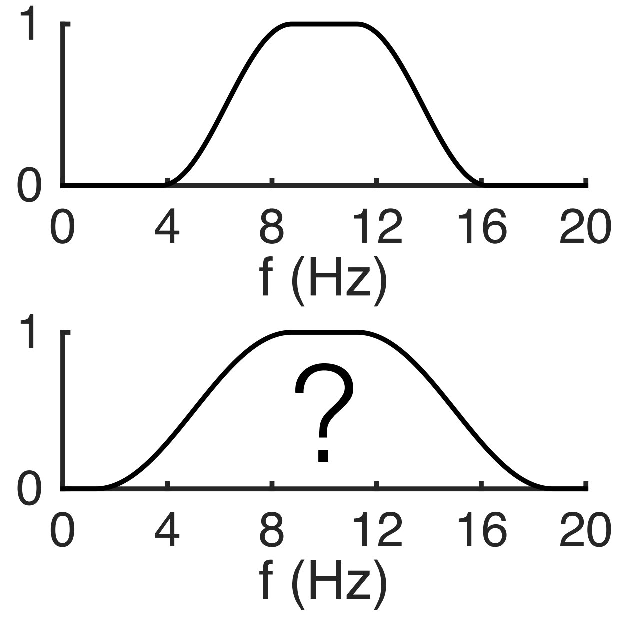
- Avoid Ringing by avoiding sharp frequency transitions



- If sharp frequency transitions are necessary (as for notch filtering), ringing may follow



- Don’t overly soften frequency transitions or you’ll lose frequency selectivity



FIR vs. IIR

- FIR (finite impulse response): Feedforward only
  - Examples: Moving Average (*avoid, in general*), Parks-McClellan ("Optimal"), others
- IIR (infinite impulse response): Feedback also incorporated
  - Instability a potential issue
  - Examples: Butterworth (*not awful, but not great*), Chebyshev, Elliptic (*very good*), others

FIR vs. IIR: How to choose?

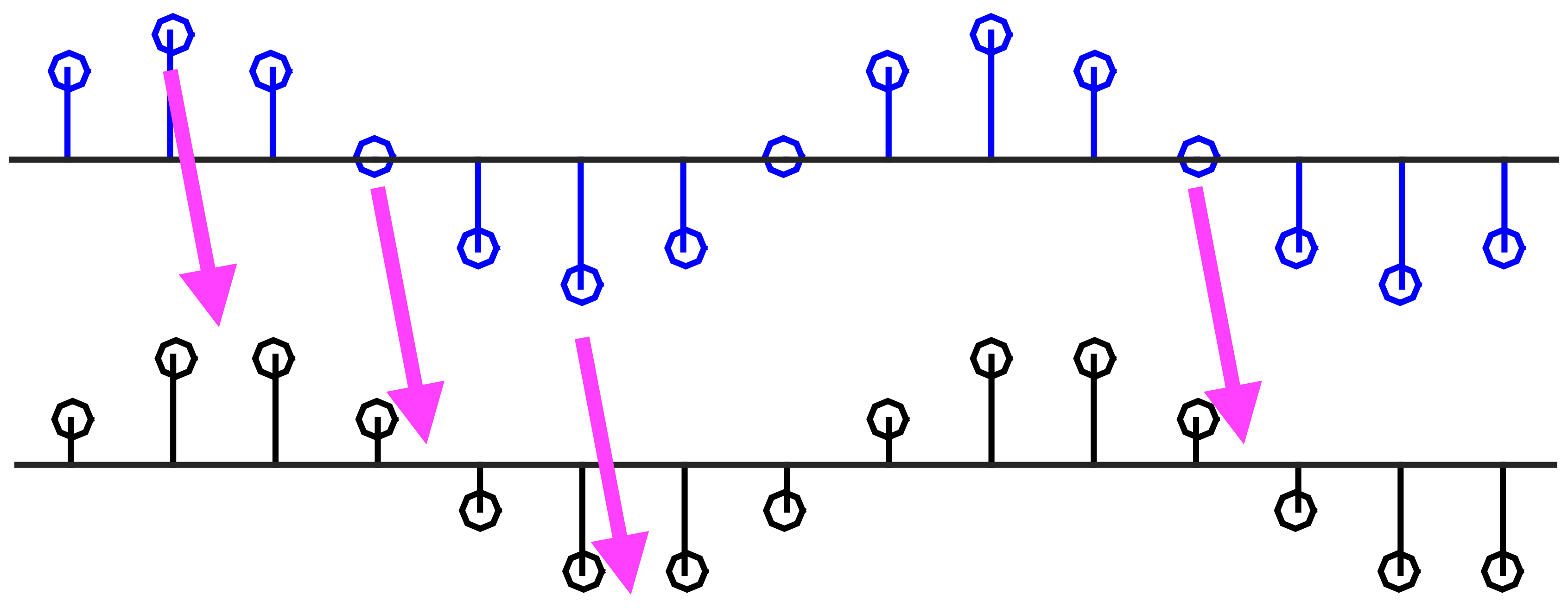
- No universal answer. It may depend on:
  - *group delay (*signal delay intrinsic to filter): group delay value and group delay frequency dependence
  - signal loss due to filter startup (dependence on signal values before signal starts)
  - stability concerns (if IIR filter)
  - more…

Group Delay

- Intrinsic to filtering—cannot be removed
- Filtering changes signals by design—all filters change temporal features of the signal
- Causal filters always incur delay

Group Delay Examples

$$y[t] = \frac{1}{2}x[t] + \frac{1}{2}x[t - \Delta t]$$



τ_D: $\frac{\Delta t}{2}$ (?)

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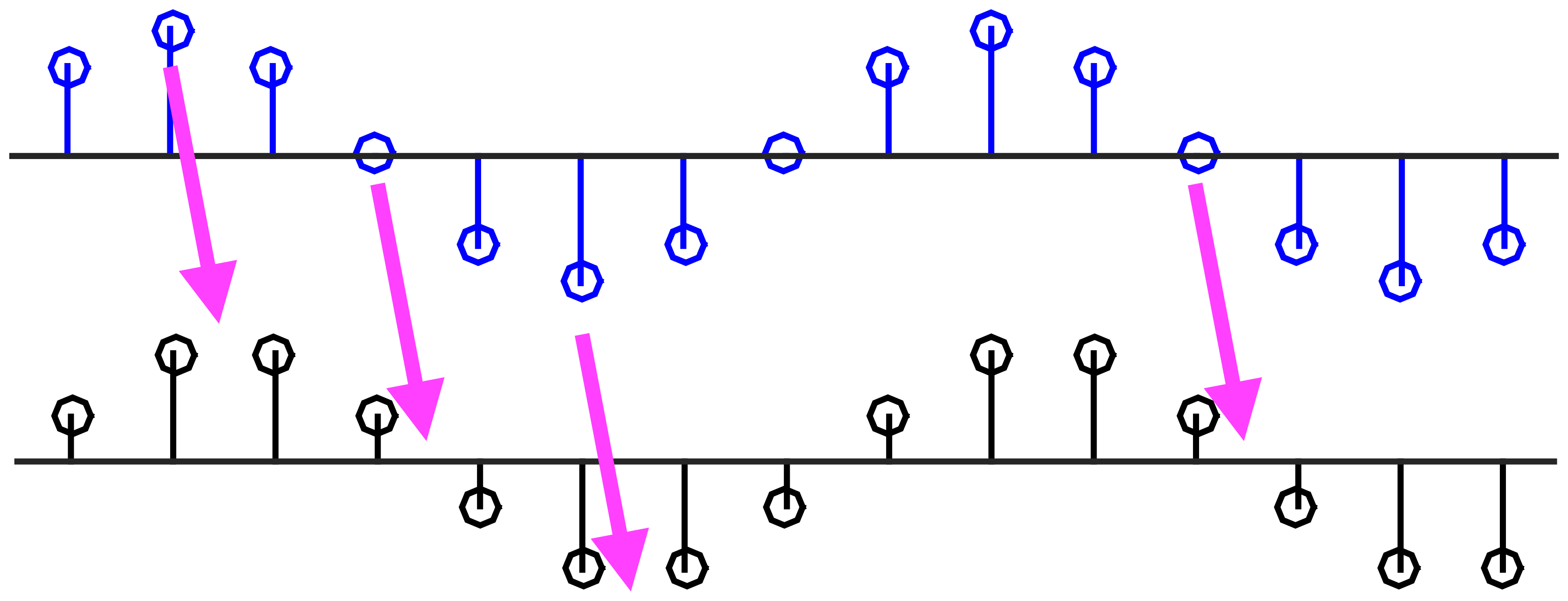

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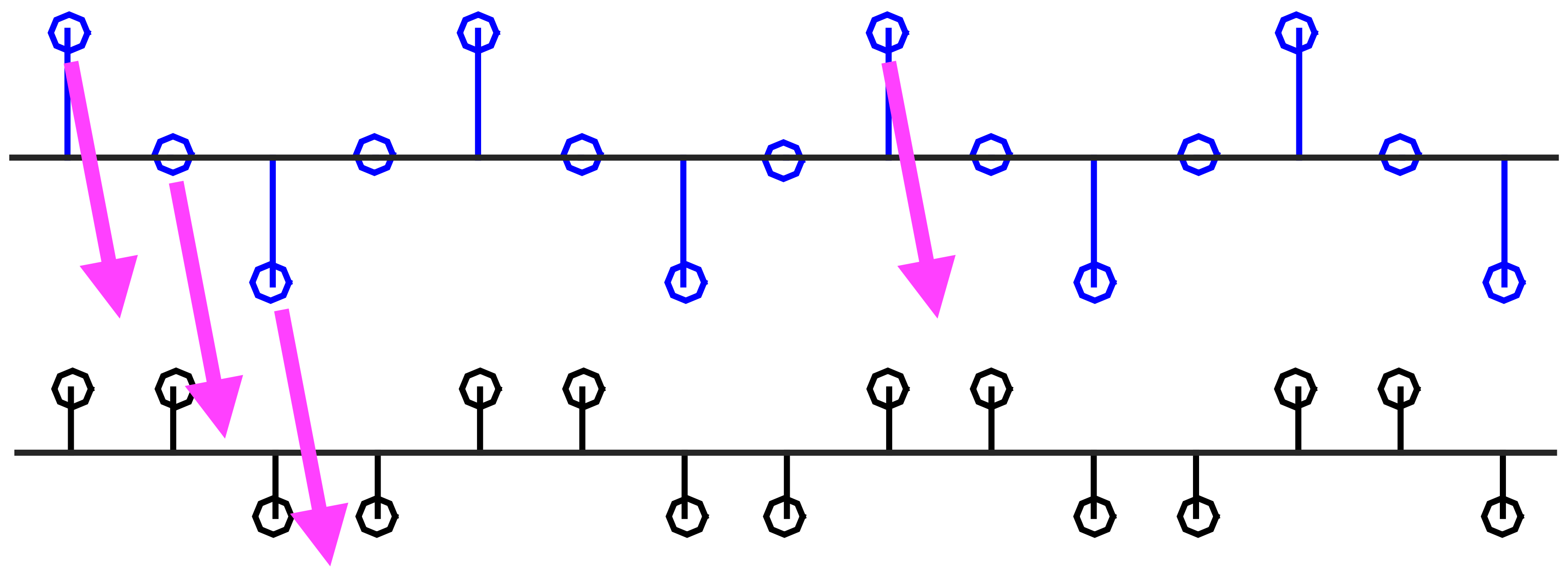
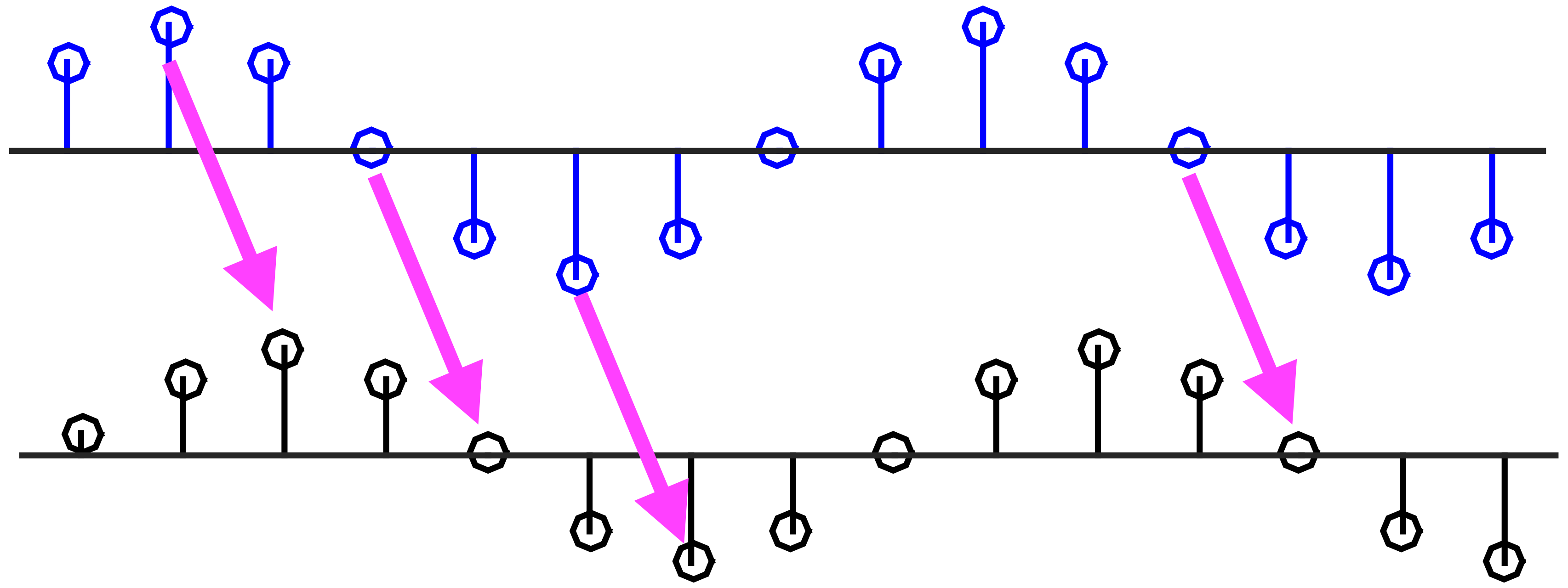


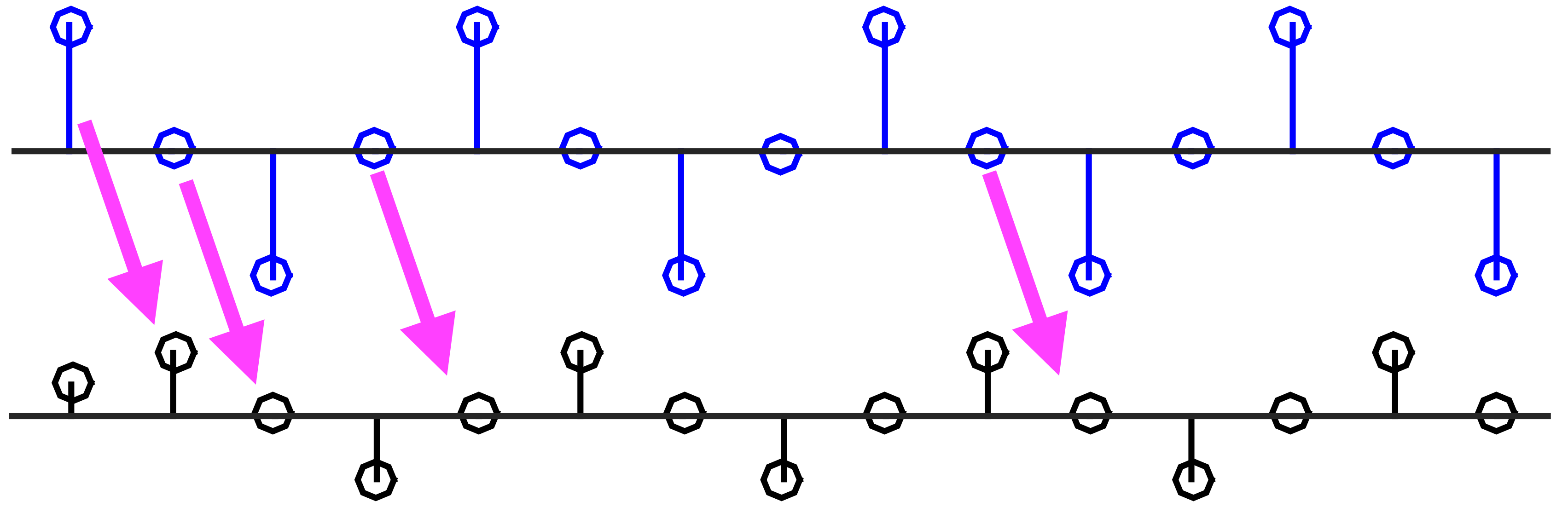
τ_D: $\frac{\Delta t}{2}$(?)

Group Delay Examples

$$y[t] = \frac{1}{4}x[t] + \frac{1}{2}x[t-\Delta t] + \frac{1}{4}x[t-2\Delta t]$$



τ_D: Δt



τ_D: Δt

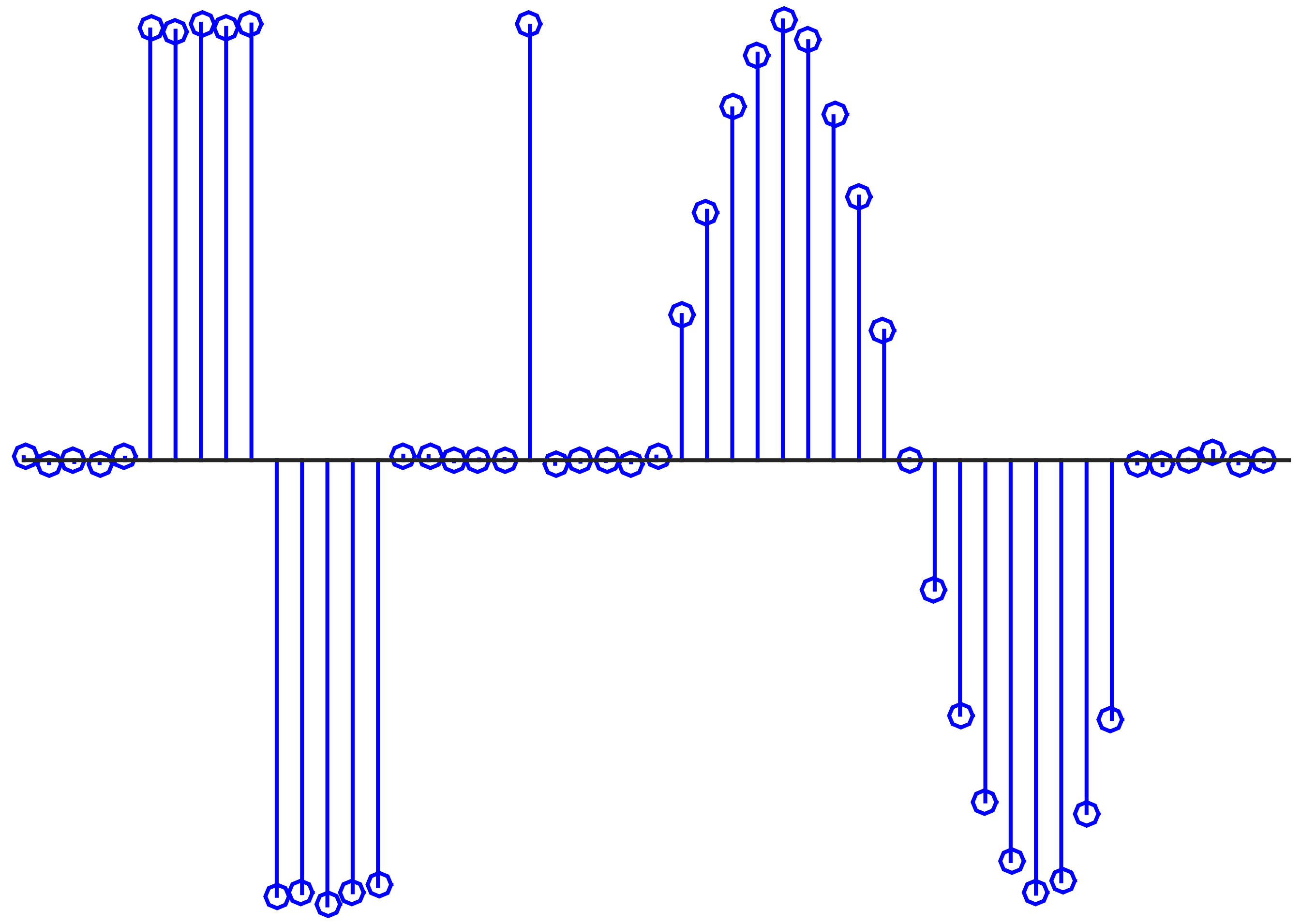
Group Delay: FIR filters

- Group delay corresponds to "average" delay imparted by *time-shifted* filter terms.
- The group delay of an FIR filter does not depend on frequency.
- The *order* of an FIR filter, N_{order}, is the number of time shifts used by the most delayed component, minus 1 (i.e. one less than the length of the filter).
- The group delay of an FIR filter is $\Delta t \times N_{order}/2$.
    - The higher the order, the longer the group delay
    - Calculating latencies? You may need to compensate (OK for *peak* latencies).
    - The smaller Δt, the smaller the delay, so if possible filter at high sampling frequency.

Group Delay: FIR filters

For non-sinusoidal (multi-frequency) signals, group delay still applies, but *how* it manifests depends on the specific signal features.

$$x[t]$$



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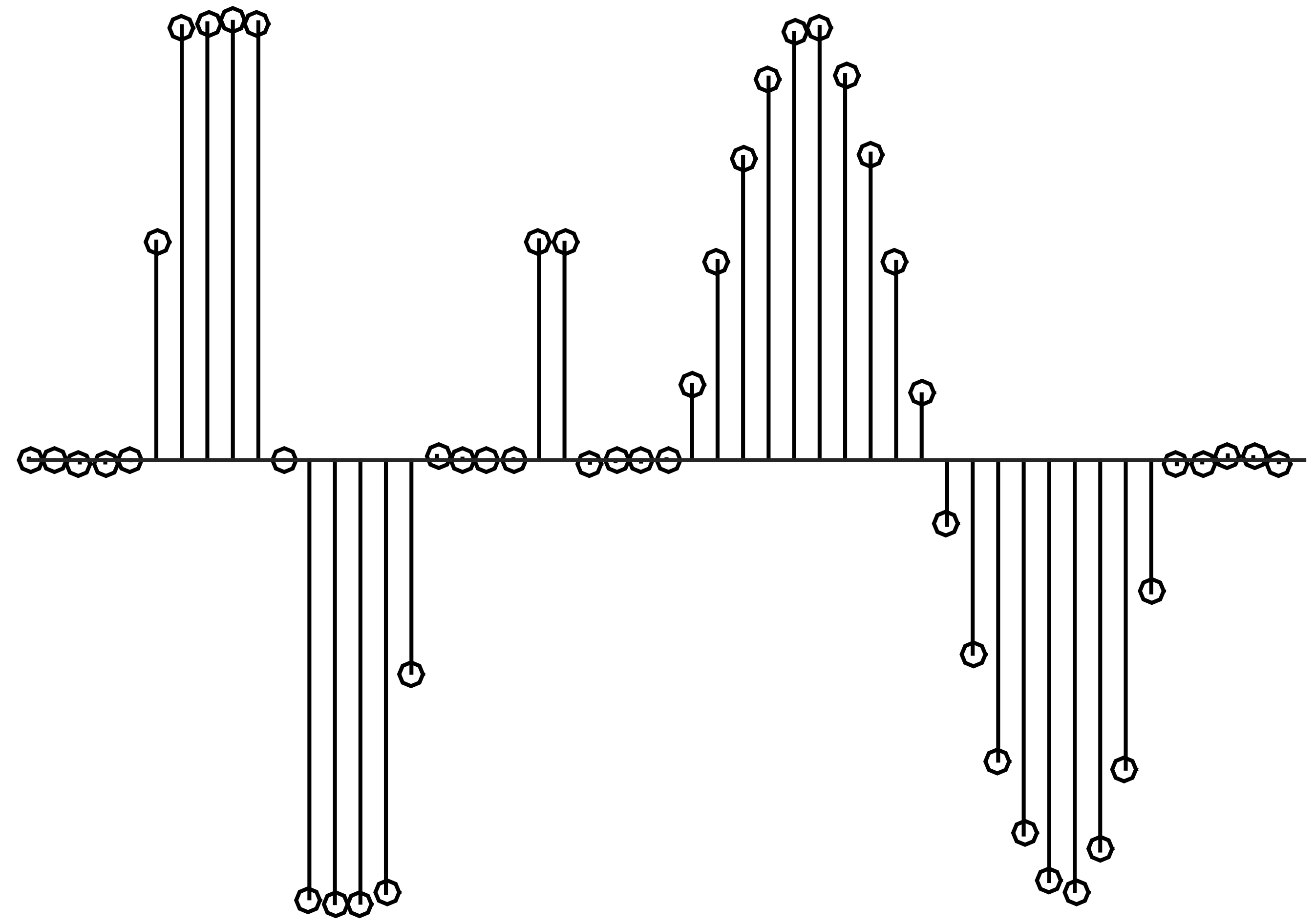

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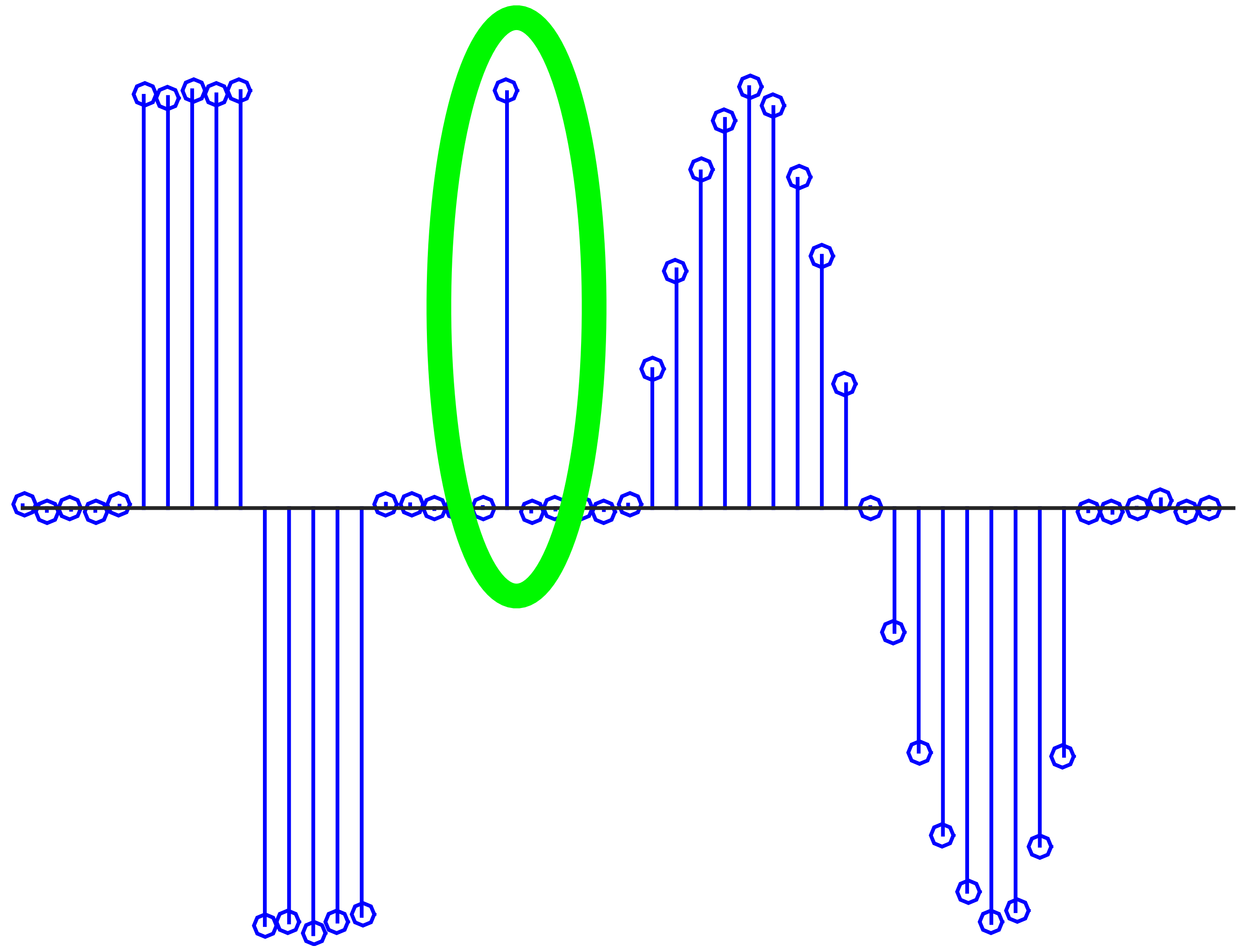

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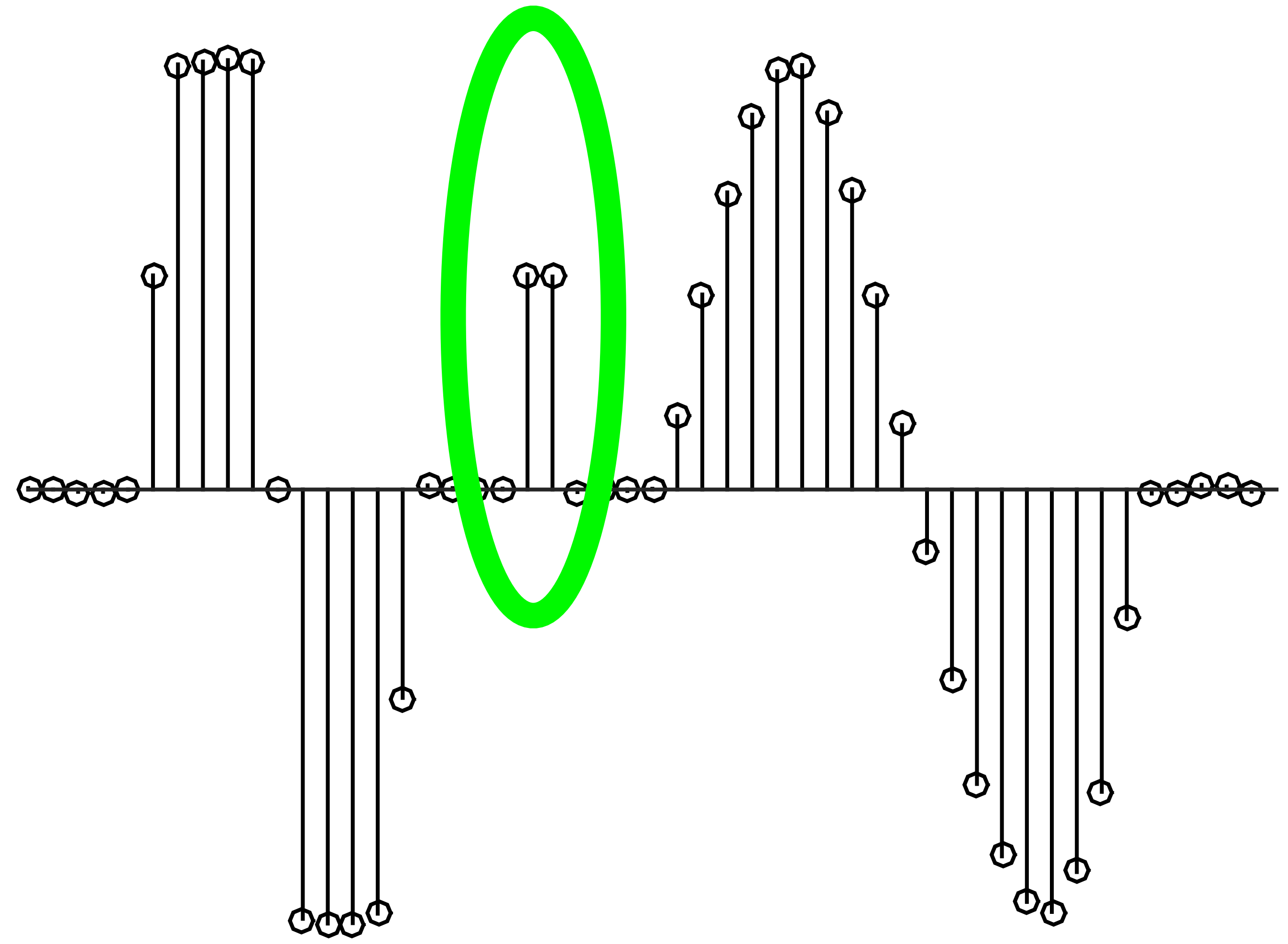

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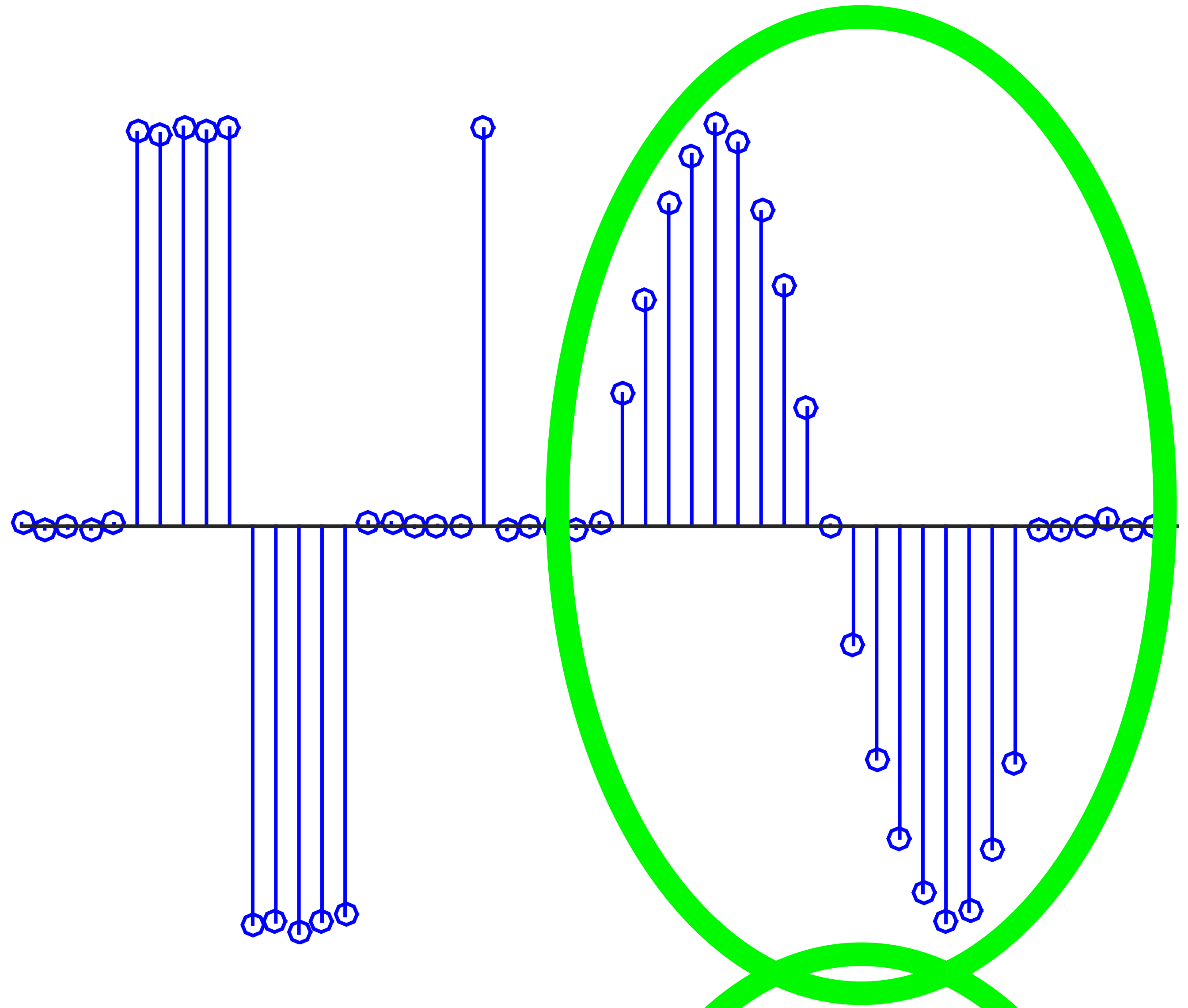

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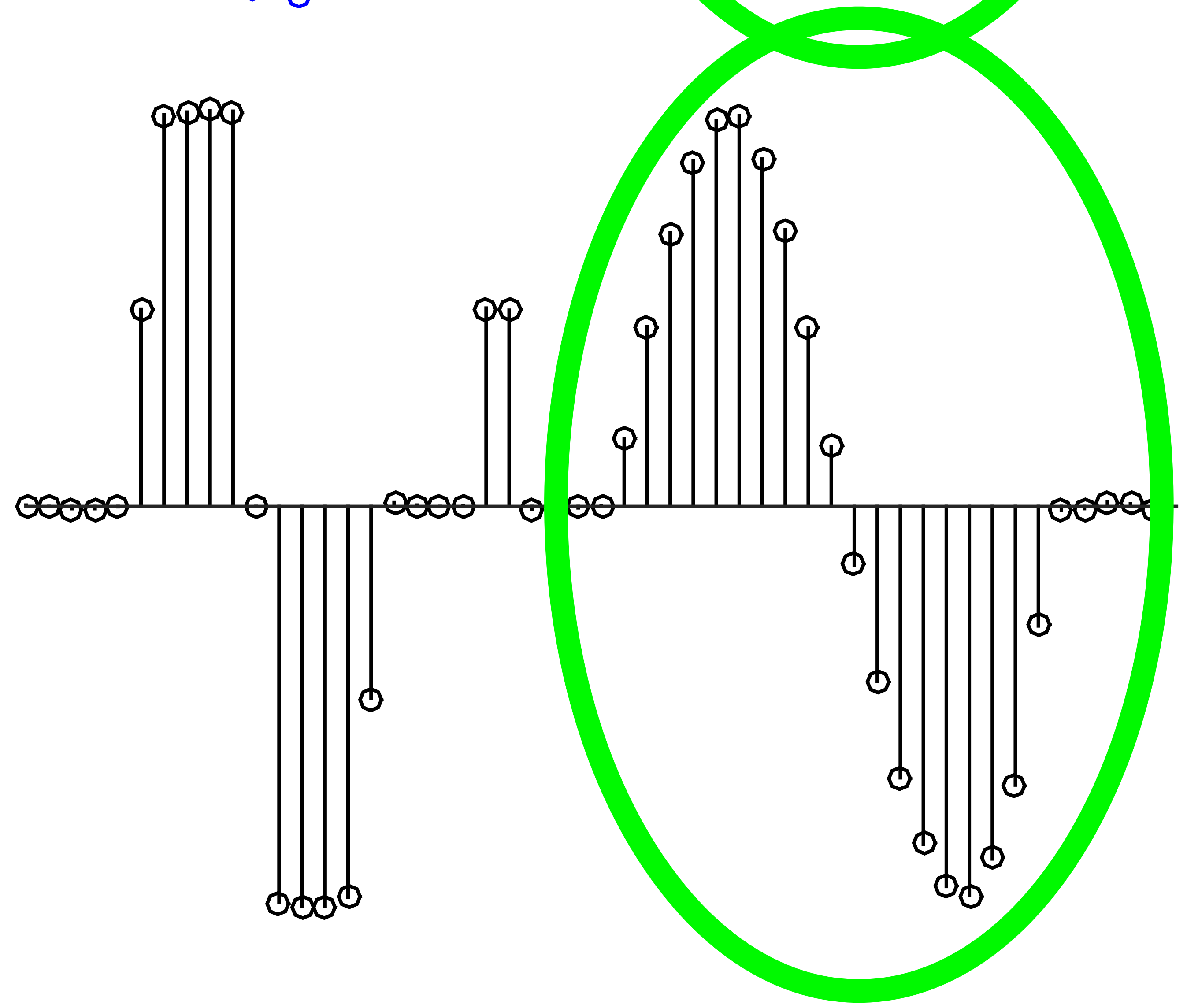
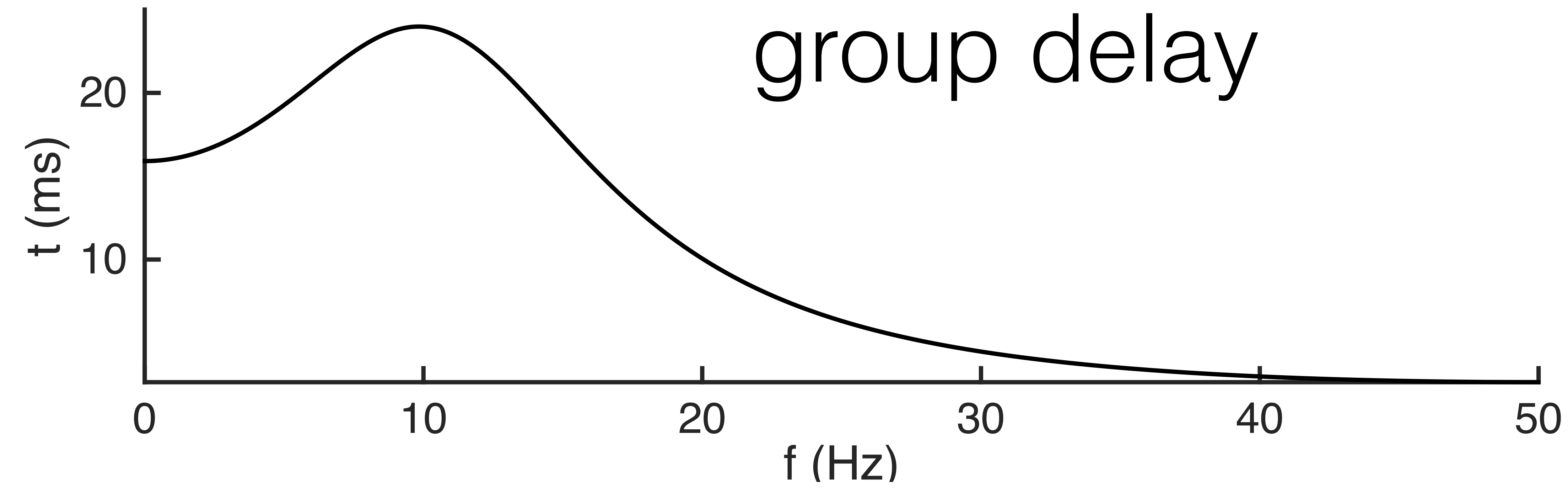


Group Delay: IIR filters

- The group delay of an IIR filter **does** depend on frequency.



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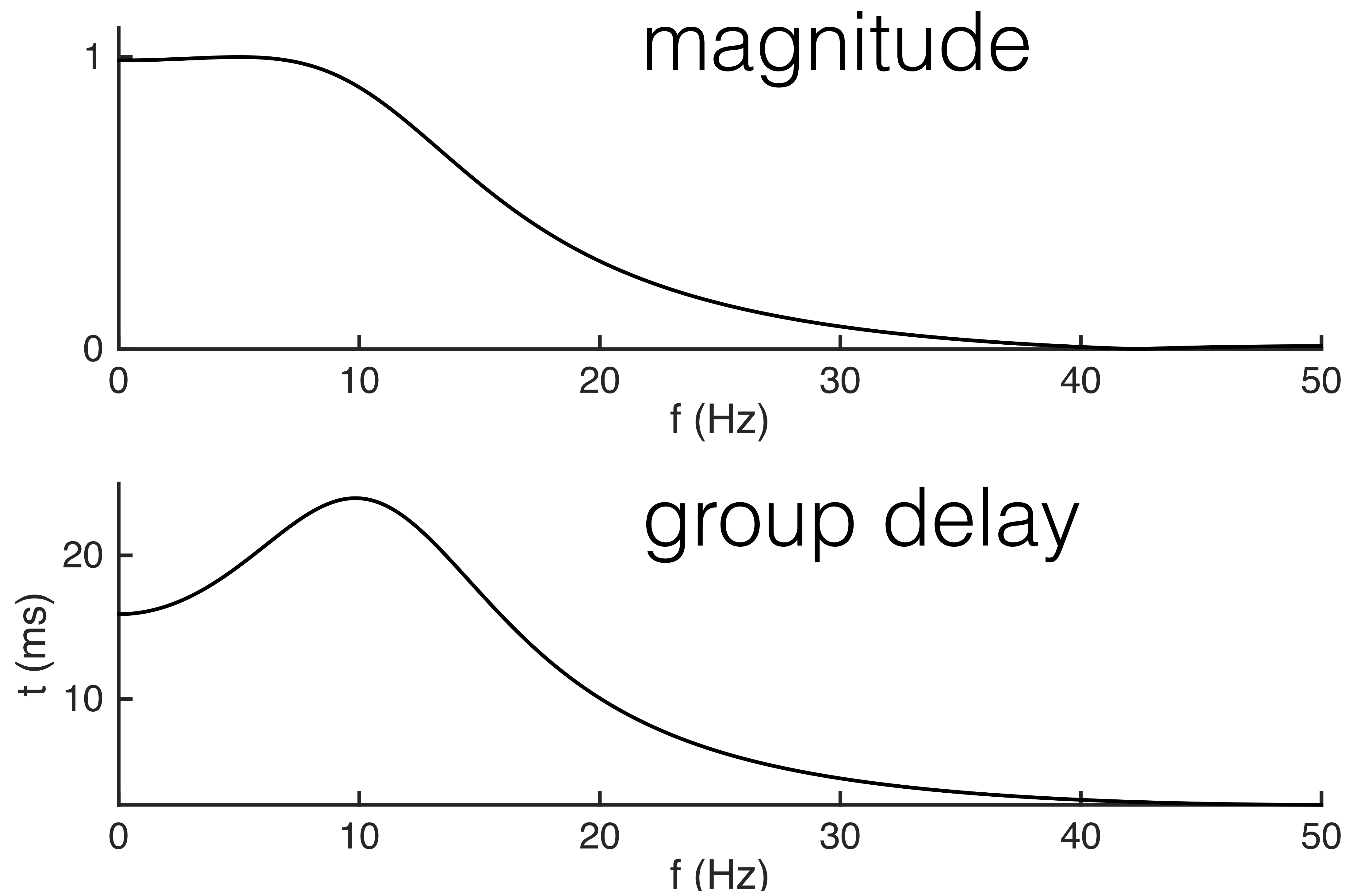

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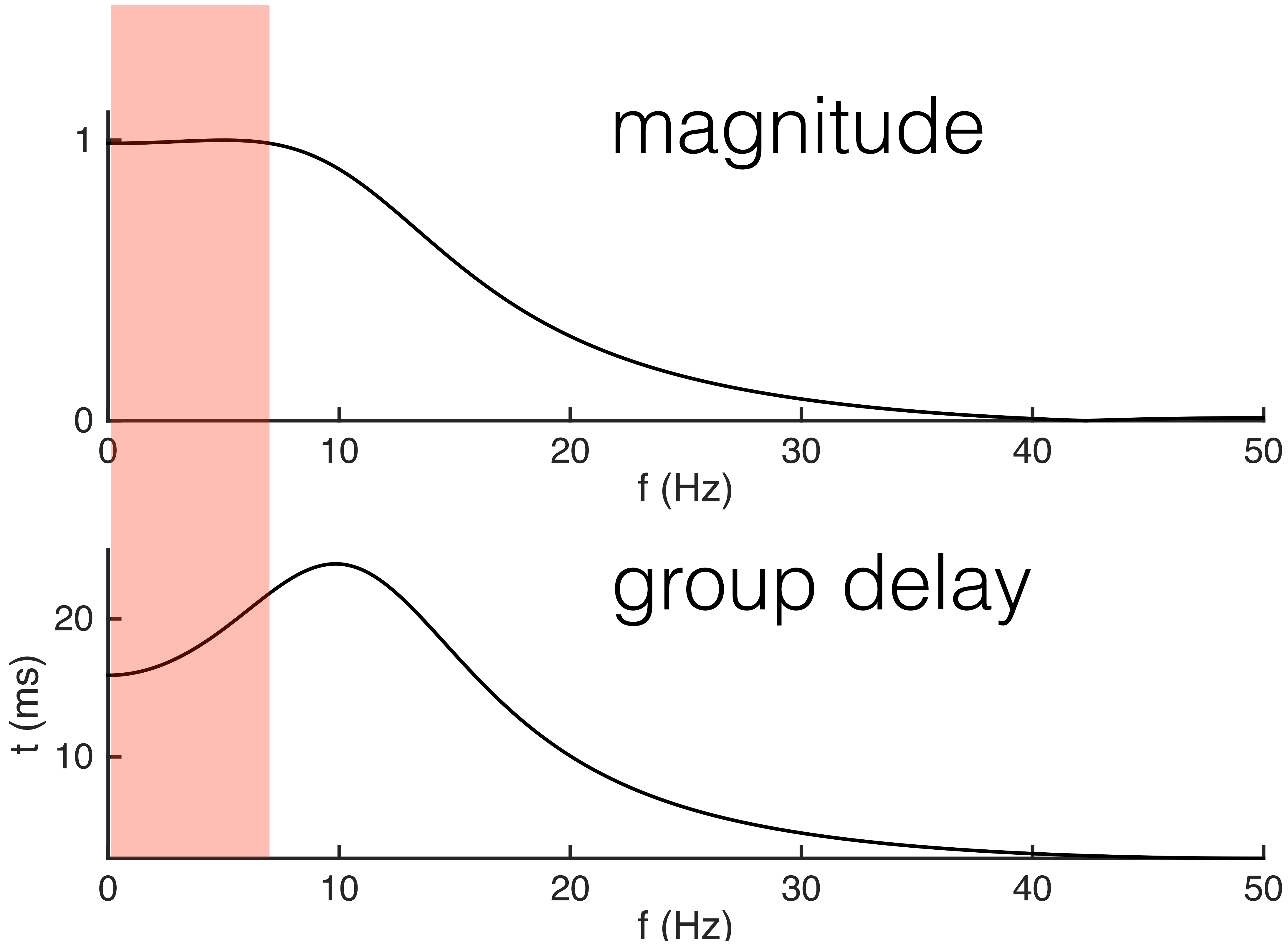

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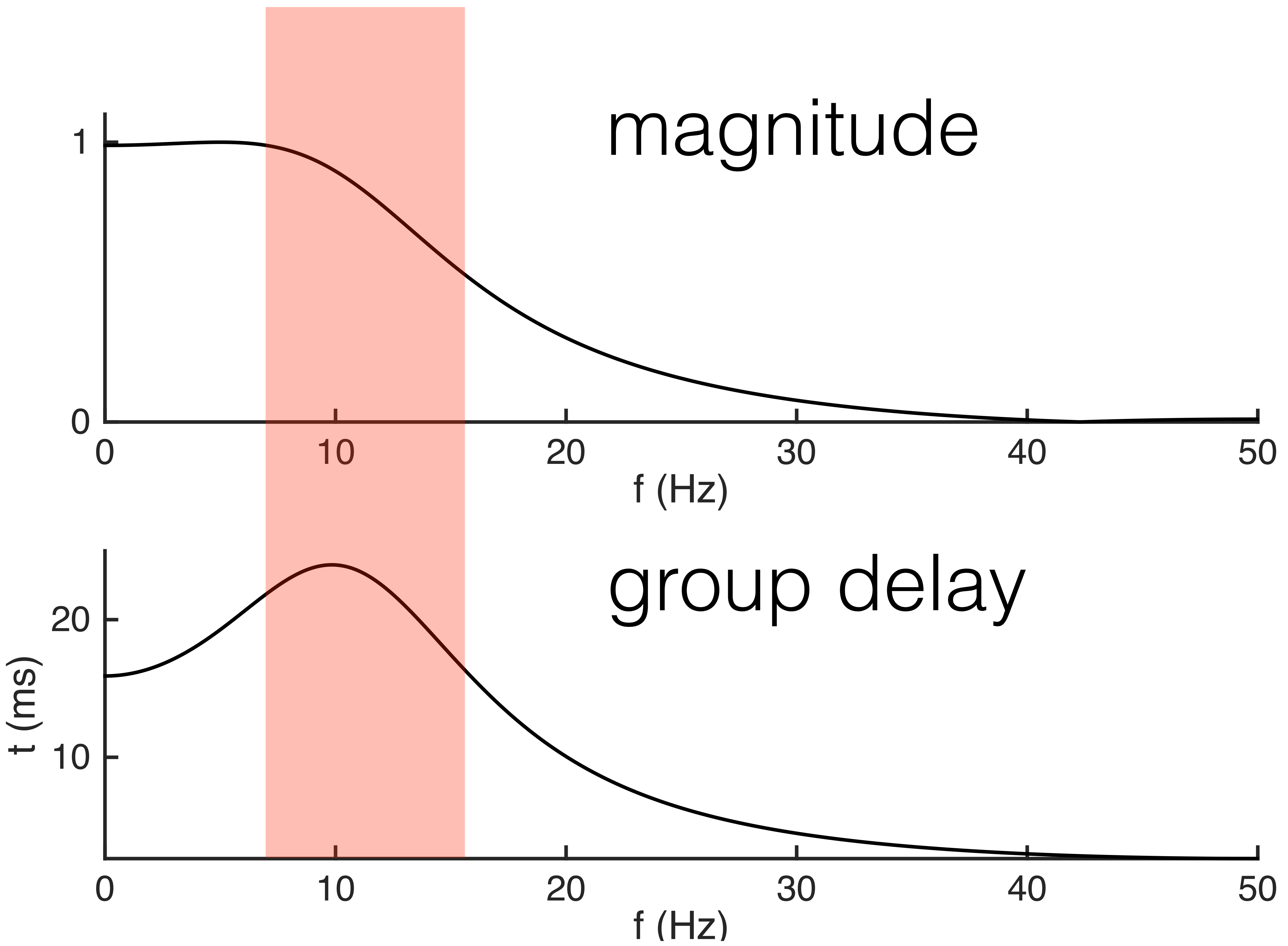

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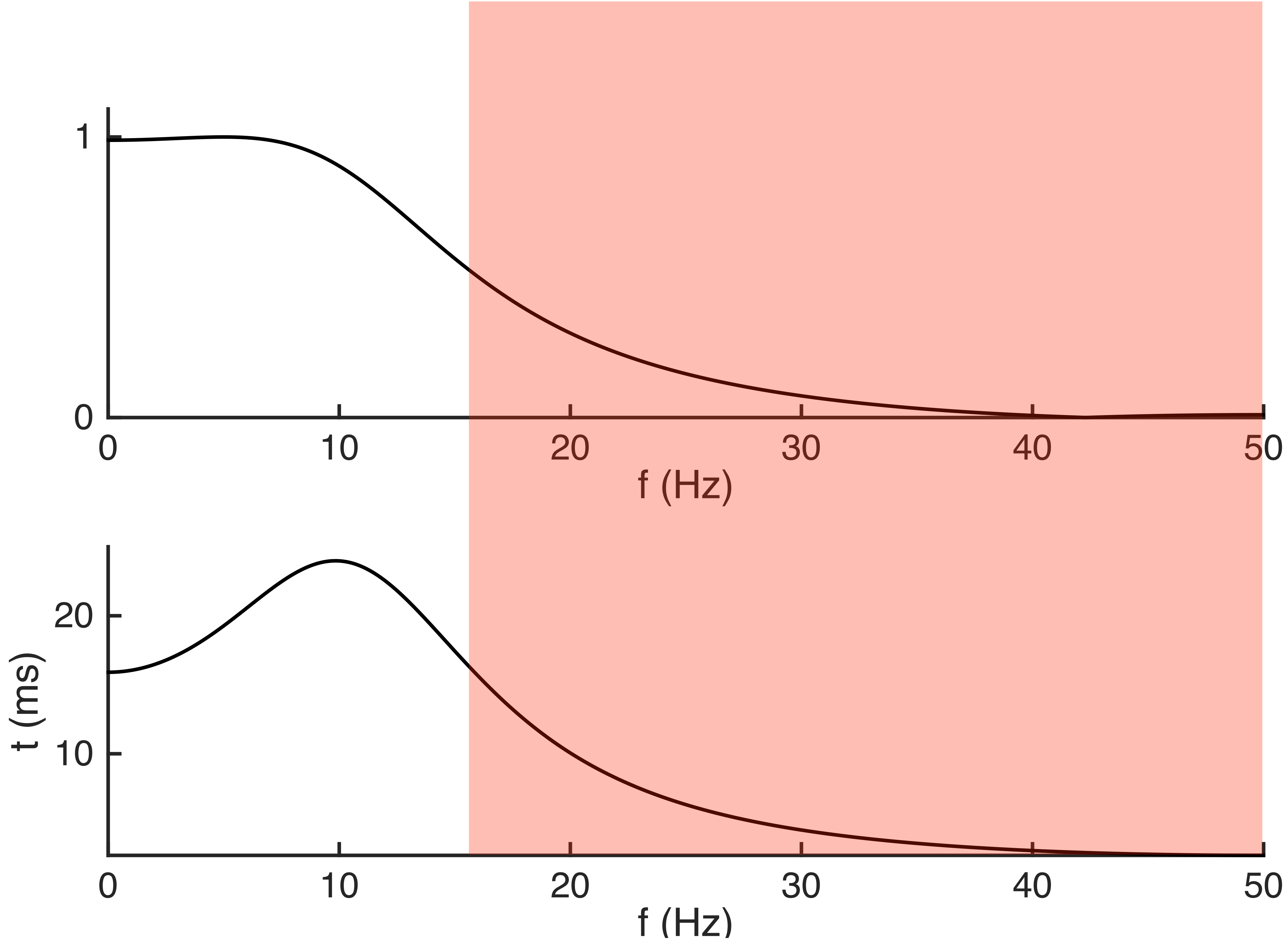
- The group delay of an IIR filter is relatively constant over frequencies that are “passed”.
- The group delay of an IIR filter is longest during the frequency transition.
- The group delay of an IIR filter may be irrelevant over frequencies that are “stopped”.



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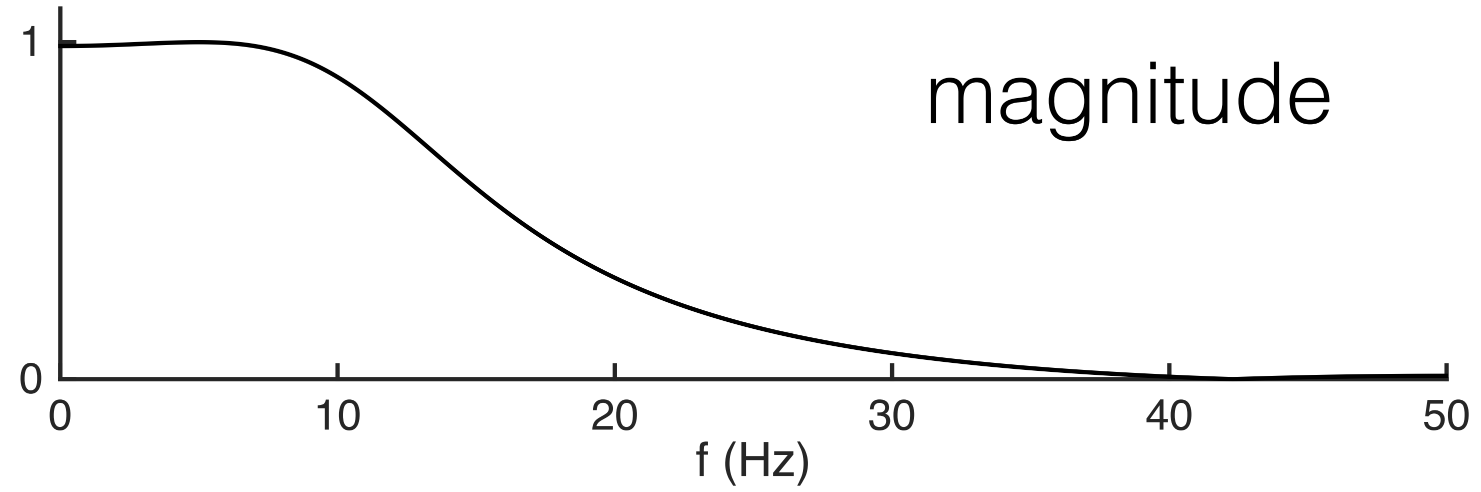
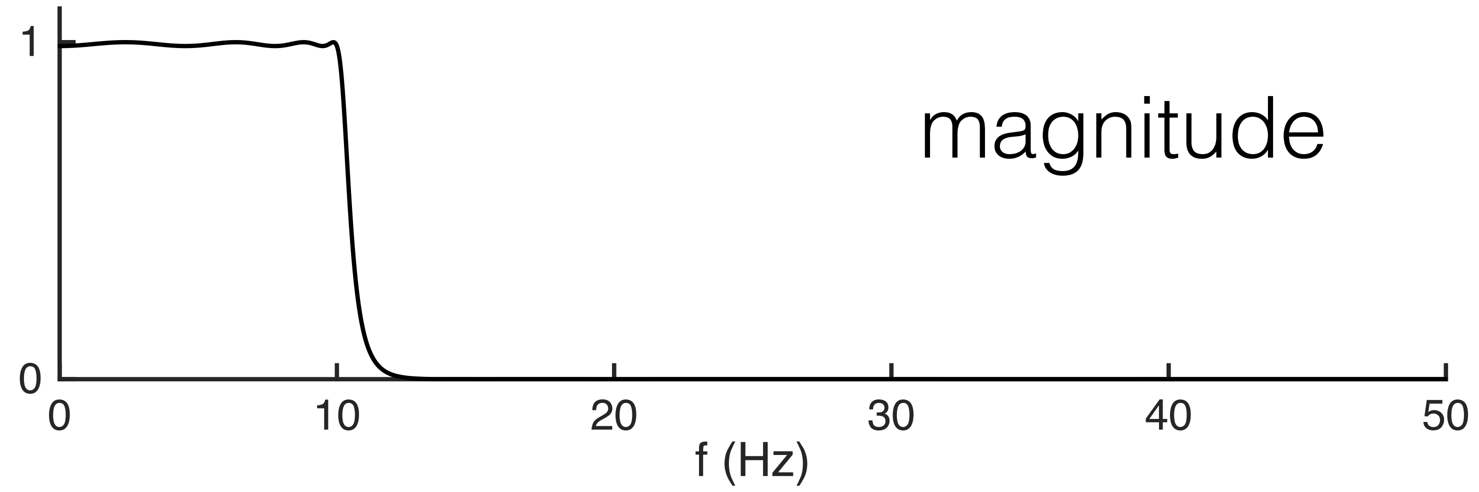
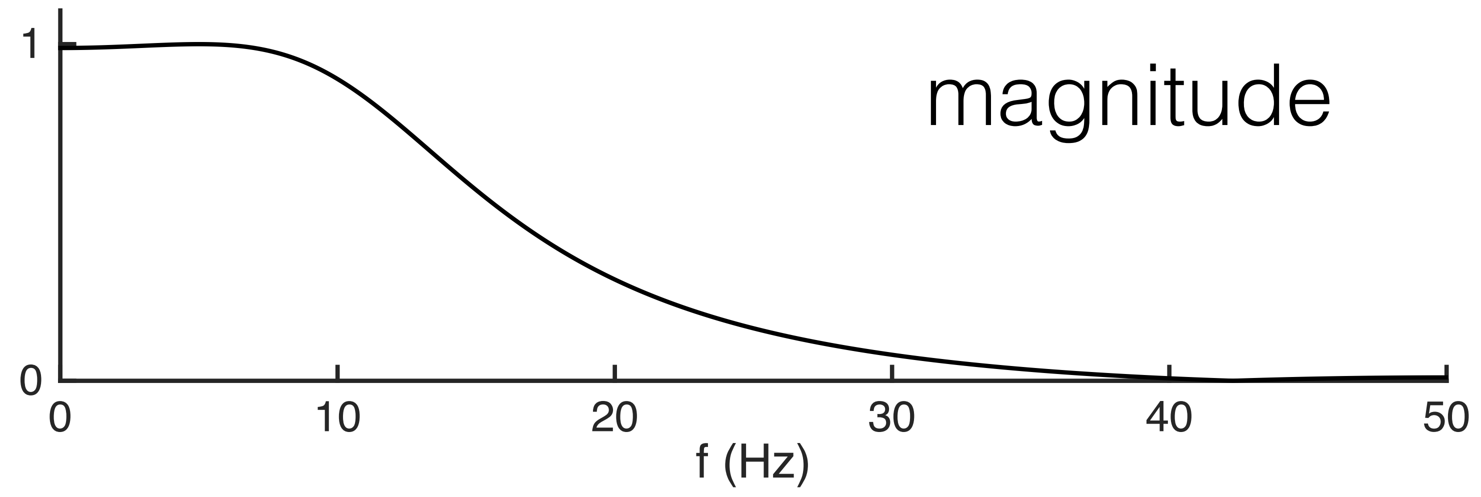
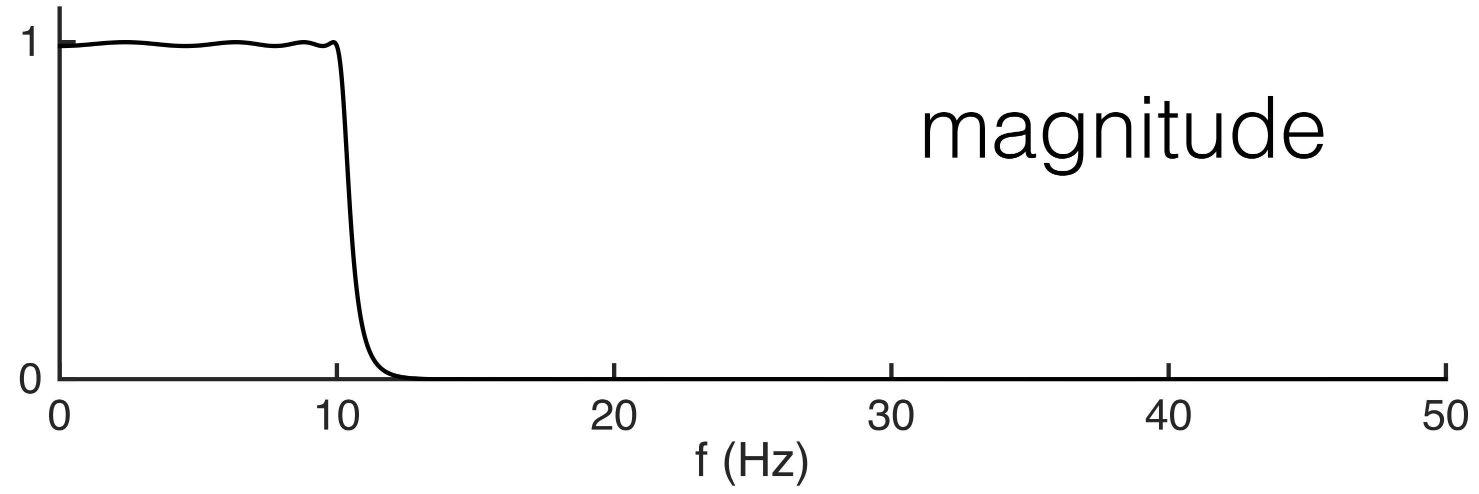




Group Delay: IIR filters

- The group delay of an IIR filter is longest during the frequency transition.
  - The sharper the transition, the longer the group delay





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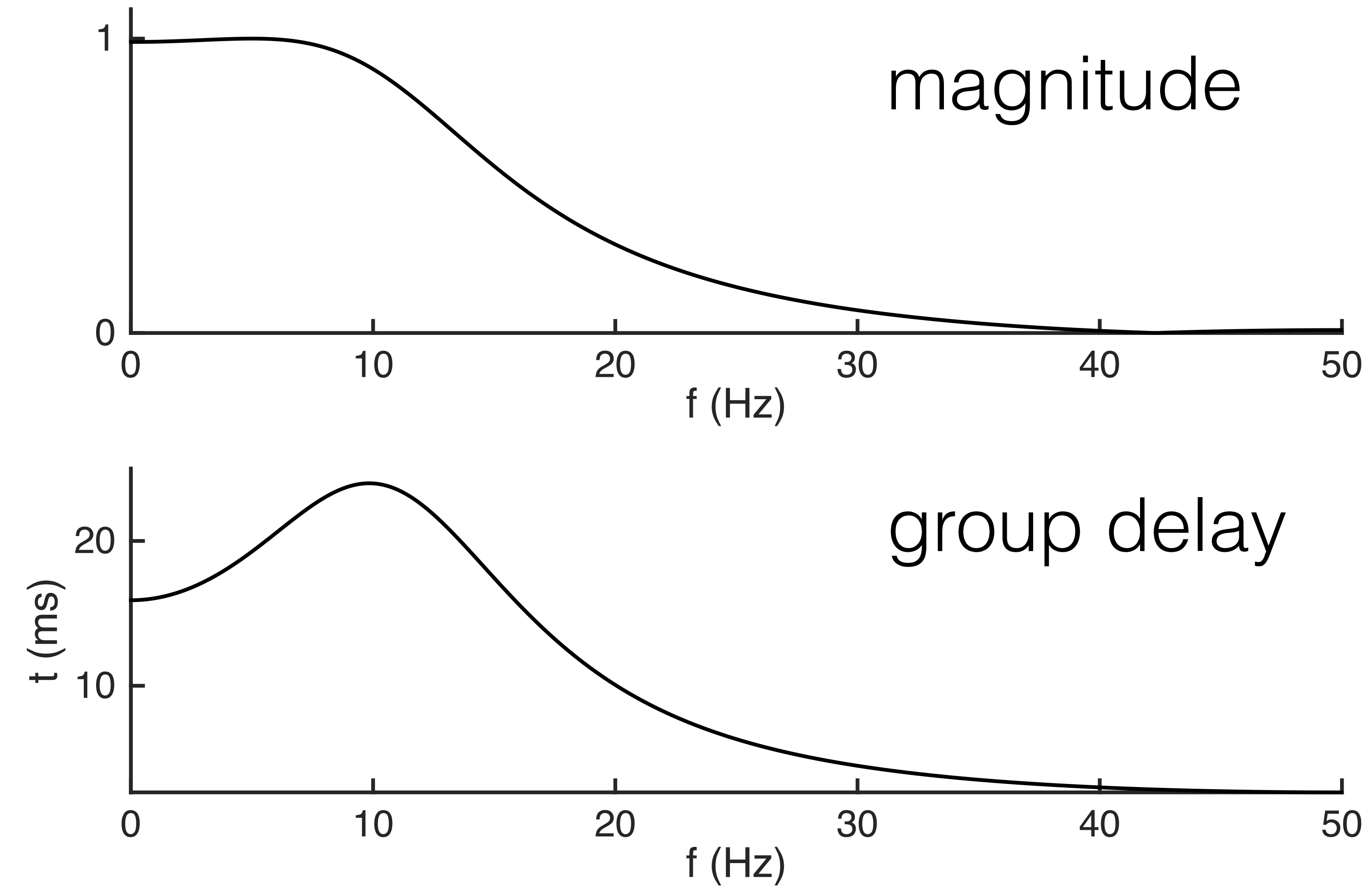

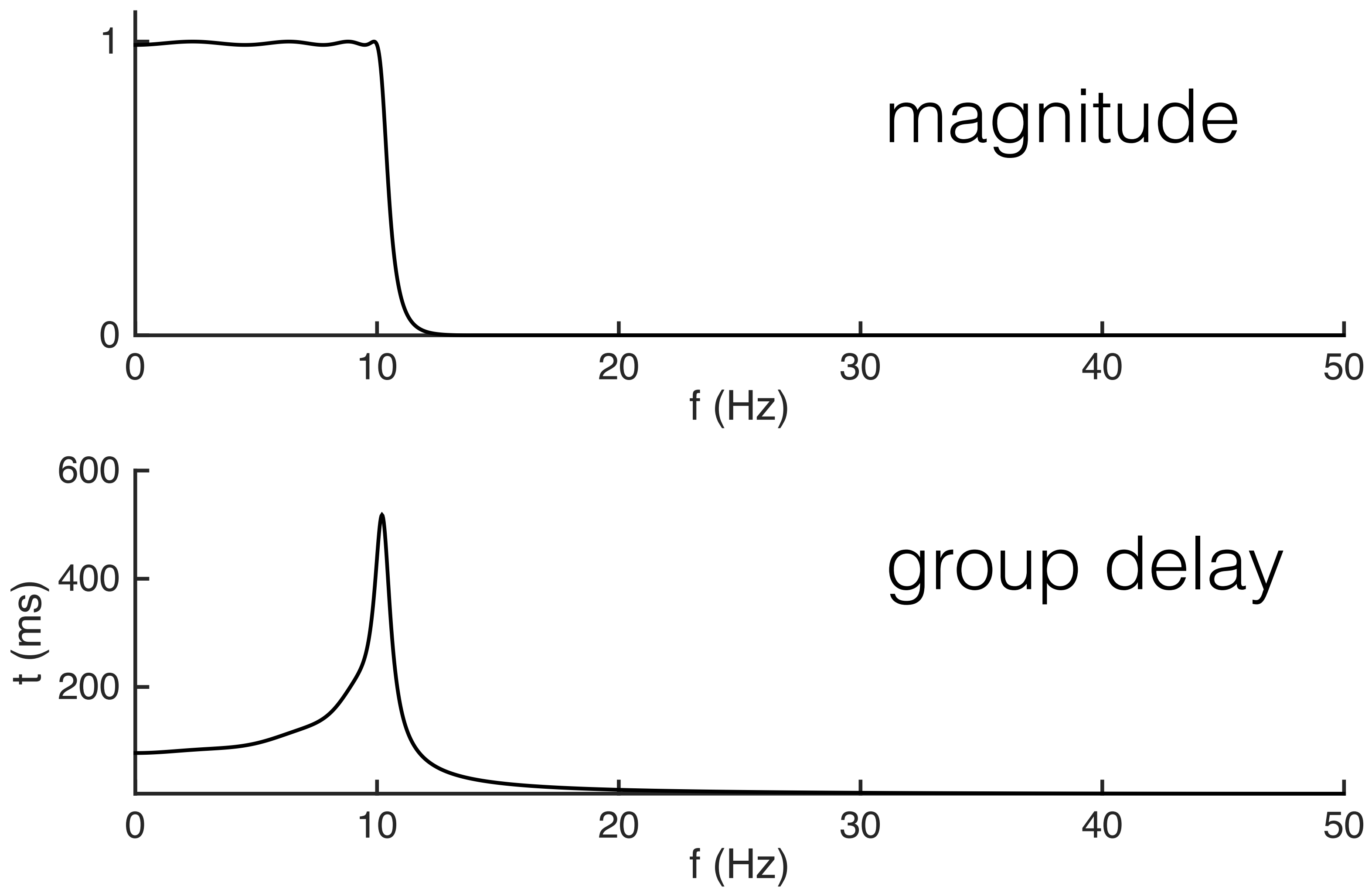

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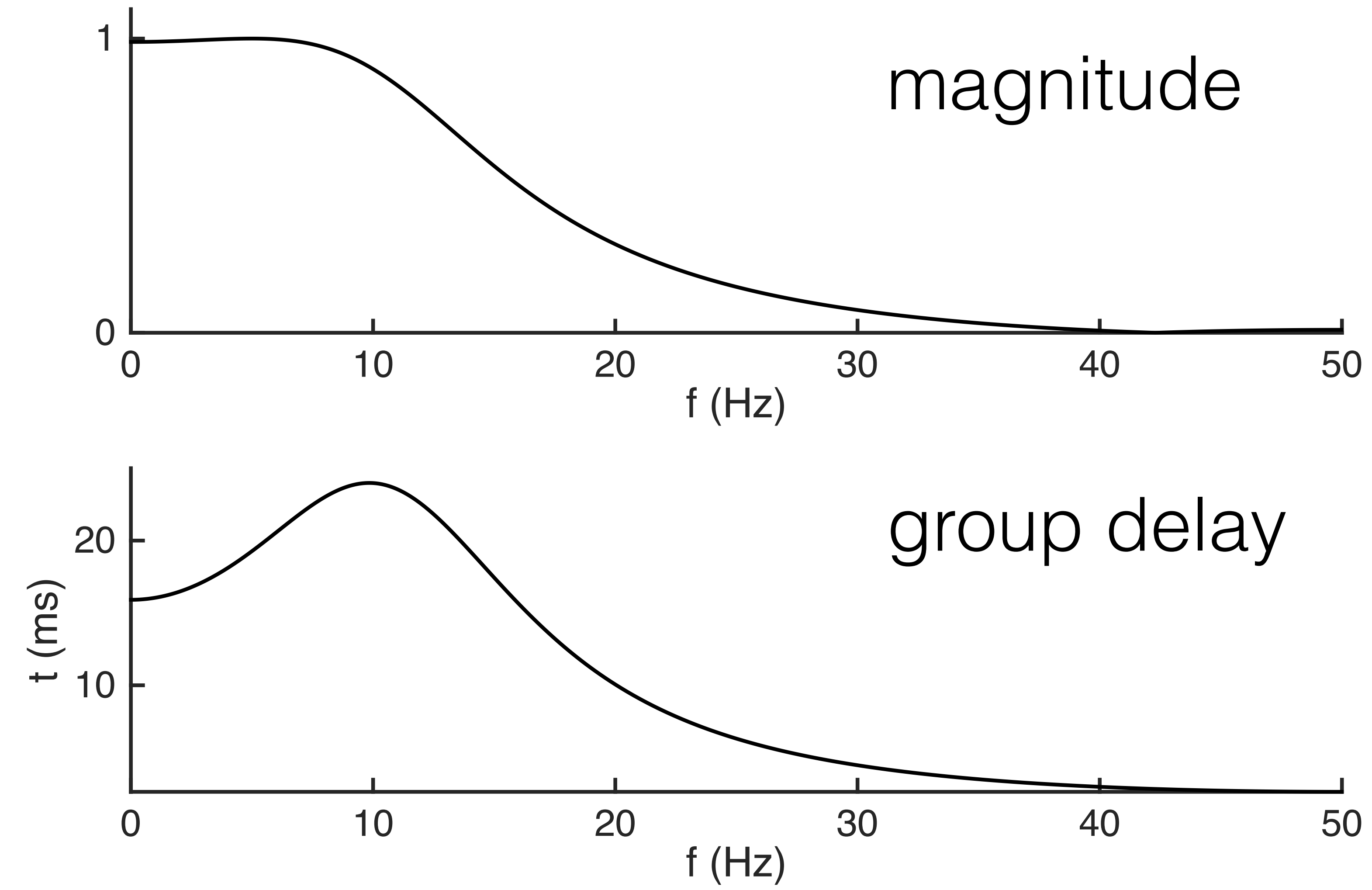

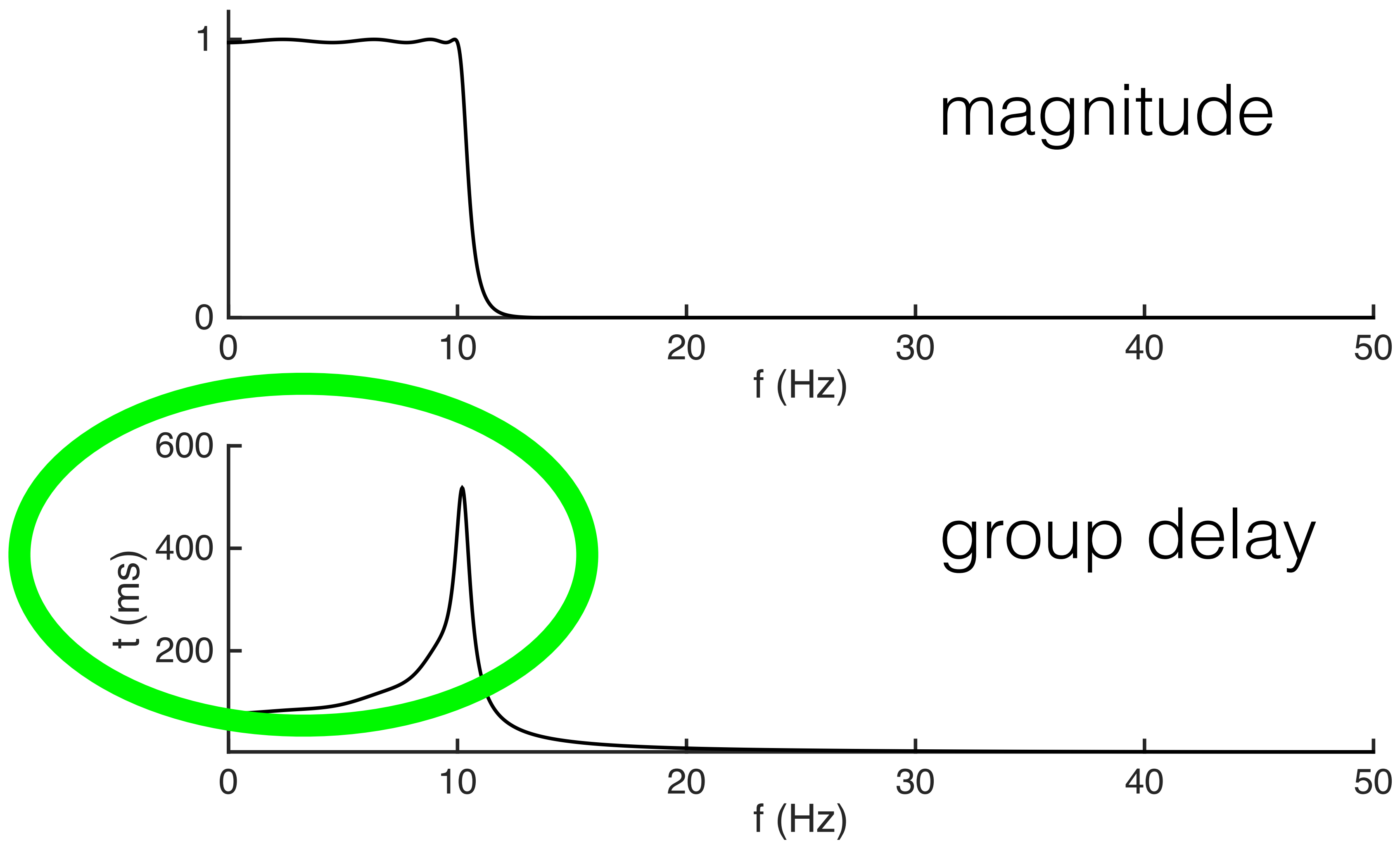

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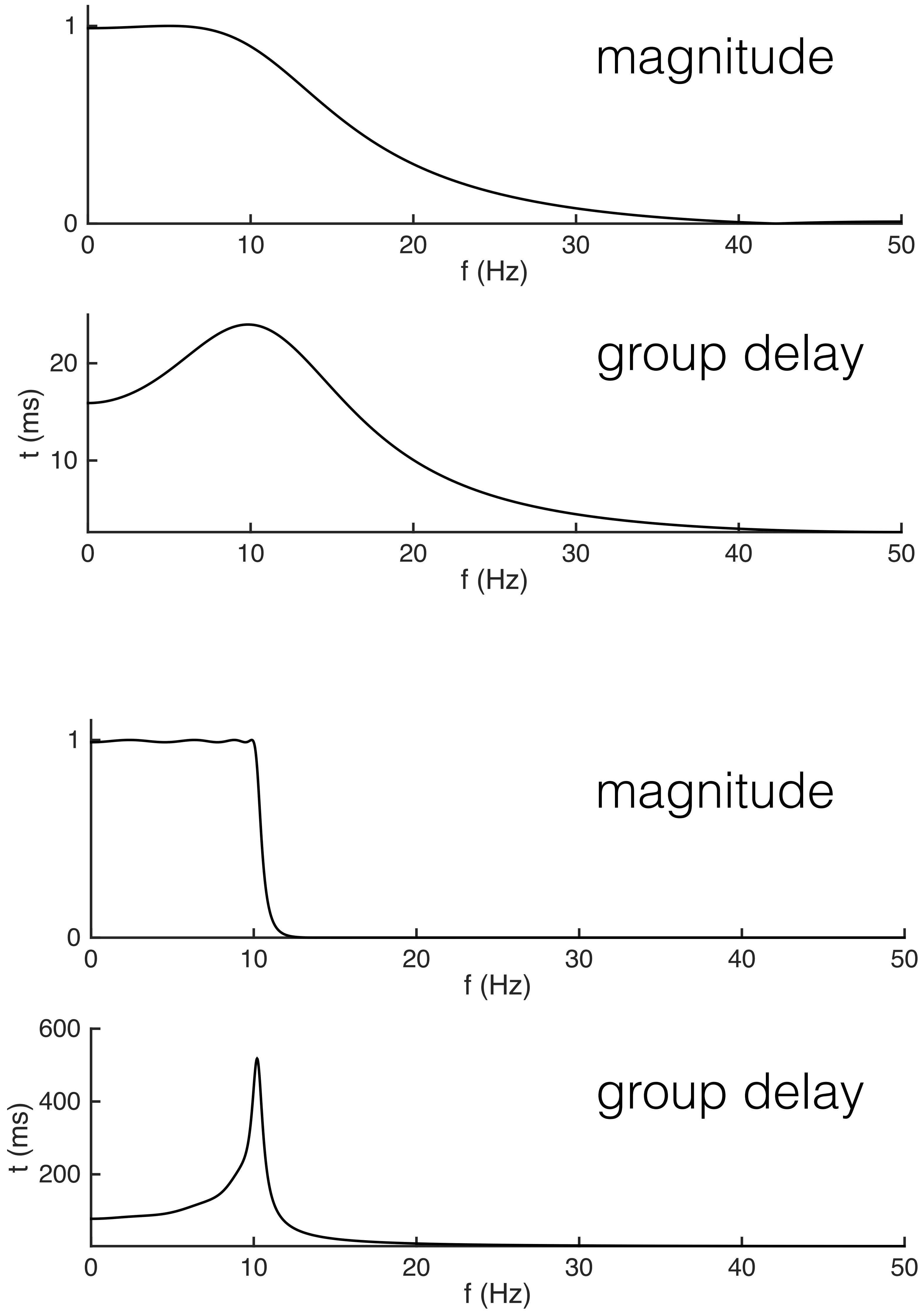
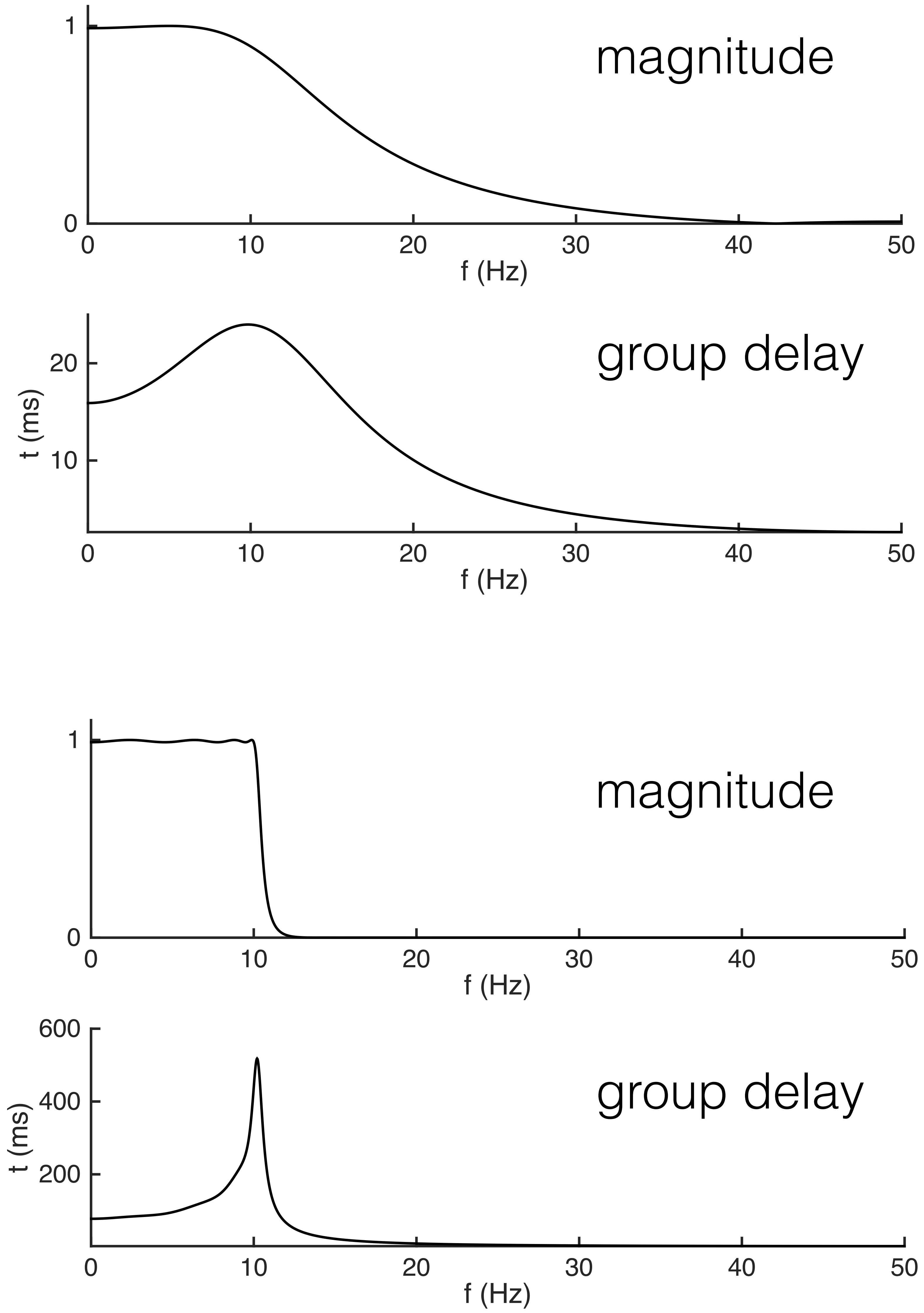


Group Delay: IIR filters

- The group delay of an IIR filter is longest during the frequency transition.
  - The sharper the transition, the longer the group delay
  - Calculating latencies? You may need to compensate (still possible for peaks dominated by frequencies far from the transition).
  - The group delay of an IIR filter does not linearly scale with Δt(!), so no penalty for filtering at low sampling frequency.



Signal Loss due to Filter Startup

- Output signal value depends on signal values in the past
- When calculating output at the very first moment of time, *there is no past to rely on!*
- Until filter output settles down, in time, the output signal is not well defined.

Signal Loss due to Filter Startup

For FIR filters, this problem goes away entirely after $N_{order} \times \Delta t$.

$$y[t] = \frac{1}{2}x[t] - \frac{1}{2}x[t-\Delta t]: \quad y[0] = \frac{1}{2}x[0] - \frac{1}{2}x[-\Delta t] \qquad y[\Delta t] = \frac{1}{2}x[\Delta t] - \frac{1}{2}x[0]$$

- Recommendation: either keep extra *earlier* data of duration $N_{order} \times \Delta t$, or prepend the same amount of zero signal (Matlab's default). Consider this "warmup" time for the filter. Then toss out this same amount from the output.

- This works well for small N_{order}.

- This is another reason to use FIR filters only of low order.

- This is another reason FIR filters may work best at high sample rates.

Signal Loss due to Filter Startup

For IIR filters, the problem is more subtle

$$y[t] = \frac{1}{10}x[t] - \frac{9}{10}y[t-\Delta t]: \qquad y[0] = \frac{1}{10}x[0] - \frac{9}{10}y[-\Delta t] \qquad y[\Delta t] = \frac{1}{10}x[\Delta t] - \frac{9}{10}y[0]$$

- The output depends not only on the input in the past, but also on the filter output of the past.

- Recommendation: again keep extra earlier data (warmup time), *however much you can afford*. Then toss out the same amount from the output.

- If keeping enough earlier data not feasible, Matlab permits supplying pre t=0 initial data. Using this with reasonable values can really help.

  - Even data from the *end* of the signal may help substantially over nothing.

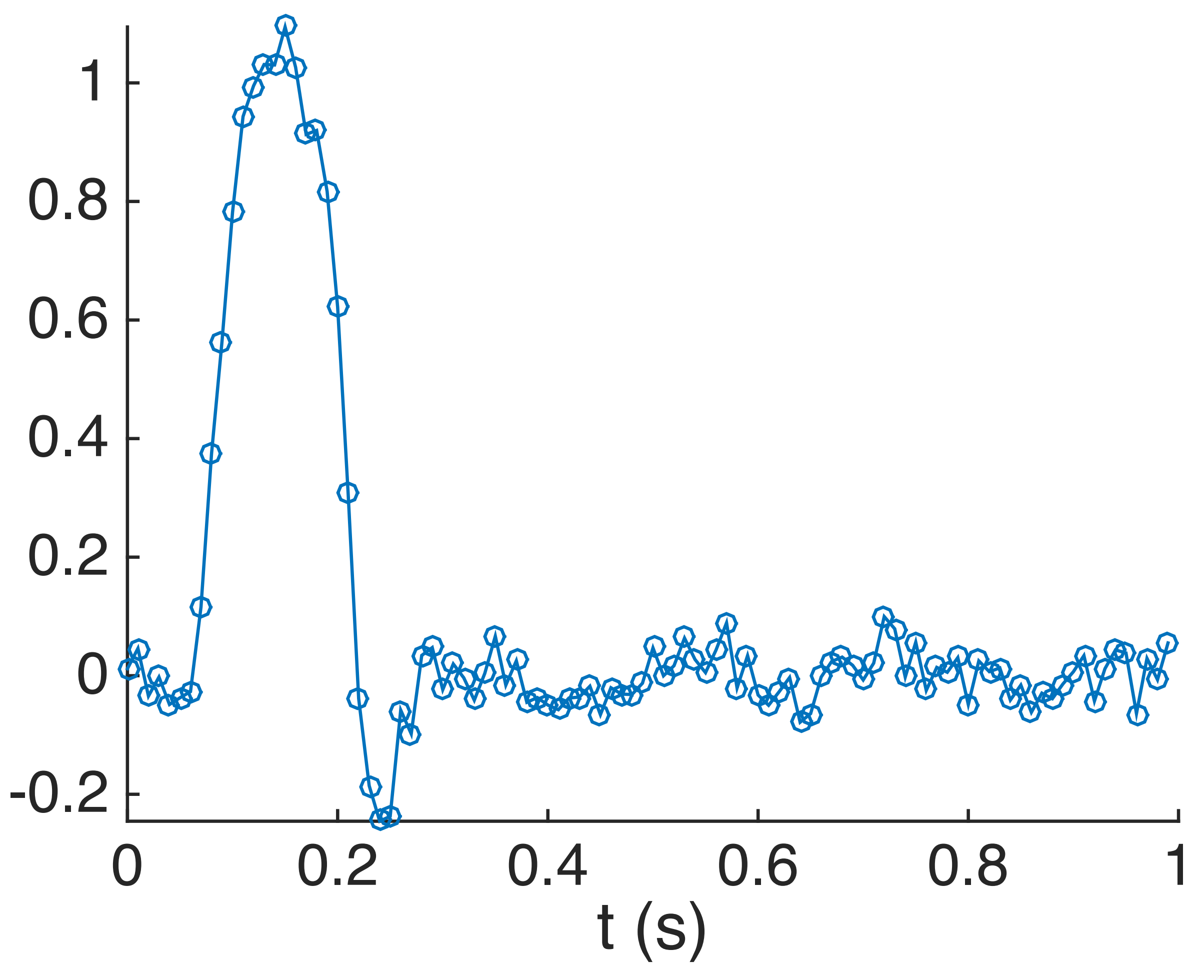
Stability concerns for IIR filters

- IIR filters employ feedback; might be negative (good) or positive (bad)
- Common IIR filters designed to be stable: all feedback negative (good)
- Design can break down due to numerical roundoff error
- Breakdown more likely for higher order filters
- Recommendation: only use low order ($N_{order} < 10$) IIR filters.
    - Lower order IIR filters also have less sharp frequency transitions, so this is rarely a burden.

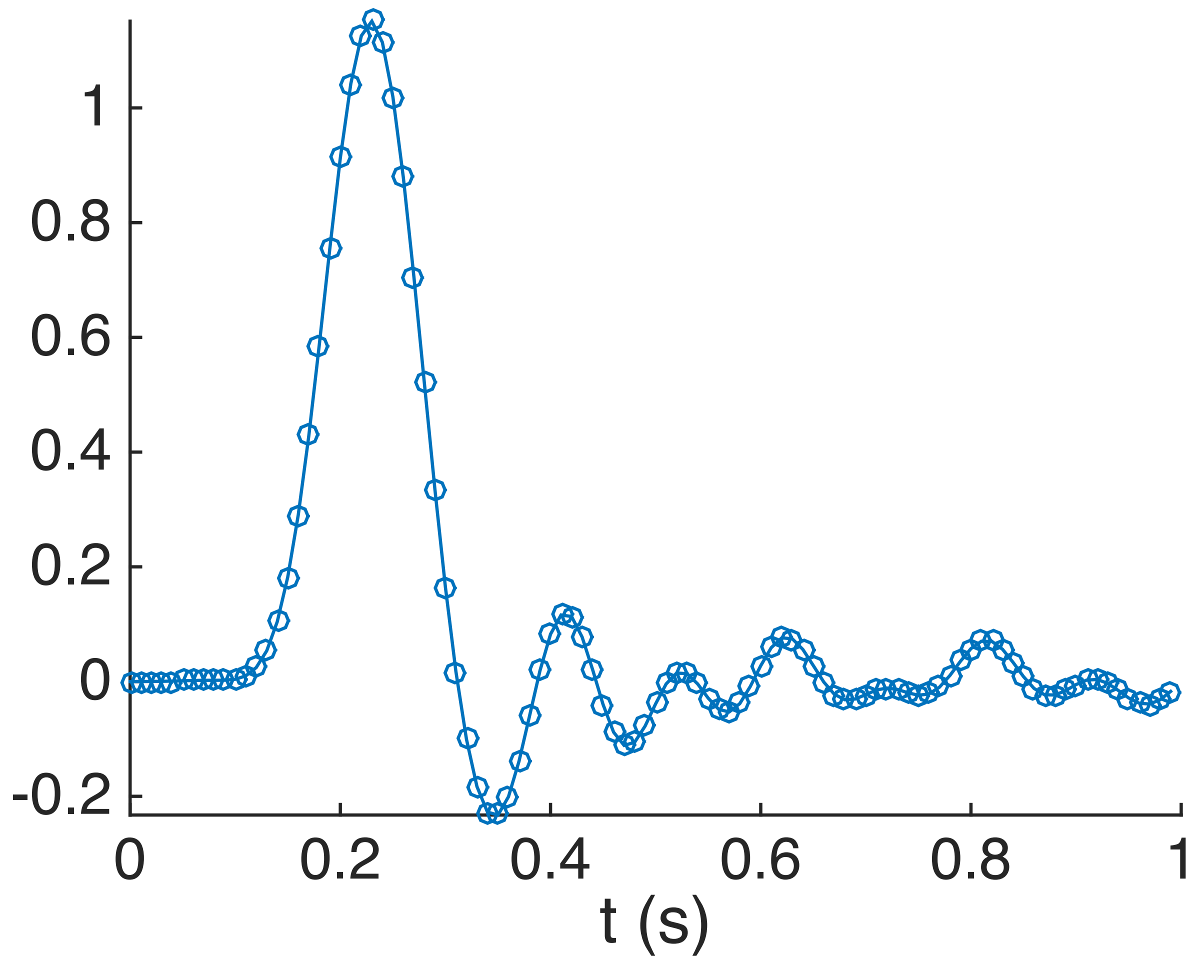
How Would I even Notice Instability?

It’s not subtle (but only if you know where to look)

Raw Signal



Stable Filter



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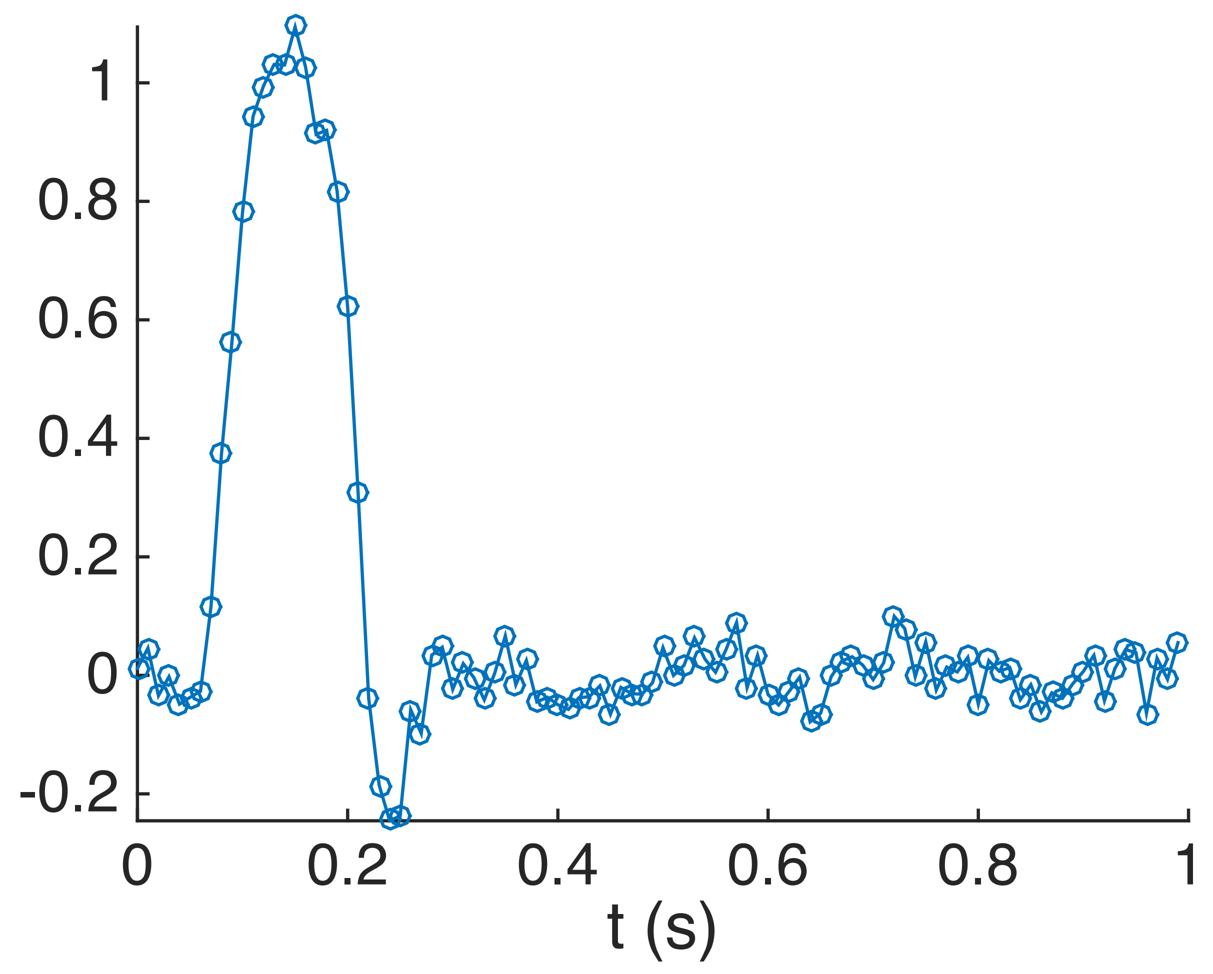

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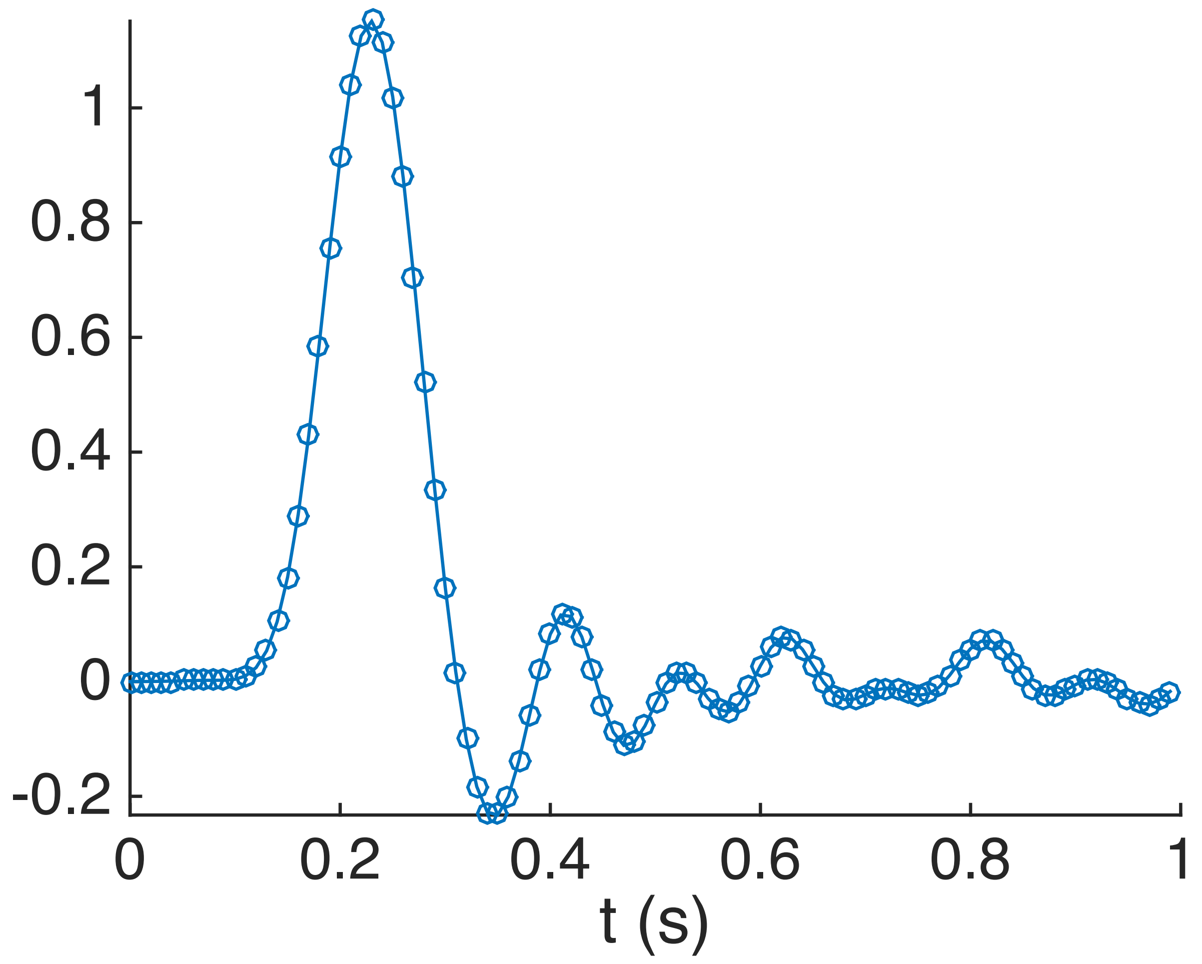
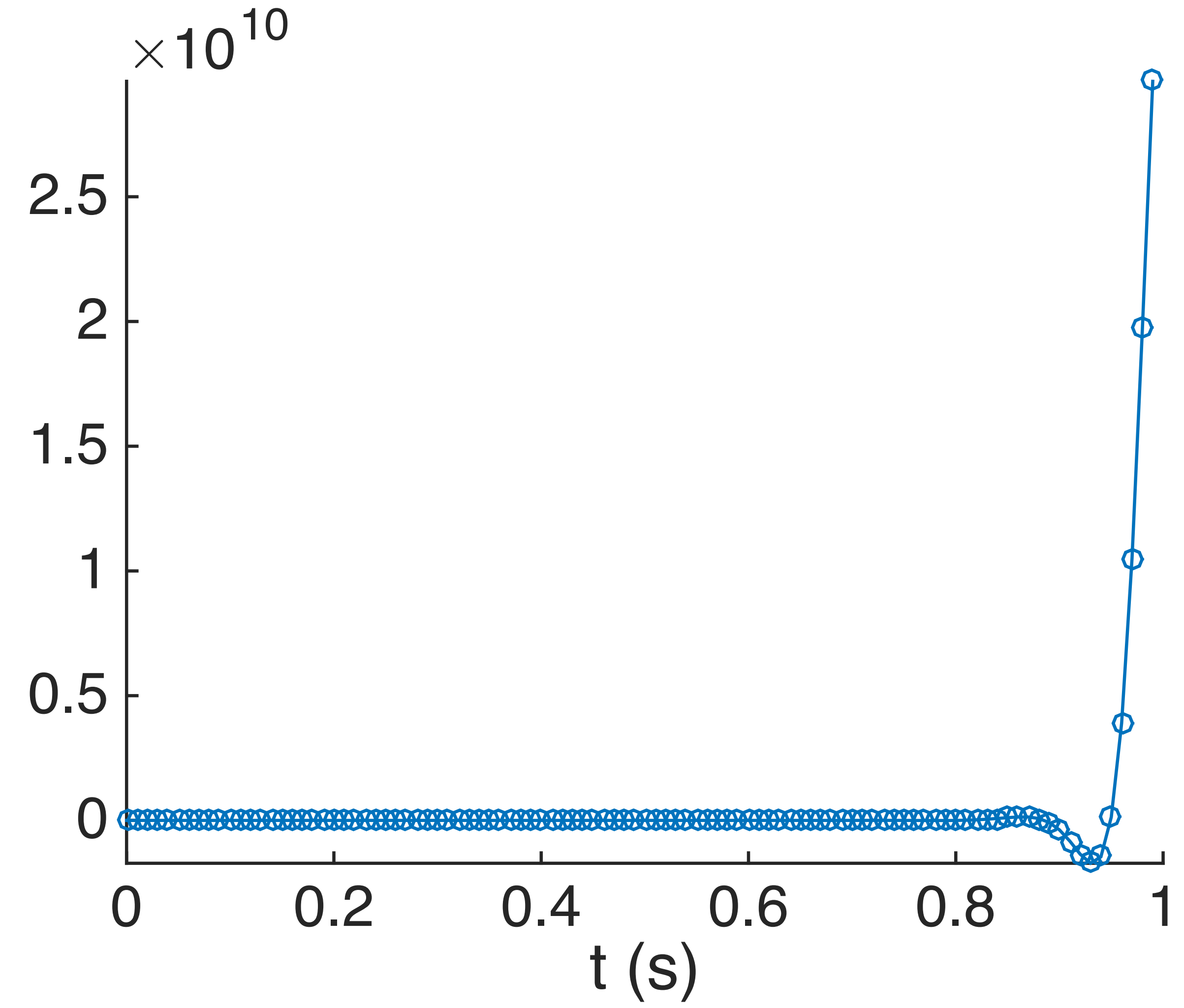


Filter gone Unstable



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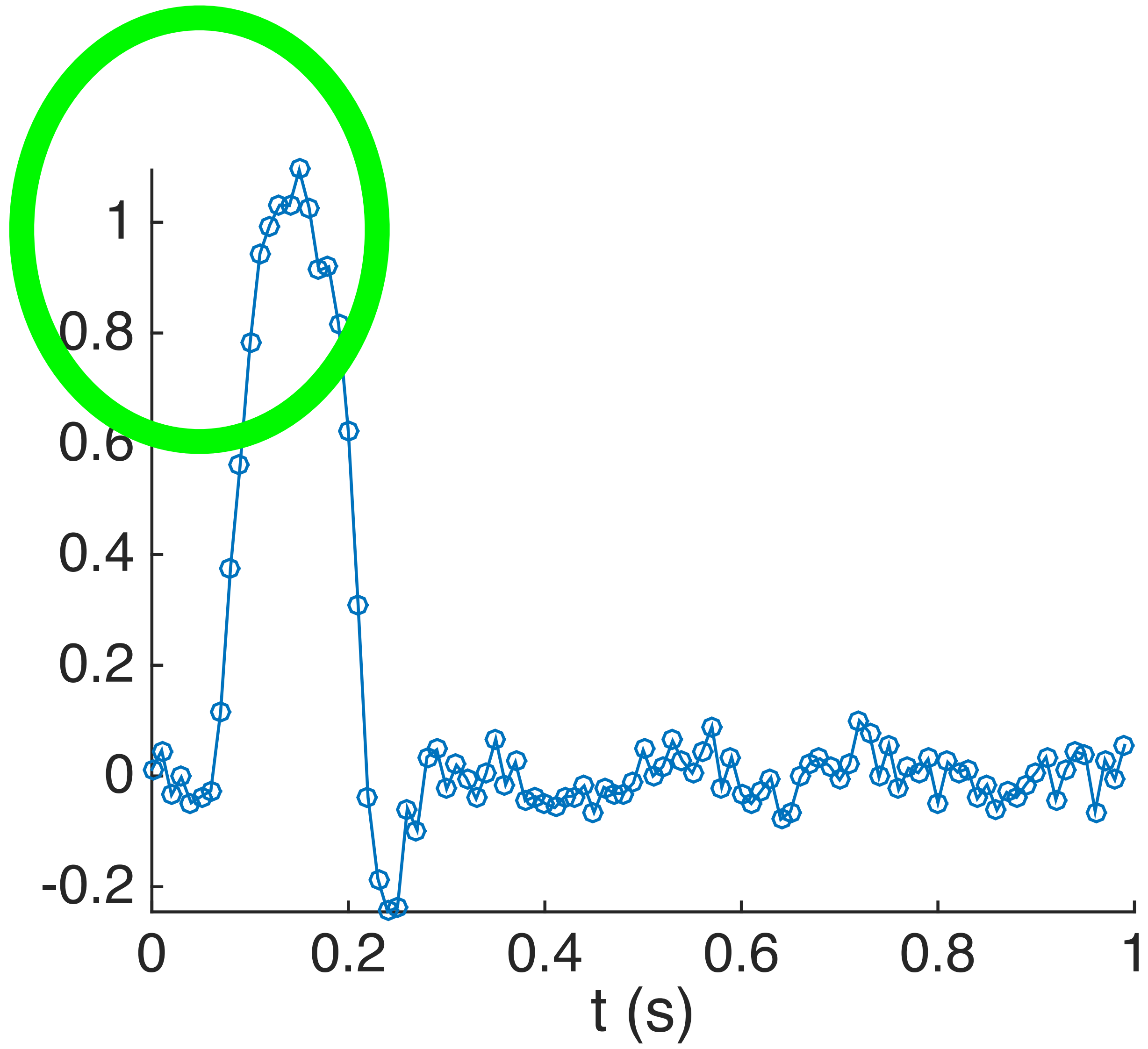

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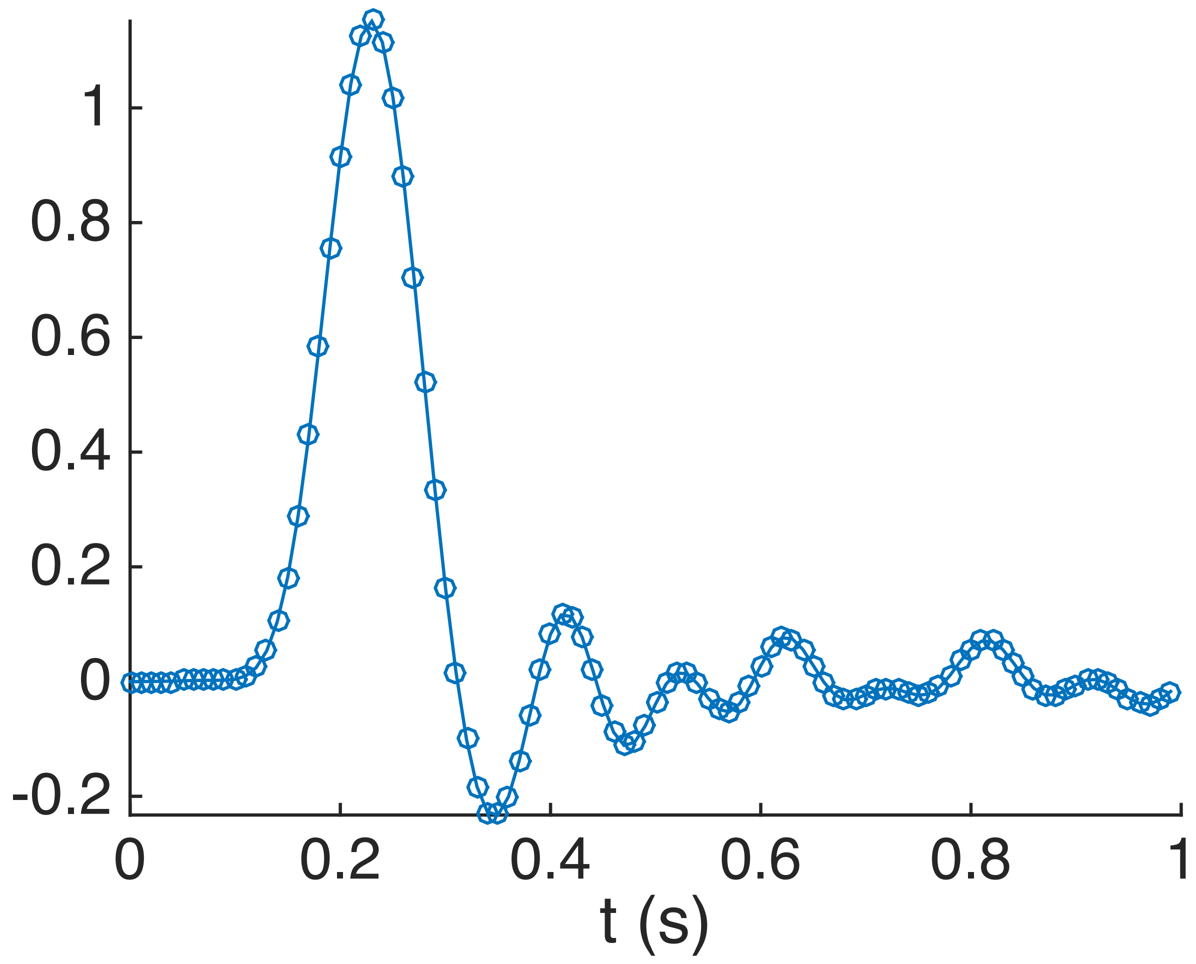

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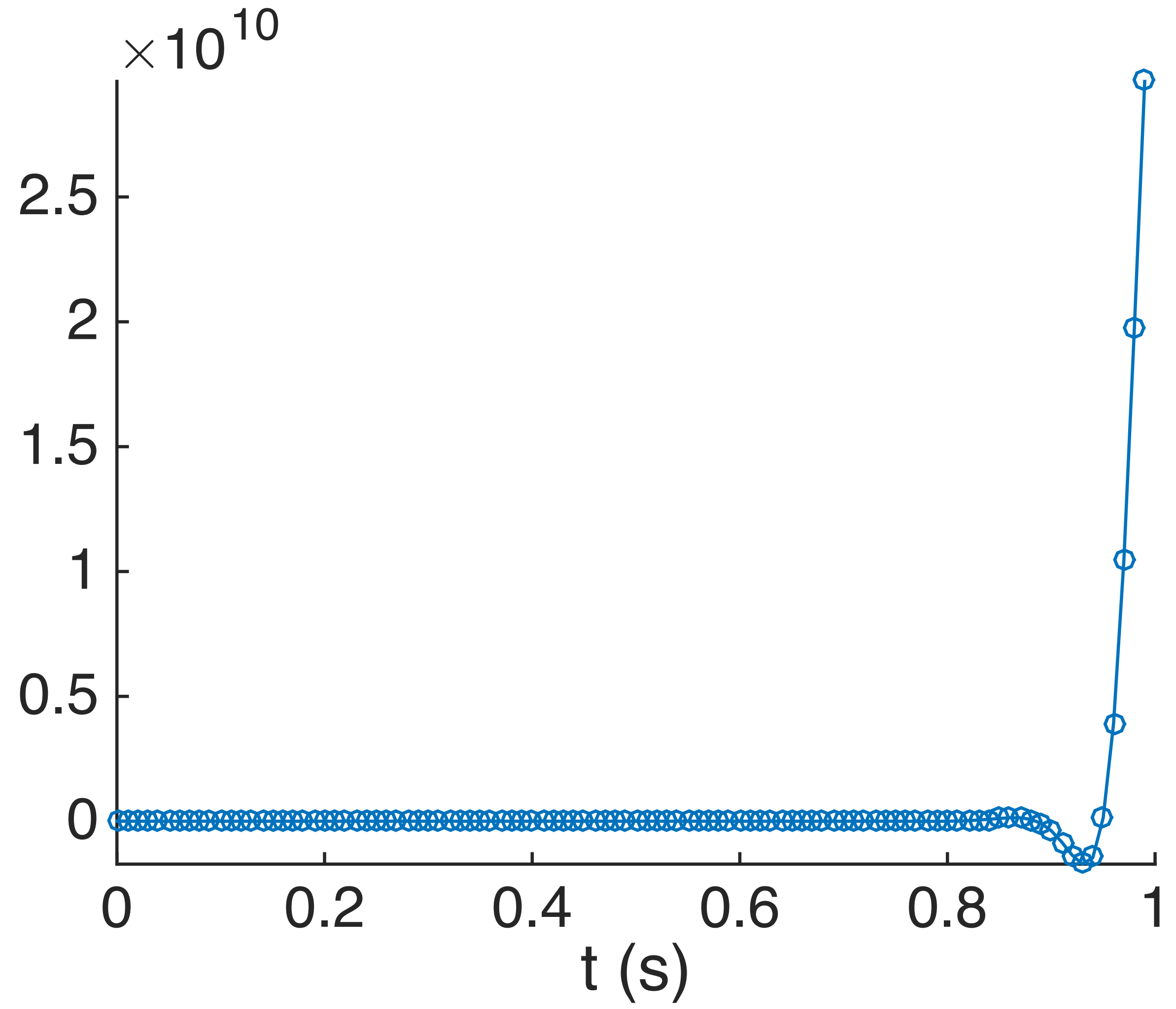

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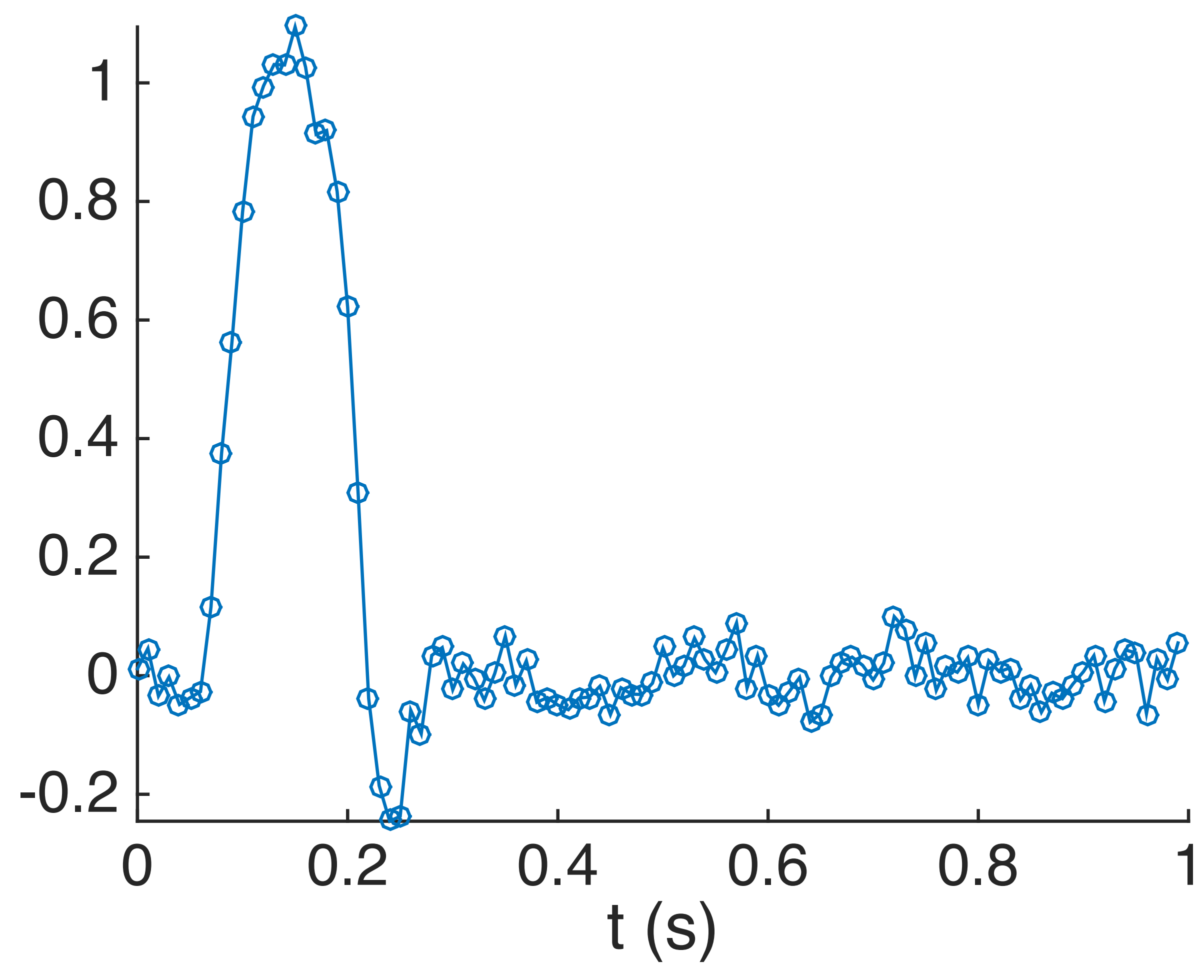

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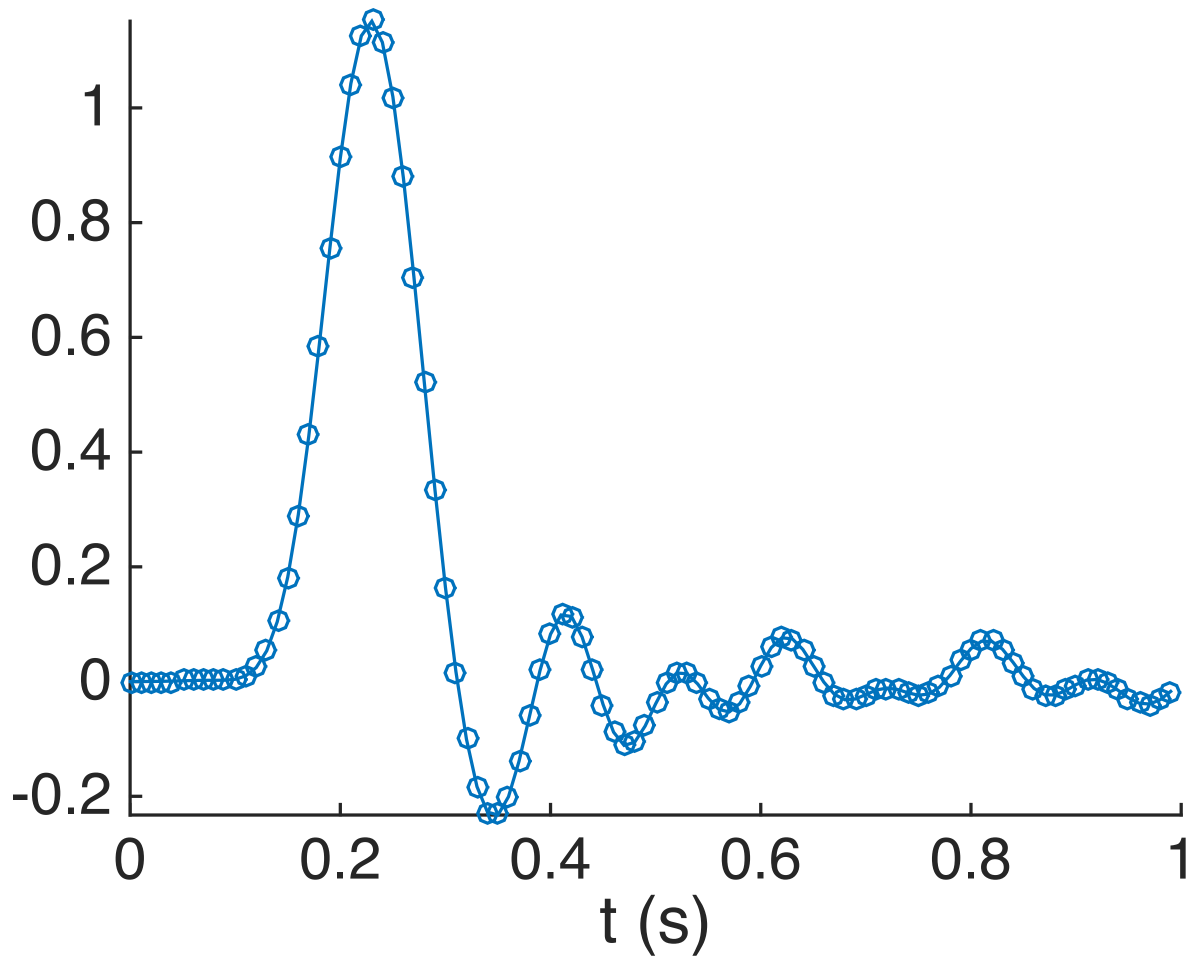
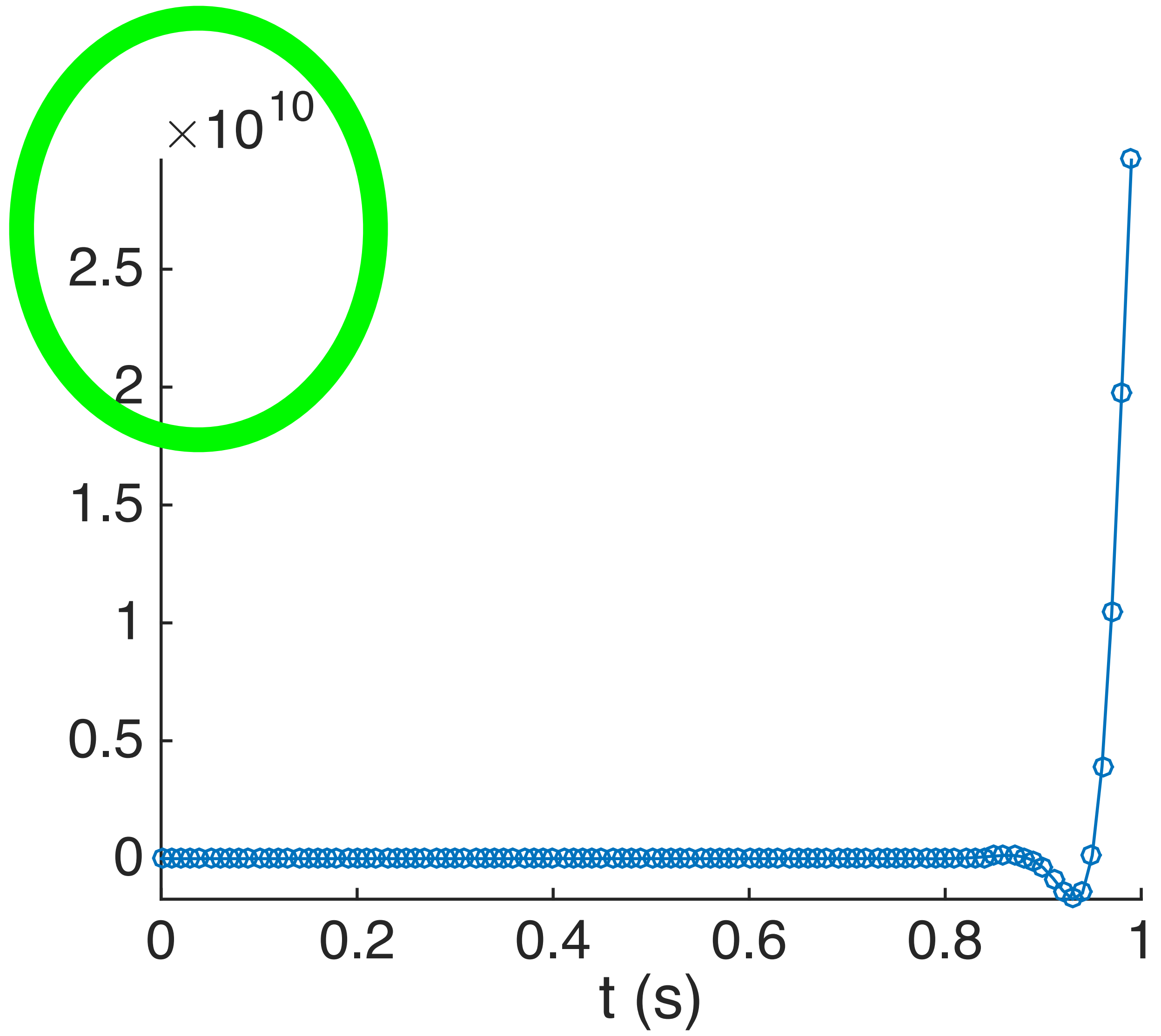


Filter gone Unstable



How Do I Choose a Filter?

For high sampling frequency and plenty of initial data, consider FIR filters

- This is typically the case for raw, un-epoched data.

- Parks-McClellen ("optimal") filters work well. Can choose soft frequency transitions.

  - (Report the filter choice and order, as well as all cutoff frequencies and any other specified parameters, in your *Methods* section.)

- Take care with software "black-box" FIR filters. Maybe good, maybe not.

  - *How much quality signal processing does the software author know?*

How Do I Choose a Filter?

Otherwise, consider IIR filters

  - This is typically the case for epoched data.
- If can't be bothered, Butterworth filters are "fine".
  - If really can't be bothered, use a 4th order Butterworth.
- If you care about your frequency bands, consider using an Elliptic filter.
  - (Report the filter choice and order, as well as all cutoff frequencies and any other specified parameters, in the *Methods* section.)
- Software "Black-box" IIR filters usually not worrisome.

How Do I Choose a Filter?

If you care about your frequency bands, consider using an Elliptic filter

- Needs “slop” factors/tolerances
  - In the *pass* frequency band, how close to “1” (100% let through) do you *really* need? If your peak height were off by only 1%, would you even notice?
    - Matlab requires this (“passband ripple”) to be in dB: 1% ≈ 0.1 dB
  - In the *stop* frequency band, how close to “0” (0% let through) do you *really* need? If your noise is suppressed only by 100x, would you even notice?
    - Matlab requires this (“stopband attenuation”) to be in dB: 100x = 40 dB

Outline

- Fourier Transform: *Why It's Useful*, and *What it Can/Cannot Do For You*
- Filters: *What They Do*, and *How They Do It*
- Filters: *Why So Many Different Kinds? Which Should I Use and When*?
- Grab Bag:
  - *Use Causal Filters; Windowing is Good*; *Low-Pass your Envelopes*

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Causal & non-Causal Filtering

All filters discussed hear are *causal.*

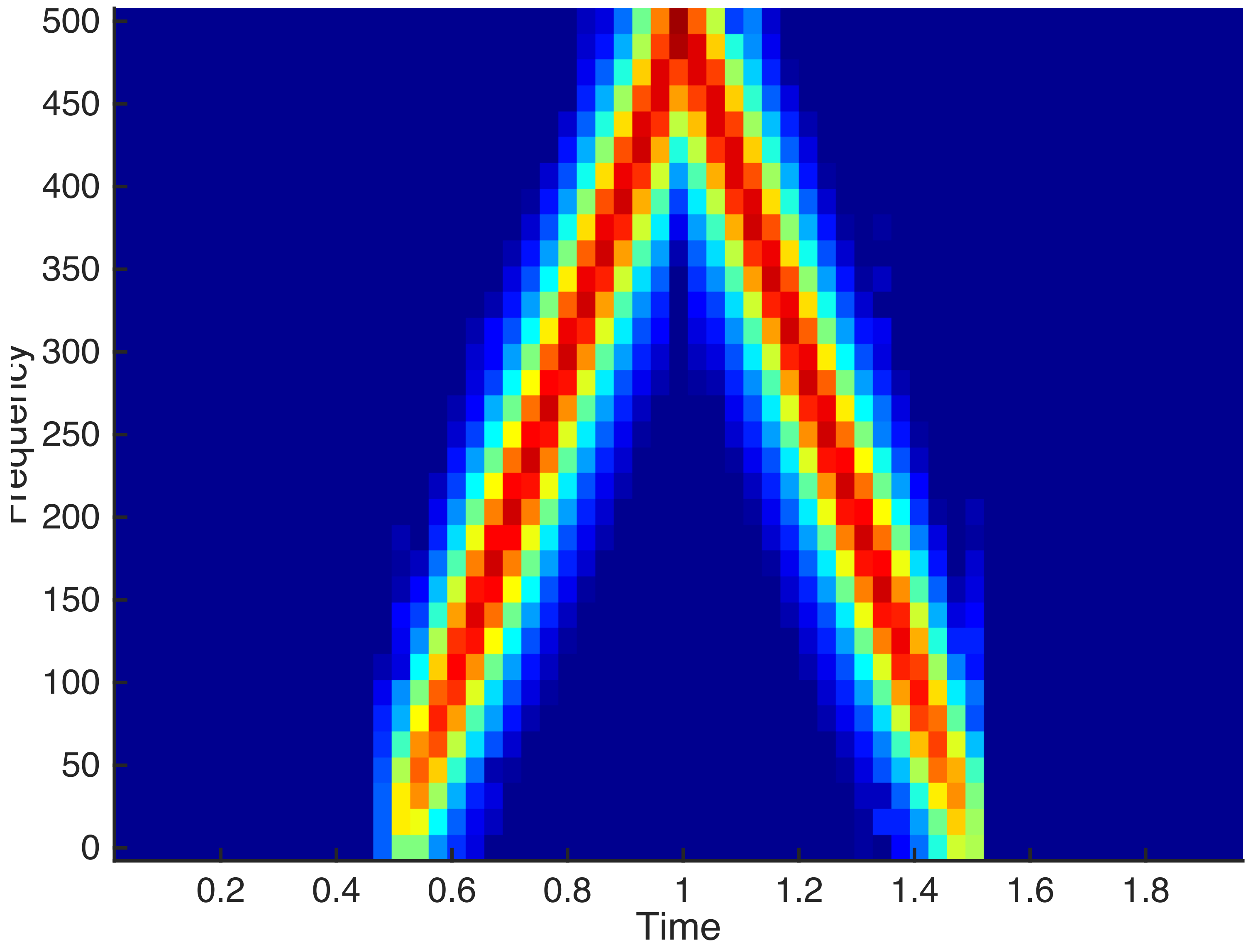
- Changes to the input signal cause changes in the output. The output changes occur at the same moment, or later, but never earlier.
    - Some output changes are desirable: using a low pass filter to slow down fast changes in the input signal.
    - Some output changes are undesirable: ringing due to increase of transition-frequency content in the input signal.

Causal & non-Causal Filtering

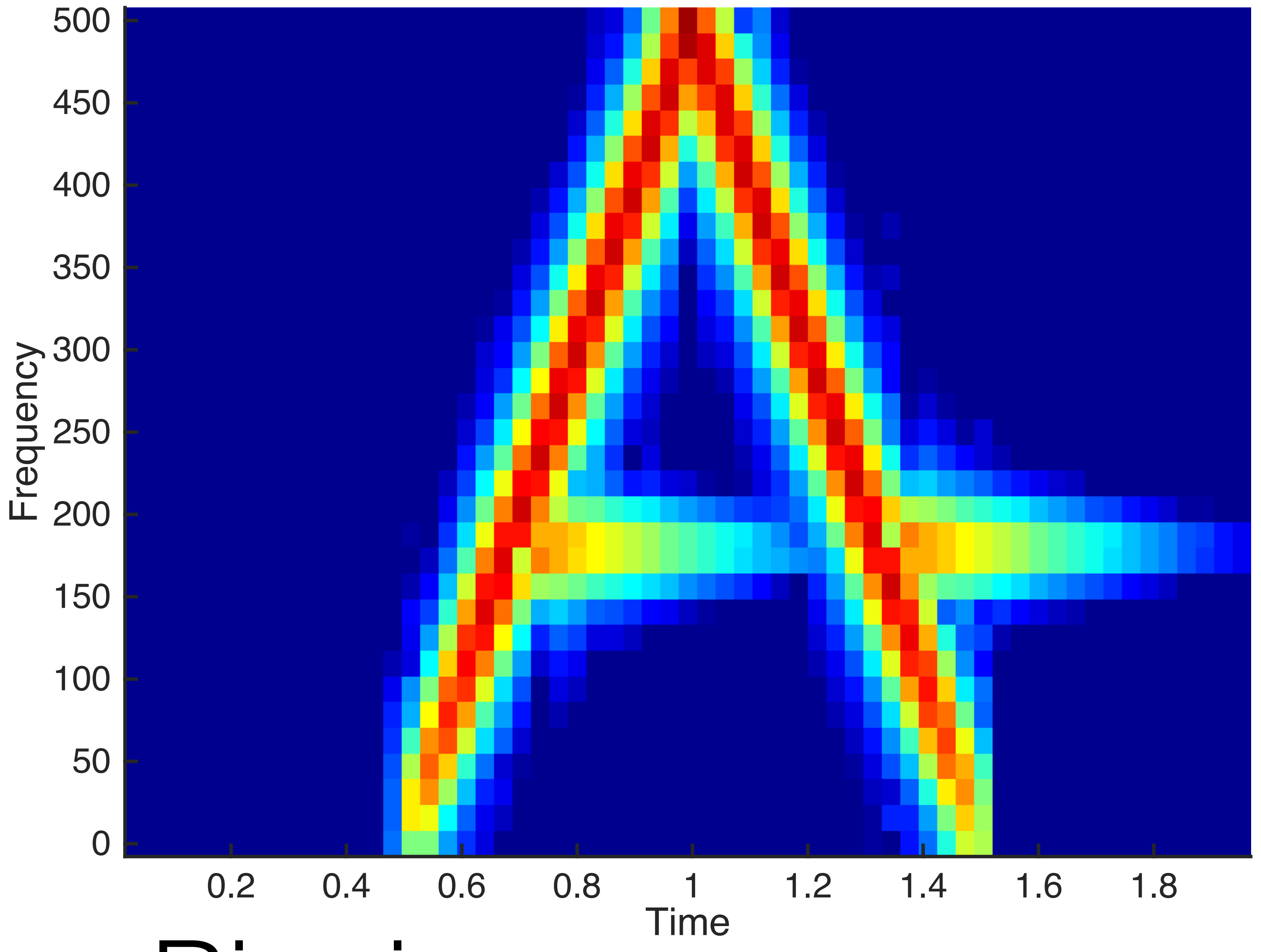
- It is mathematically possible (but biologically undesirable!), to temporally “center” all such output changes so they do not seem to be all contribute to delay.
- This (undesirable act) can be achieved with a particular kind of non-causal filtering: *zero-phase* filtering (Matlab “filtfilt”).
- Zero-phase sounds wonderful, but it is not (c.f. “ideal” filter).
- Zero-phase filters *do not remove delay-based artifacts*, and in fact they double them.

Zero-Phase Filtering Example

FM Sweep (Spectrogram)



Notched FM Sweep



Ringing:

- persistent
- causal

Zero-Phase Filtering Example

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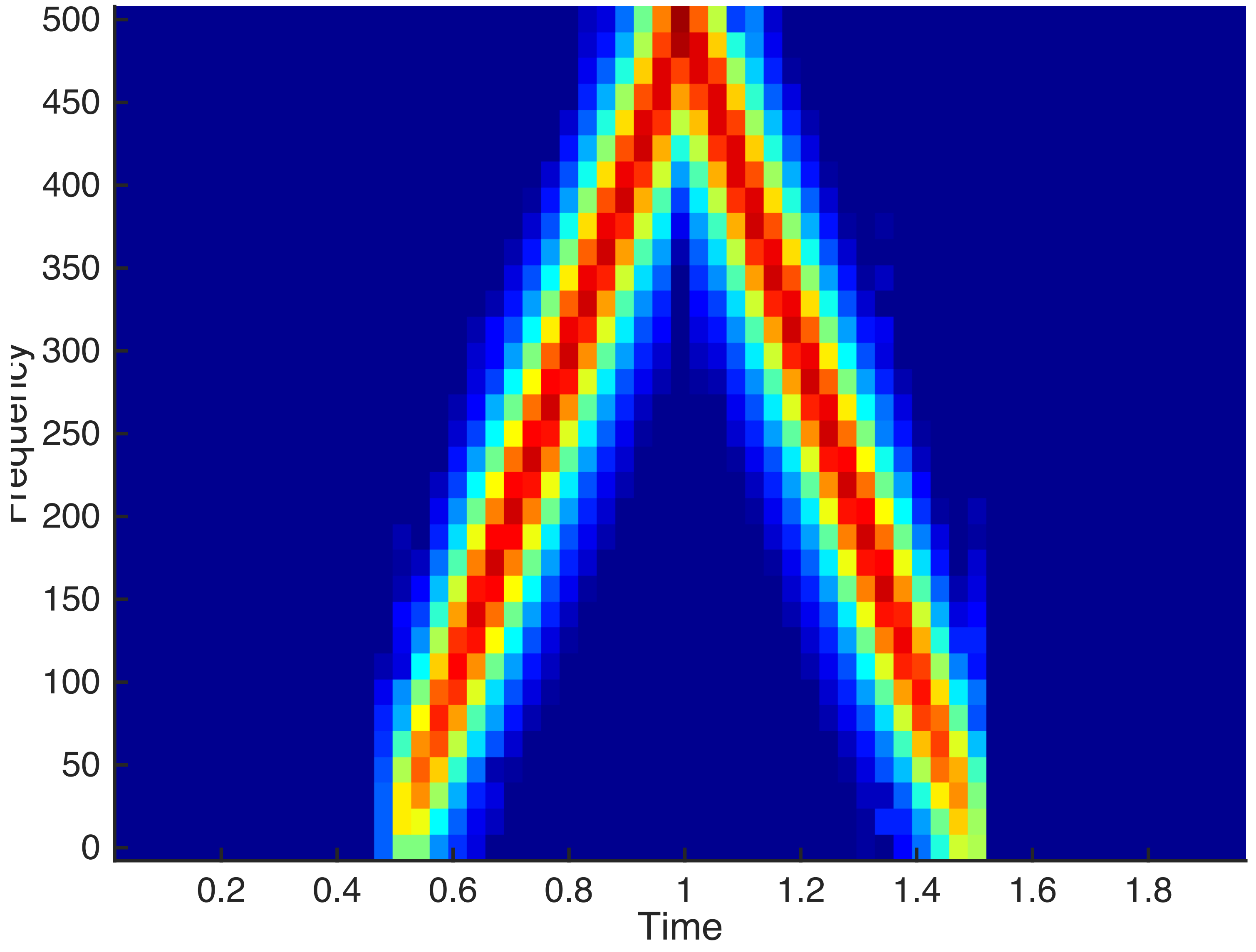

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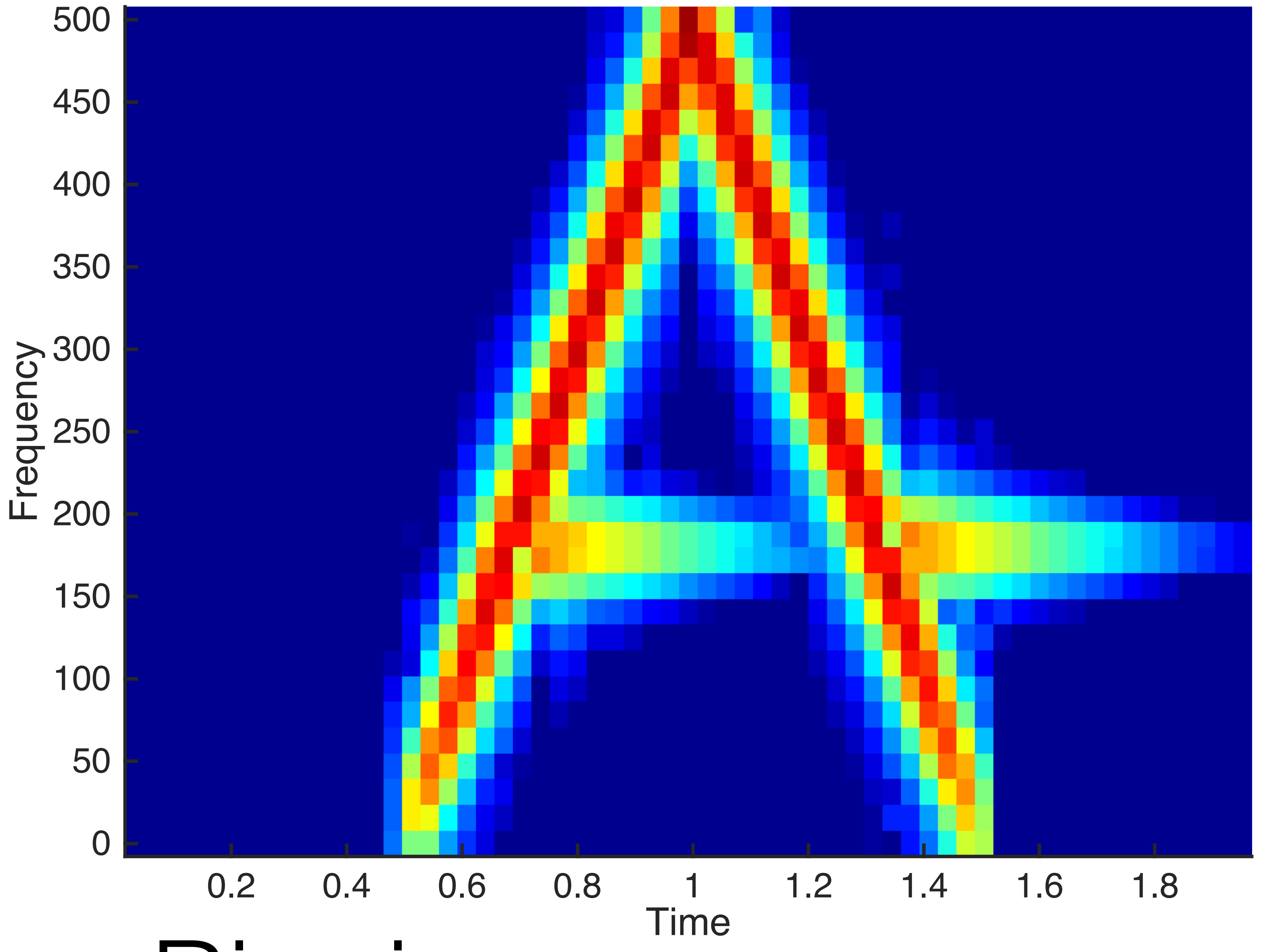

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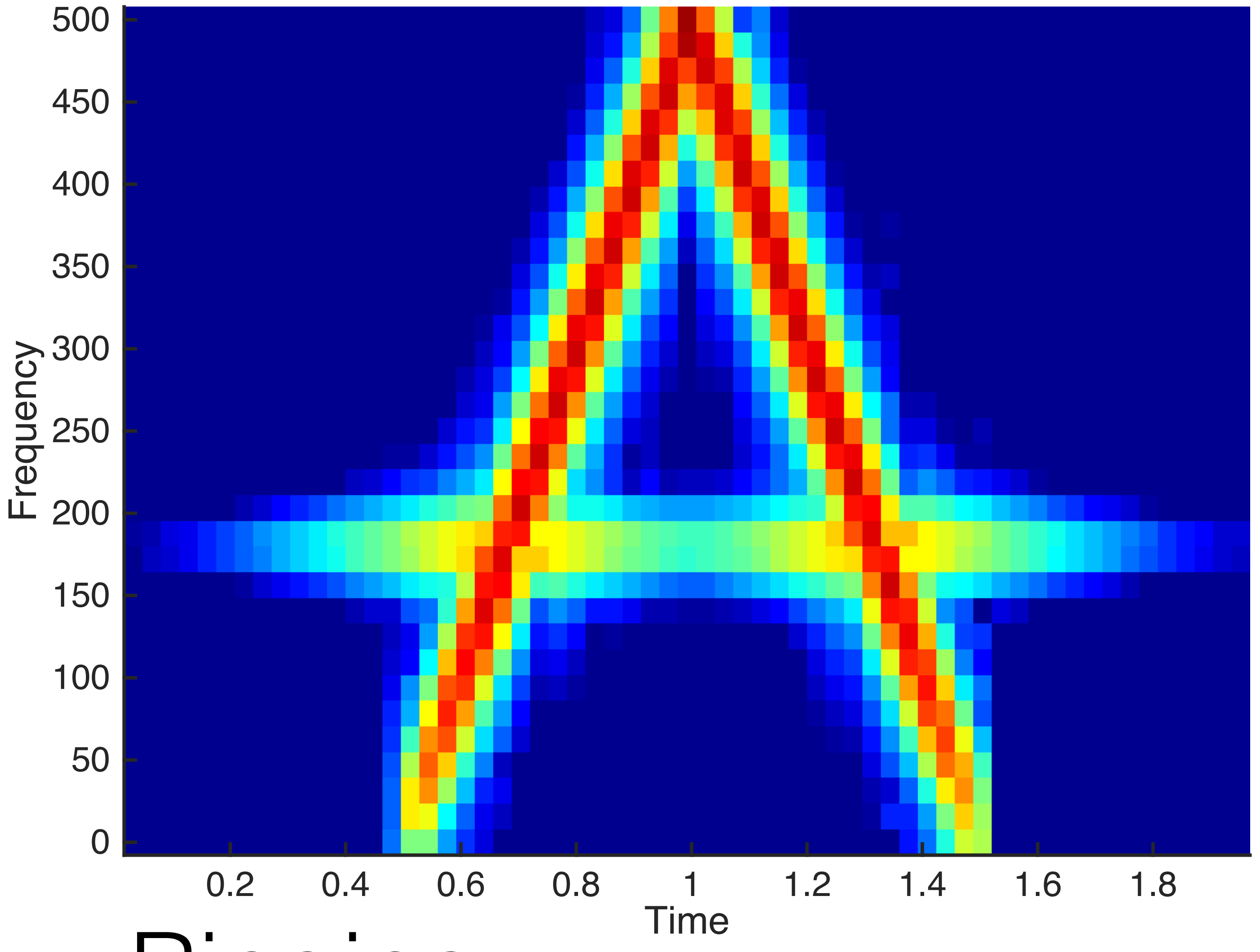
Zero-Phase Notched FM Sweep



Ringing:

- duplicated and flipped
- no cancellation (except "on average"?)

Causal & non-Causal Filtering

- Zero-phase filters do not remove distortions, but instead spread them out symmetrically.

- Spreading them out gives zero “on average” but not actually zero.

- Other temporal features (e.g. smoothing, change detection) get similarly spread out by duplication.

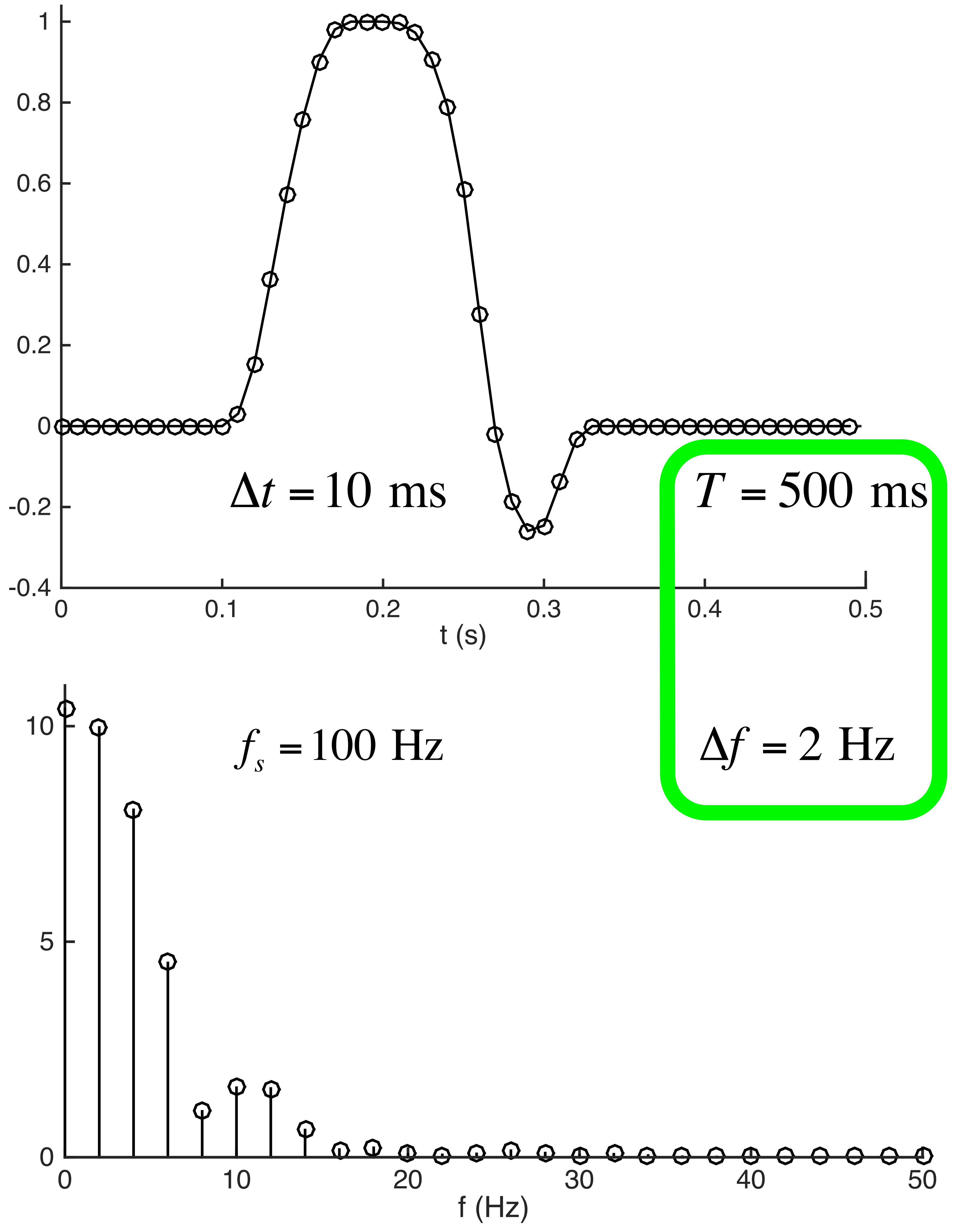
  - May remove peak delay, but replaces un-delayed rise with an anti-causal rise.

- Recommendation: Don’t use. Causes far more problems than solutions.

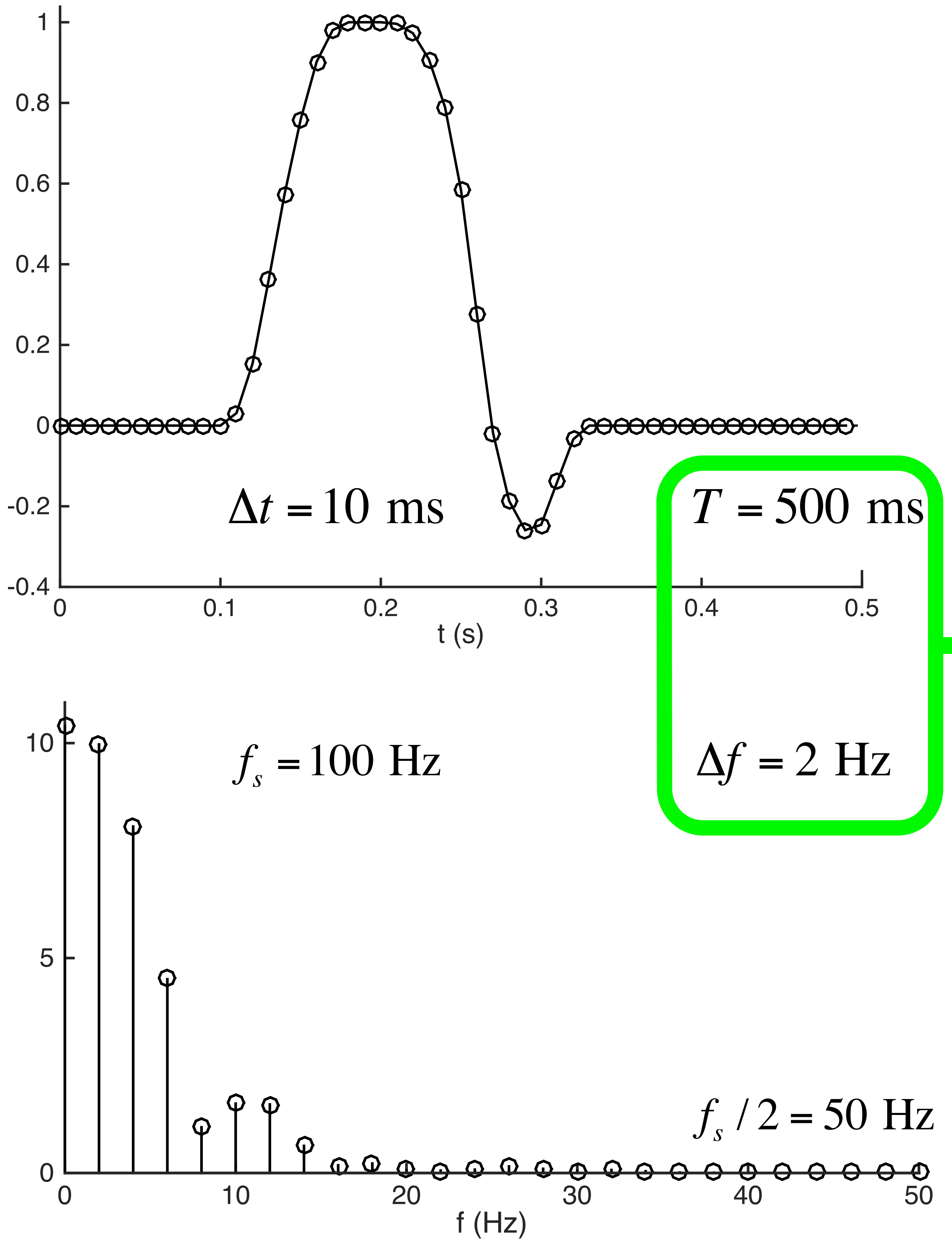
Windowing and Frequency Resolution

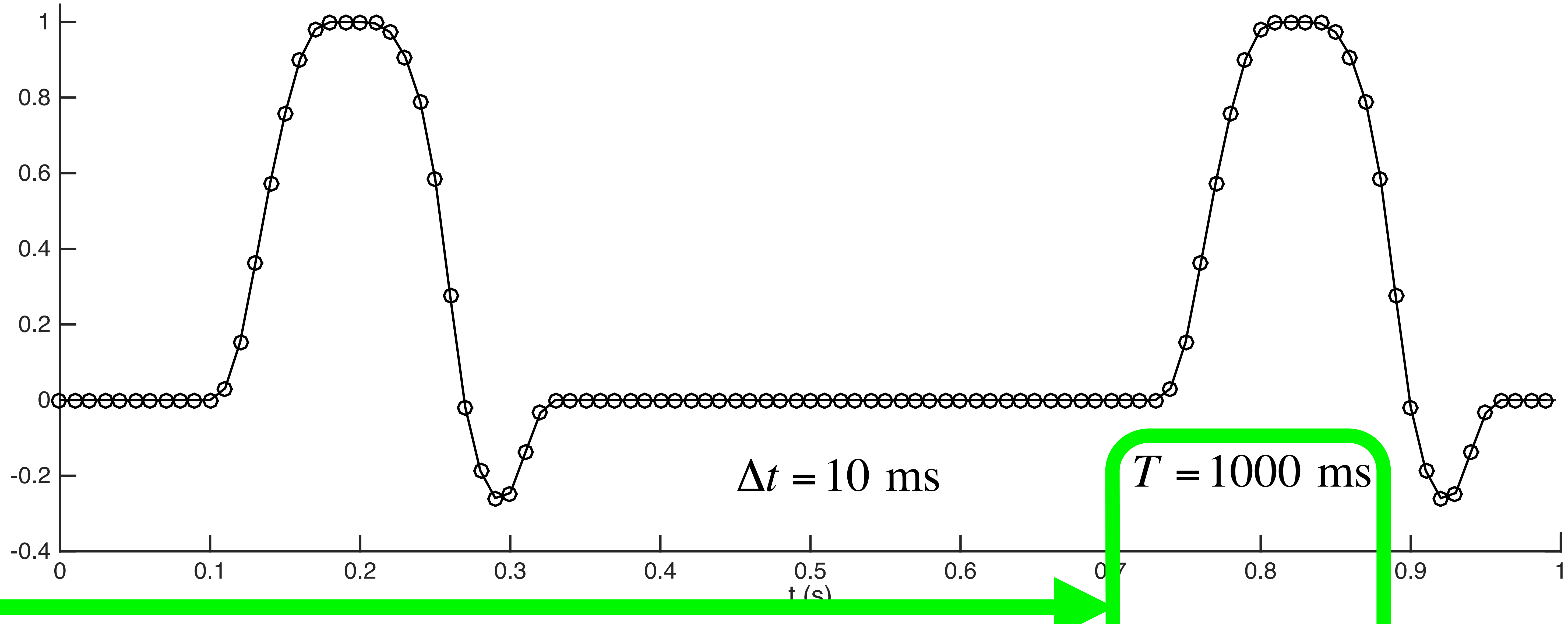
- *Frequency resolution* (Δ*f*), the limiting factor in distinguishing one frequency from another, is determined by the total *duration* of the signal (*T*).

- This relationship is the time-frequency conjugate of the relationship between and *temporal resolution* (Δ*f*) and *sampling frequency* (f_s).

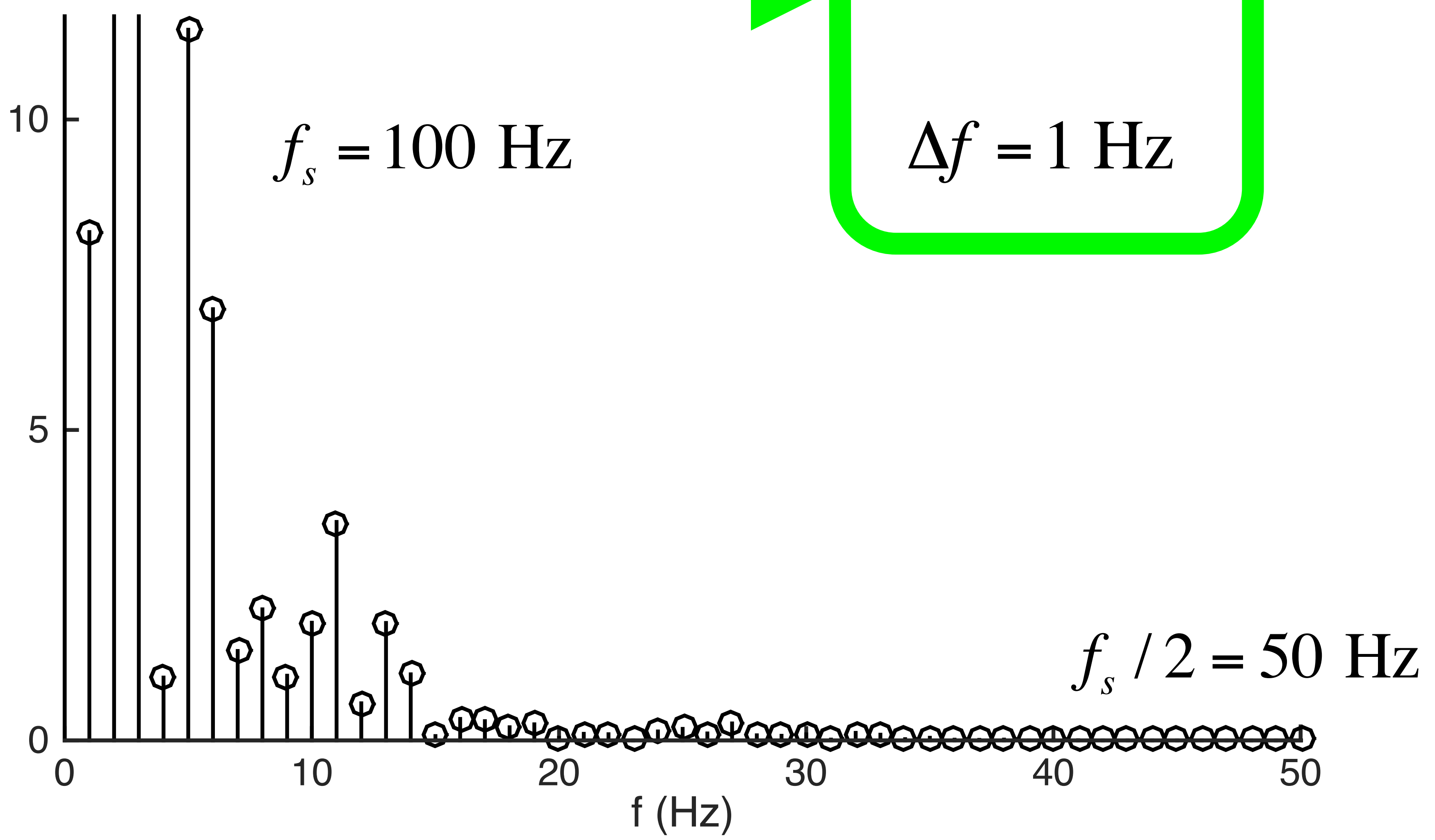


Windowing and Frequency Resolution





Finer frequency resolution is obtained by by increasing the signal duration.



Windowing and Frequency Resolution

- It is sometimes desirable to “smear” information temporally (e.g. low-pass filter in order to attenuate noise).
  - The *effective* time resolution is worse, even though Δt remains unchanged.
- Analogously, it is sometimes desirable to “smear” information over frequencies (e.g. to attenuate spectral leakage).
  - The *effective* frequency resolution is worse, even though Δf remains unchanged.
- This frequency smearing is typically accomplished by *windowing* in the time domain.

“Fourier coefficients do not always mean what you think they mean.”

–The Princess Bride (paraphrased)

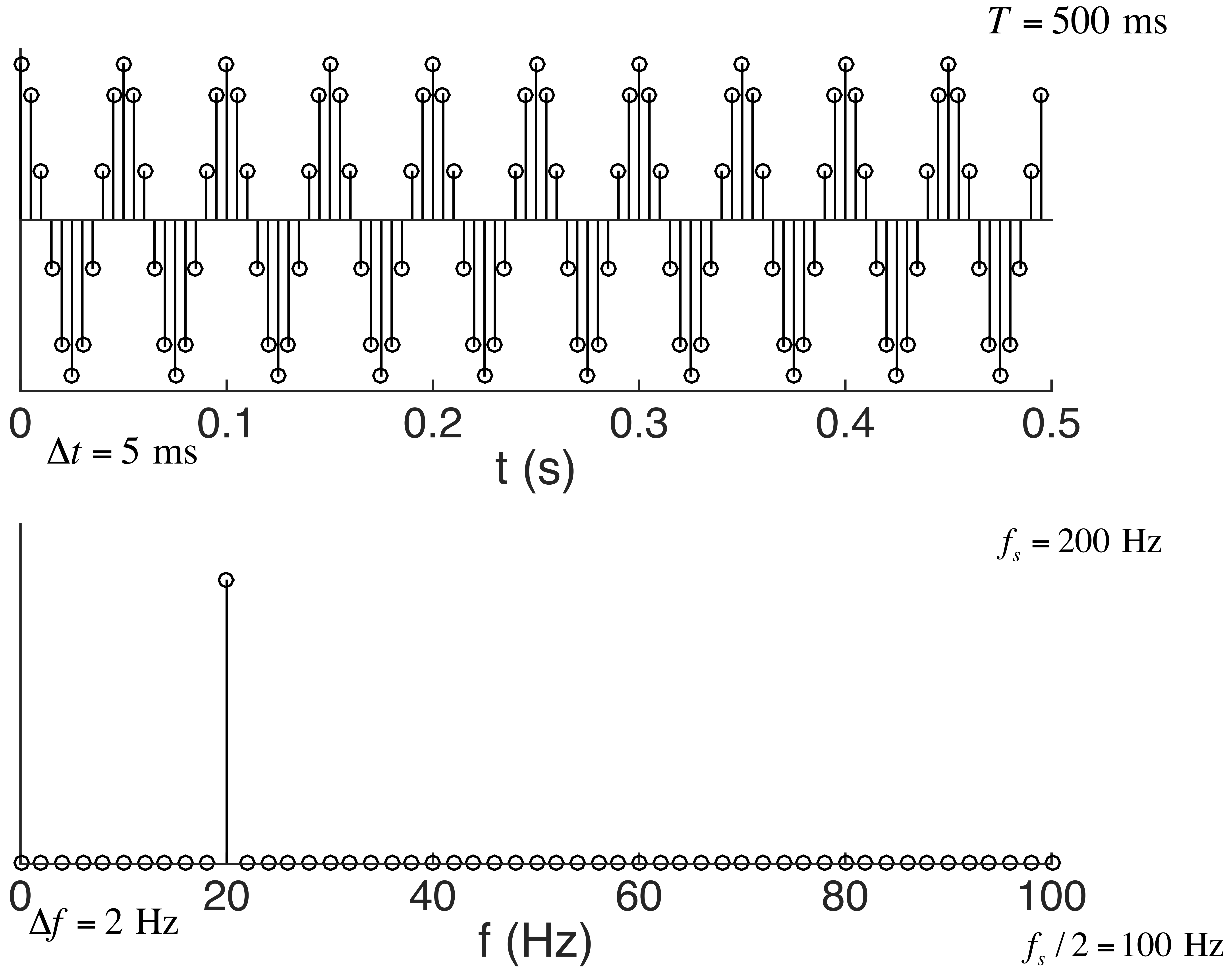
Spectral Leakage

Example 1

A pure sinusoid (single frequency).

In the Fourier domain it has a single Fourier component.

$$x[t] = \cos(2\pi f_a t)$$
$$f_a = 20 \text{ Hz}$$



Spectral Leakage

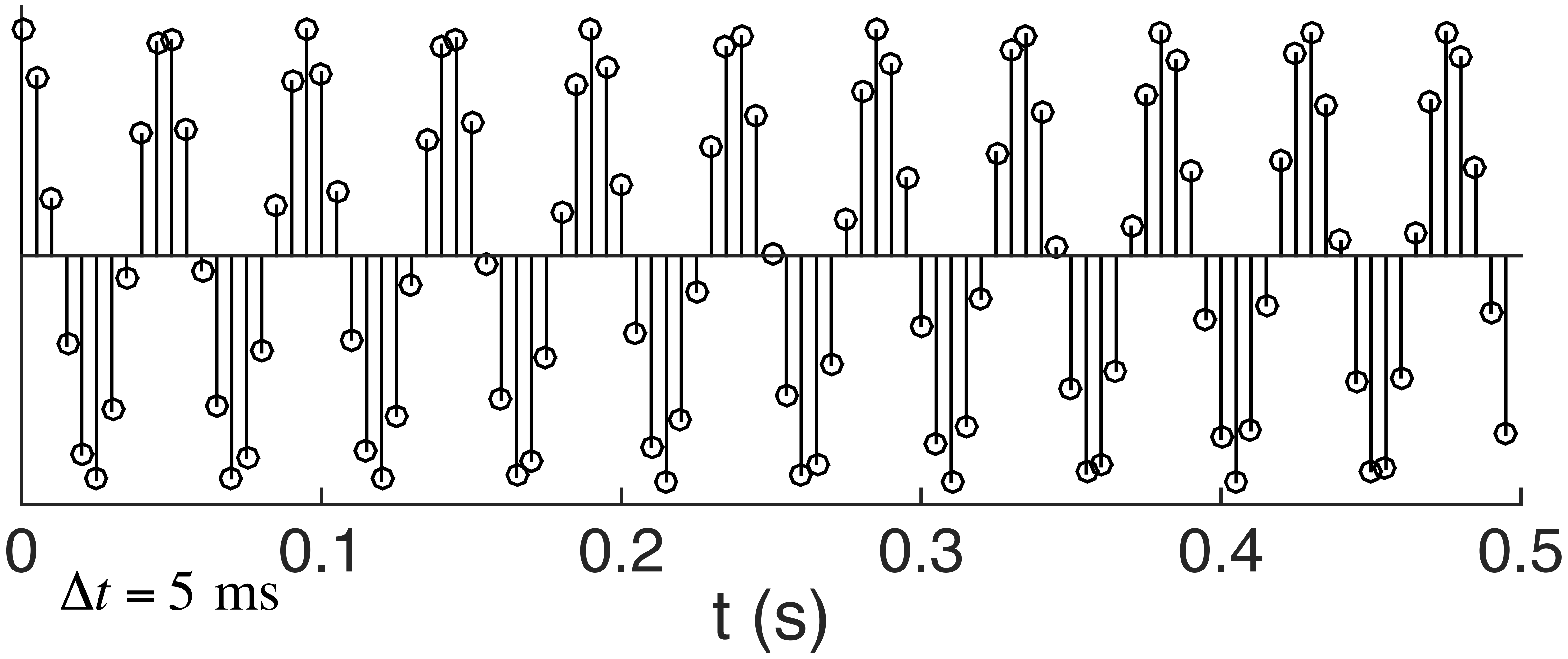
Example 2

A pure sinusoid (single frequency).

What does it look like in the Fourier Domain?

$$x[t] = \cos(2\pi f_b t)$$
$$f_b = 21 \text{ Hz}$$

$T = 500$ ms



$\Delta t = 5$ ms

$f_s = 200$ Hz

$\Delta f = 2$ Hz

$f_s / 2 = 100$ Hz

Spectral Leakage

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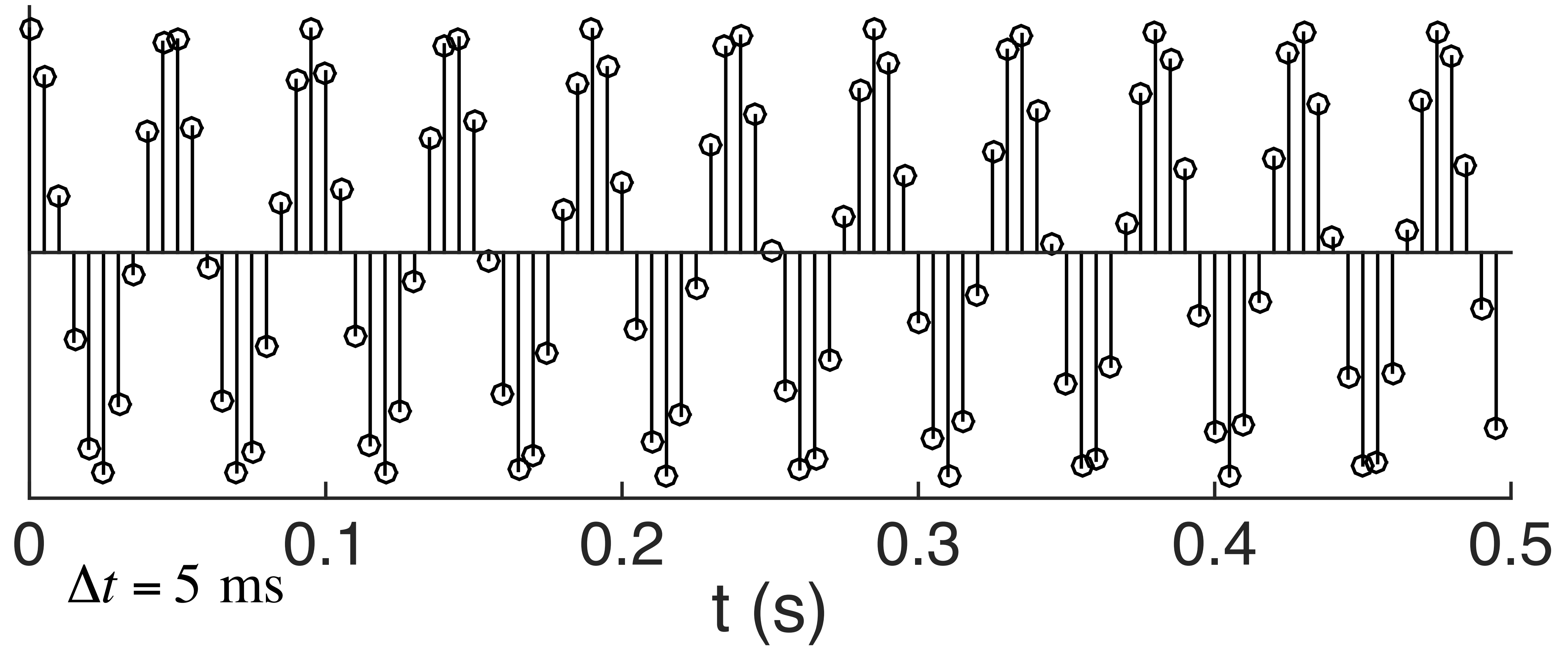

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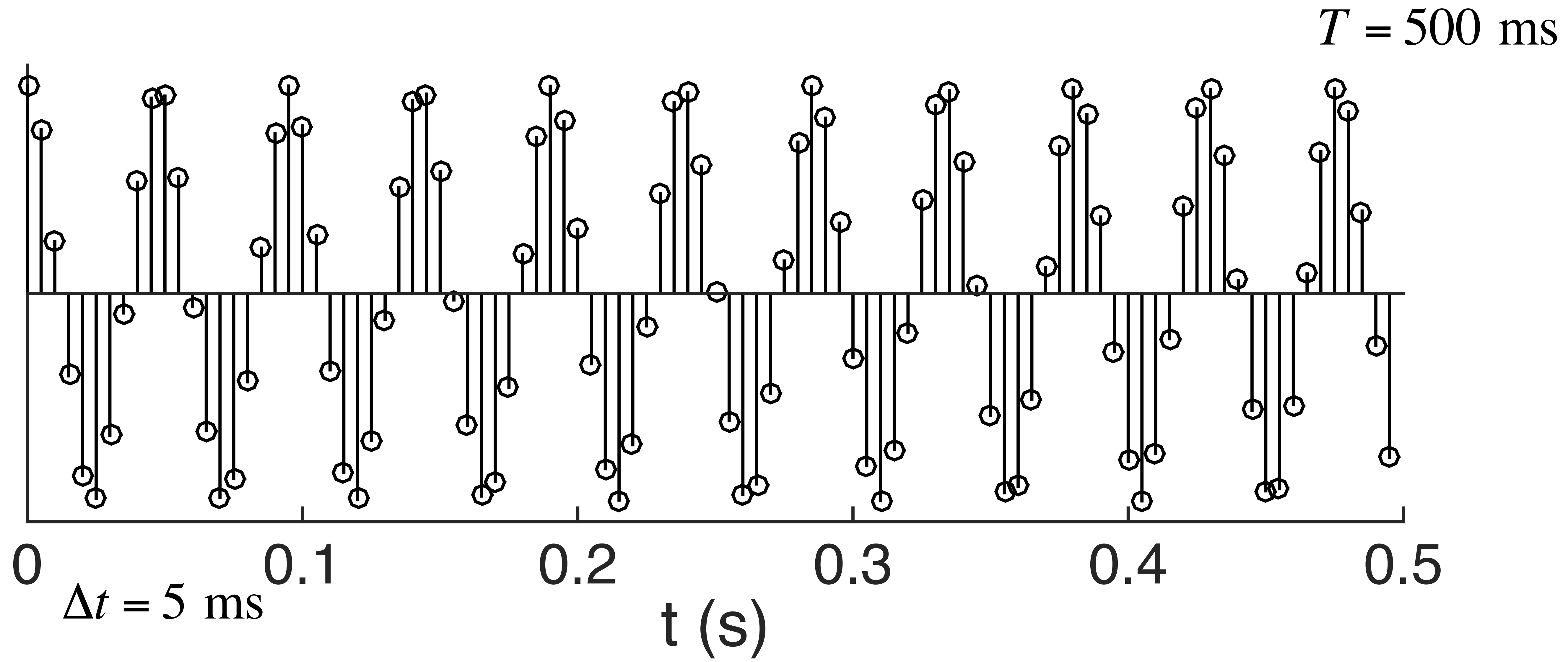

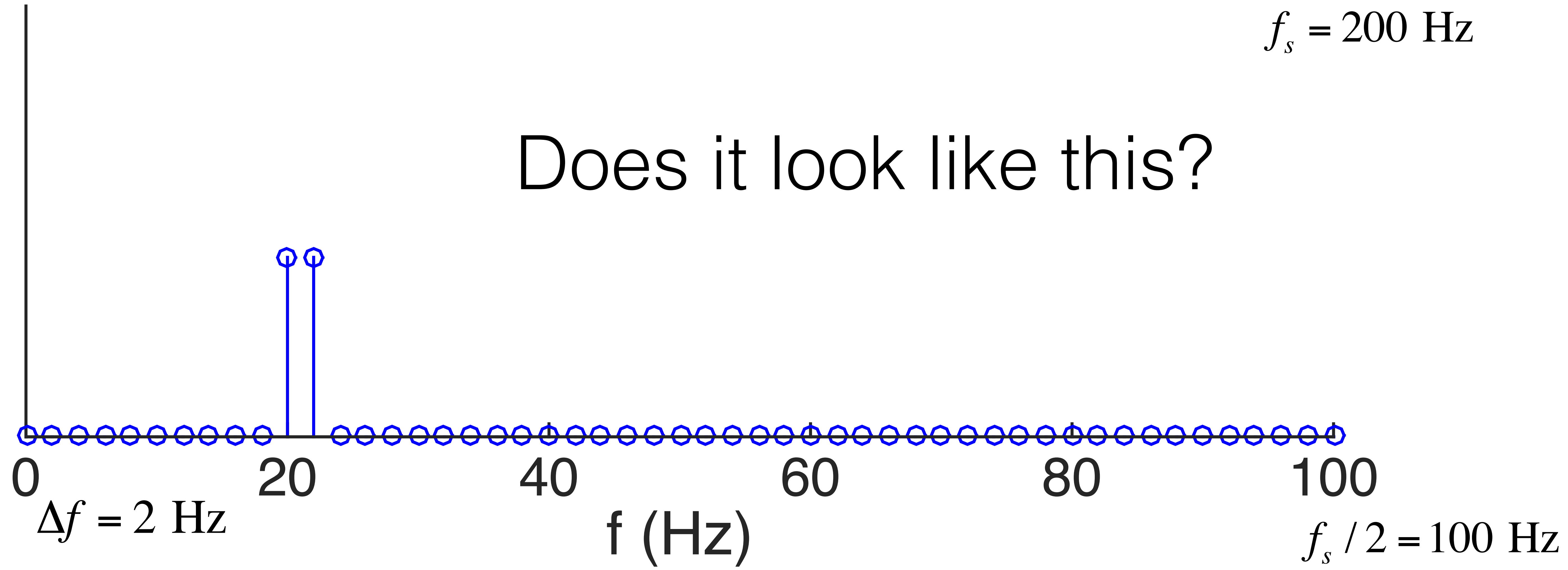

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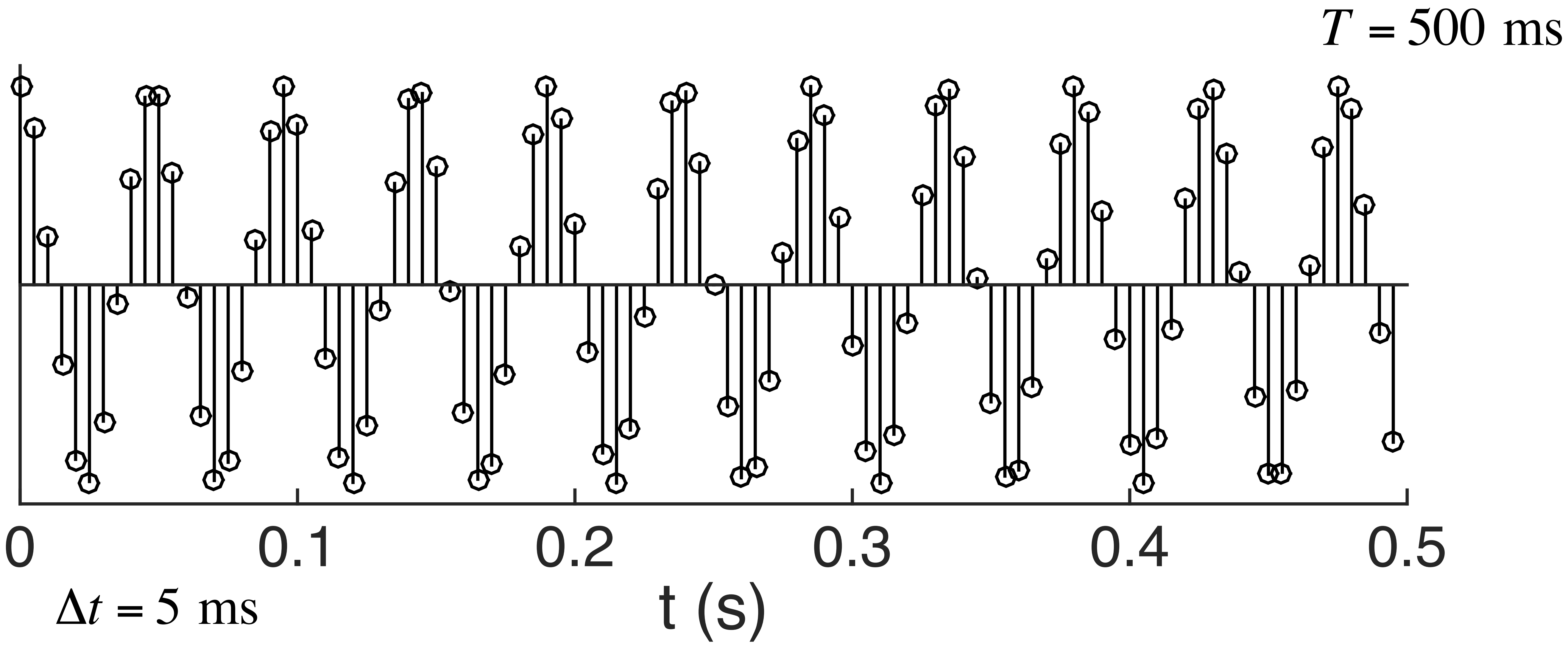

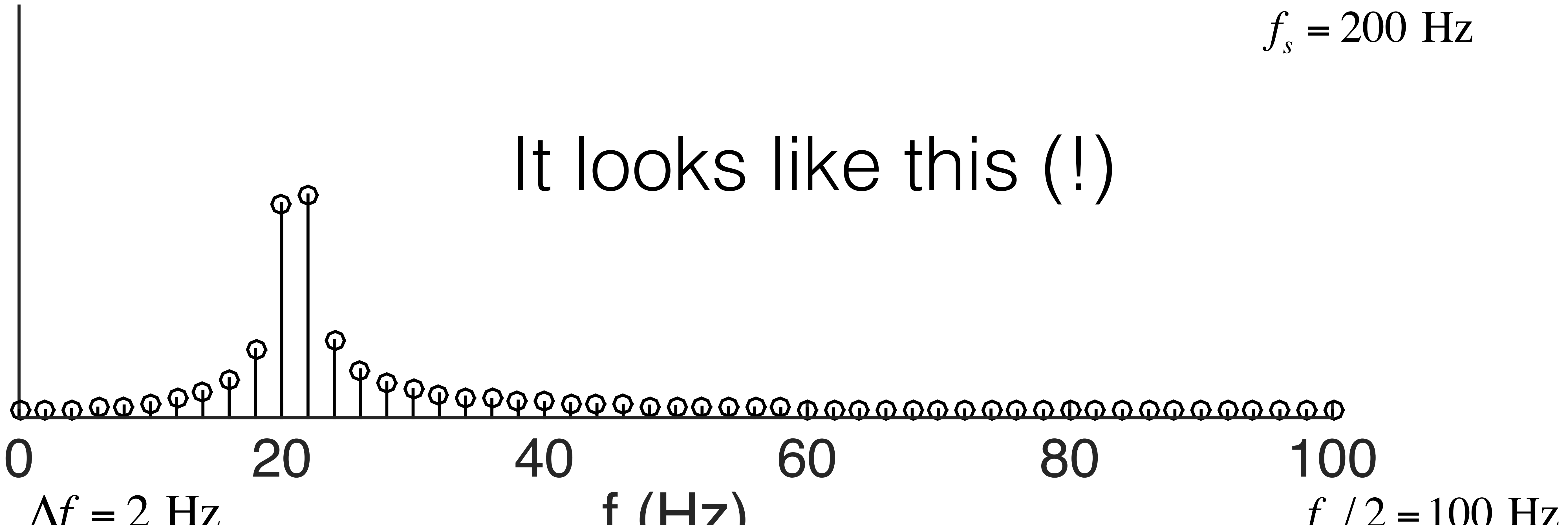

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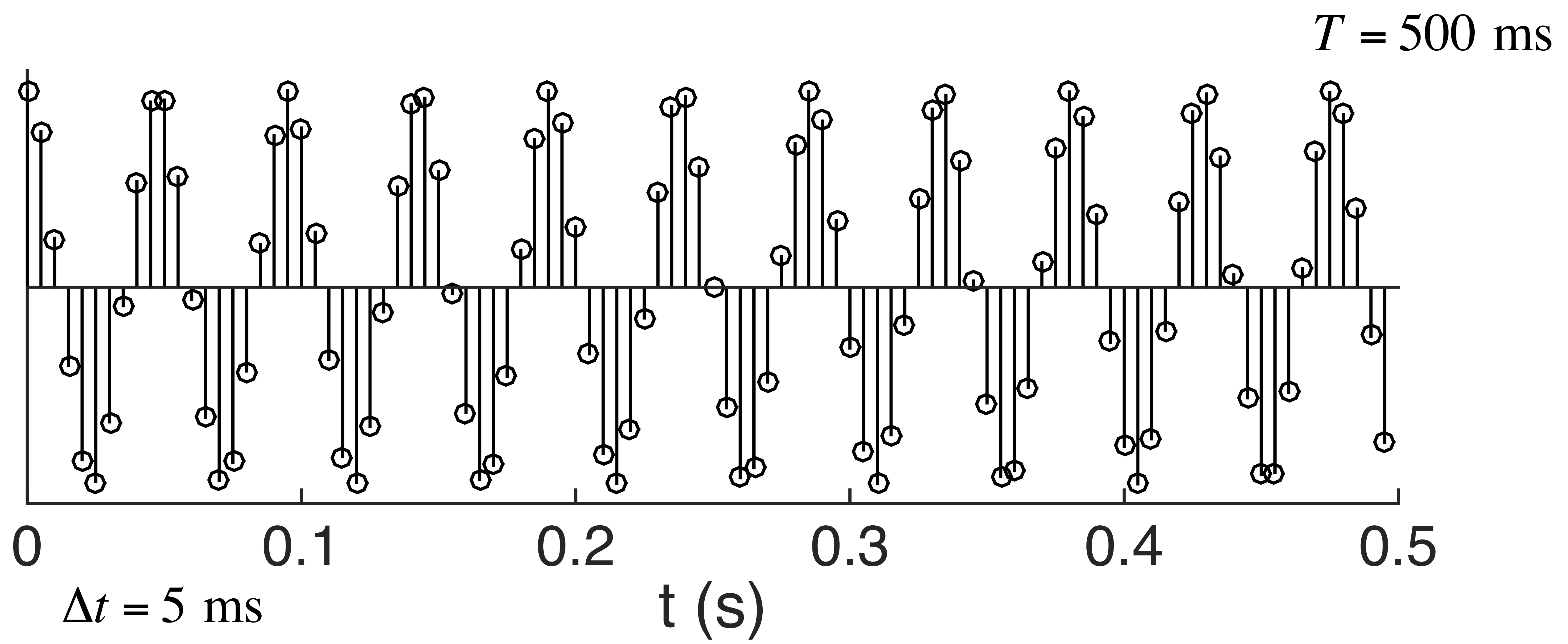

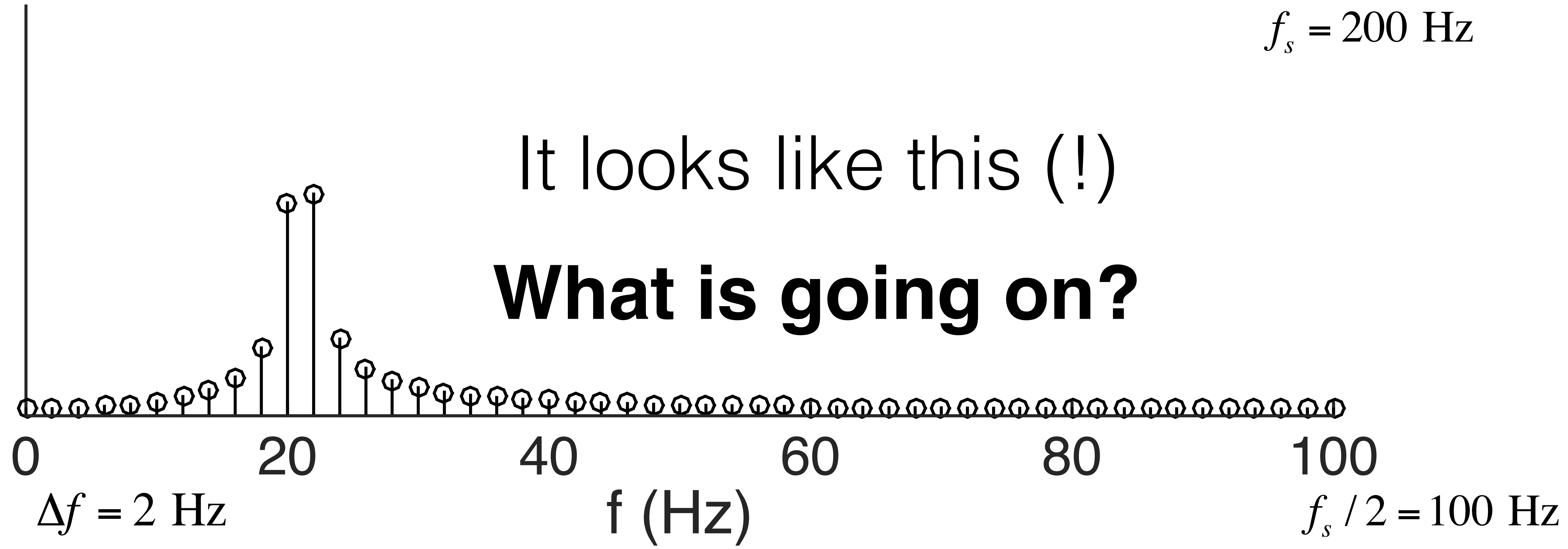

Spectral Leakage

A sinusoid whose single frequency is *not* a Fourier frequency exhibits *Spectral Leakage*.

Spectral Leakage of a strong signal component can easily overwhelm weaker nearby signal components.

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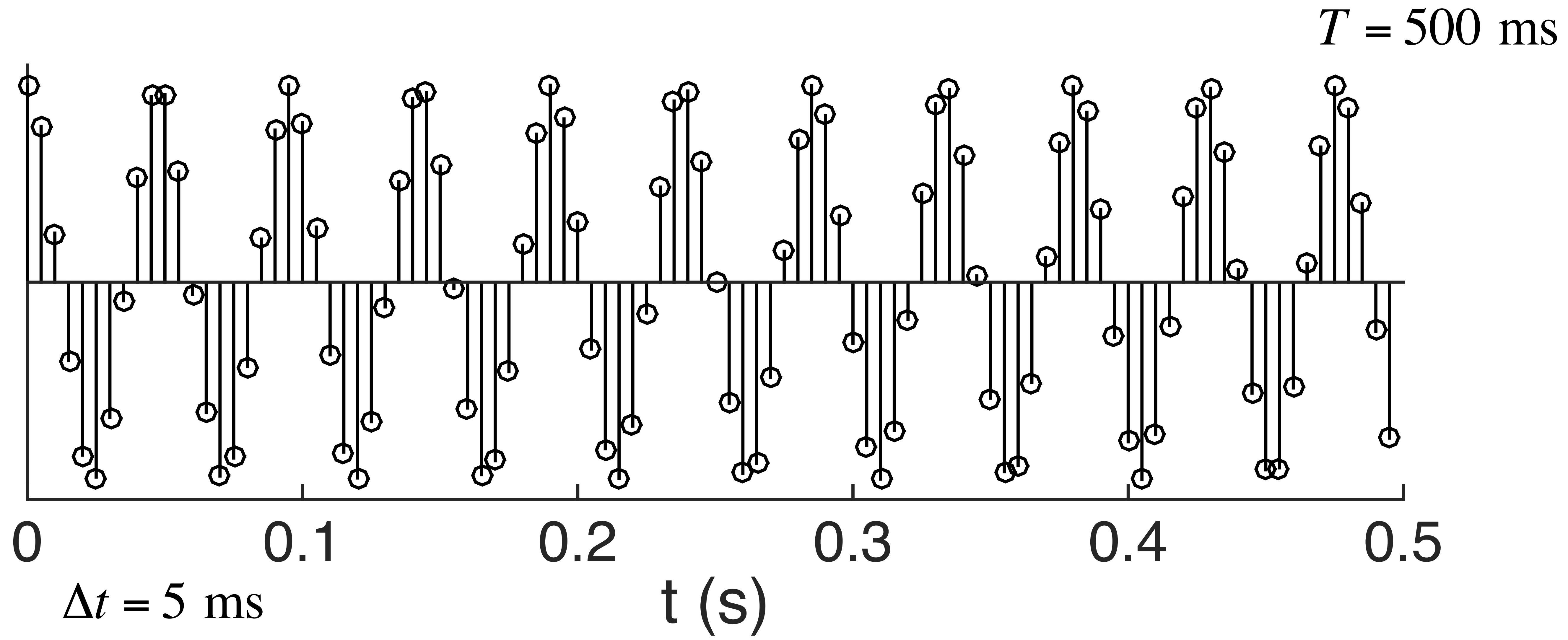
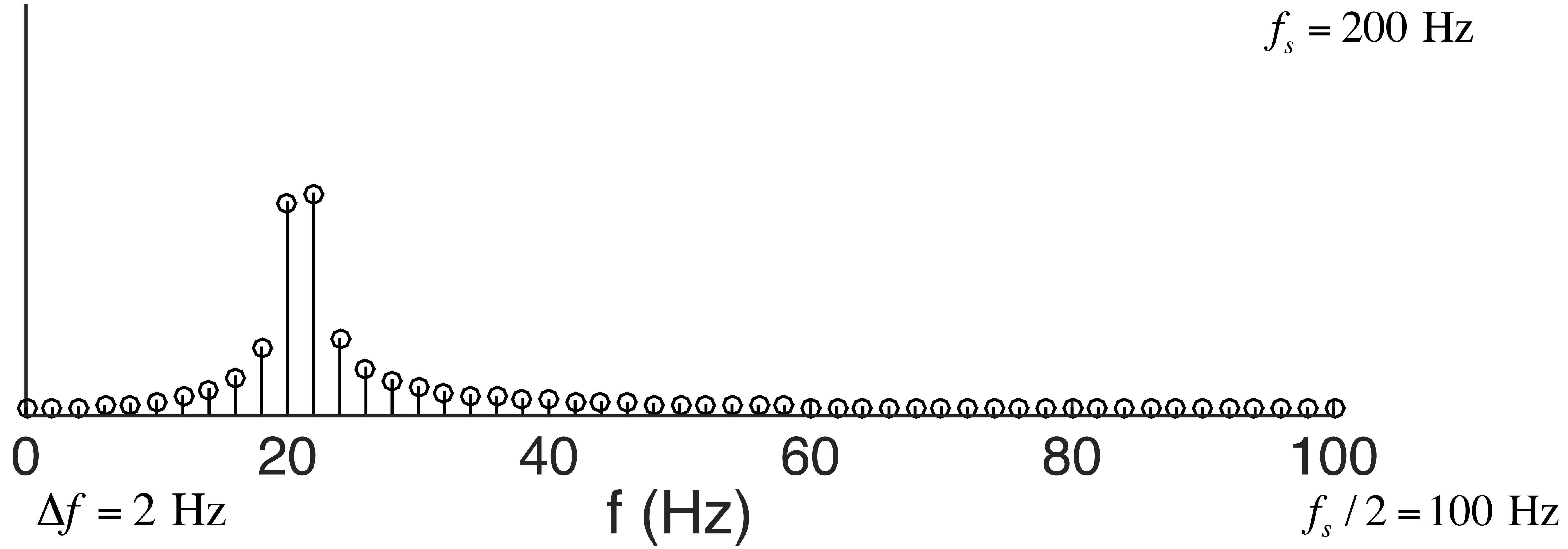




Spectral Leakage

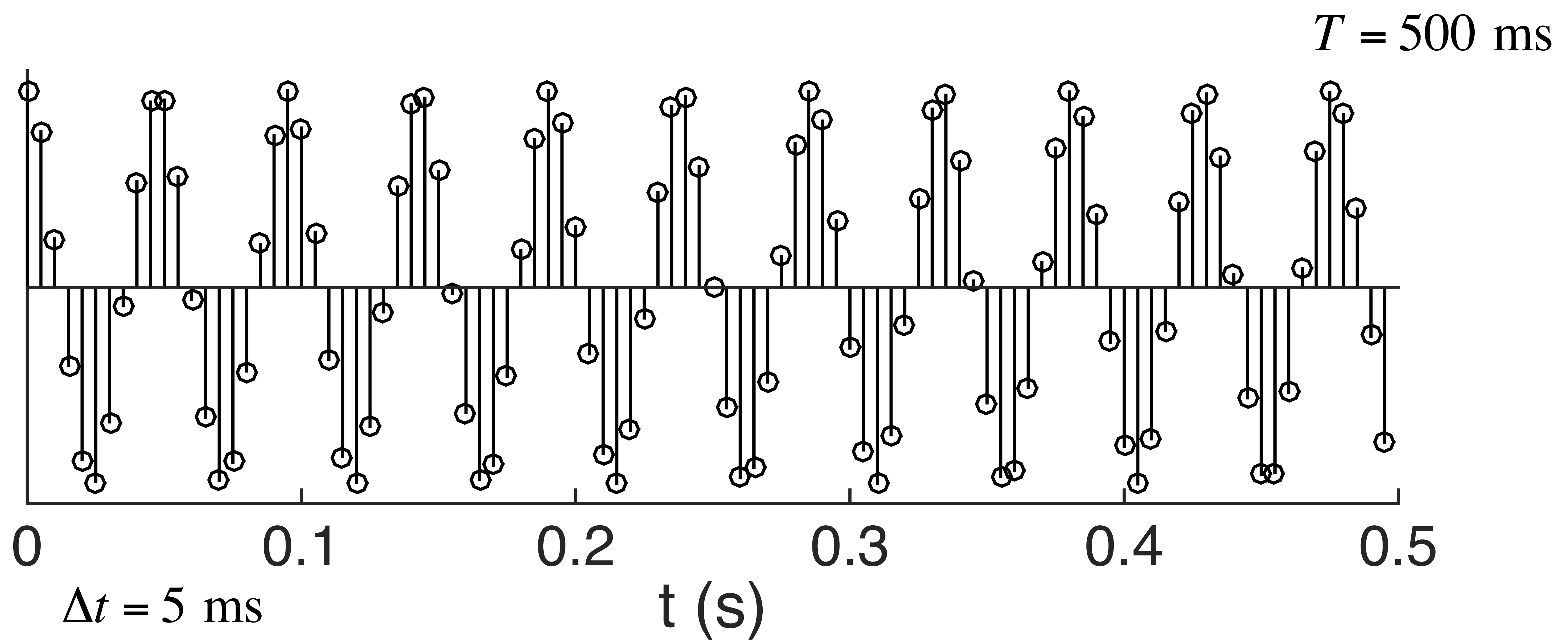
What is the origin of spectral leakage?

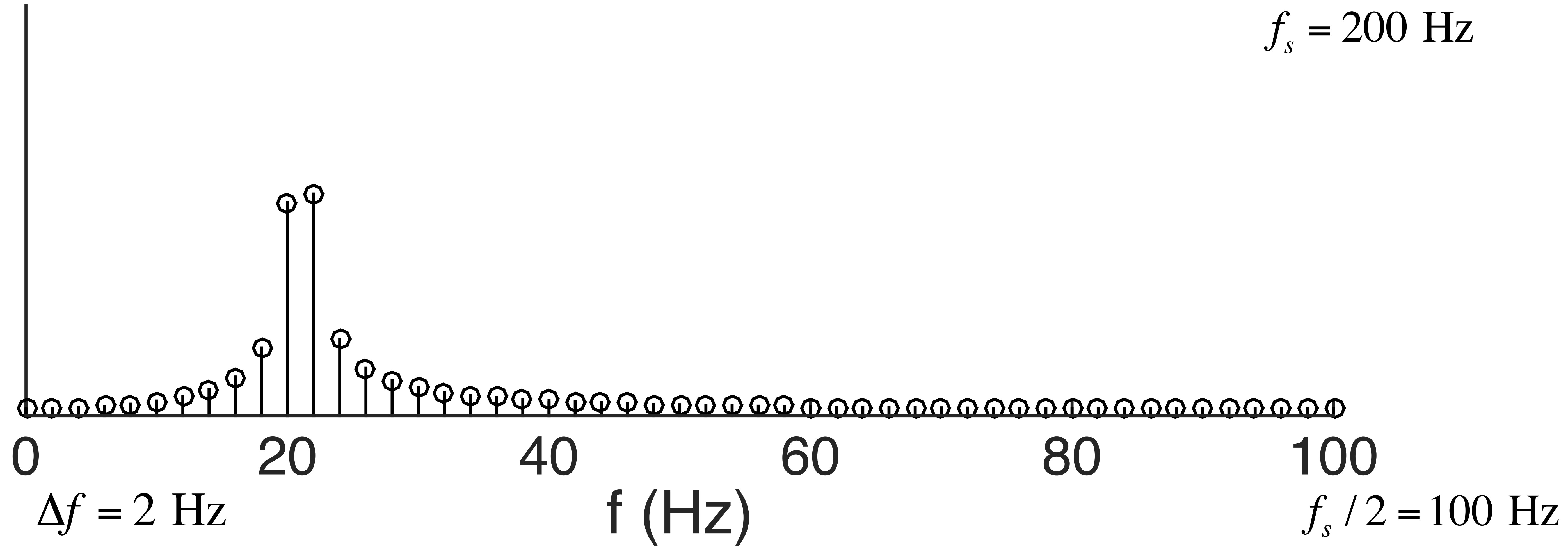
This signal is a cosine, but not periodic with period 2π. The ends do not match.

This can be seen by rotating the signal by *T*/2, which does affect the Fourier transform in magnitude.

Signal discontinuities are spectrally broadband!

$$f_b = 21 \text{ Hz}$$
$$\Delta f = 2 \text{ Hz}$$





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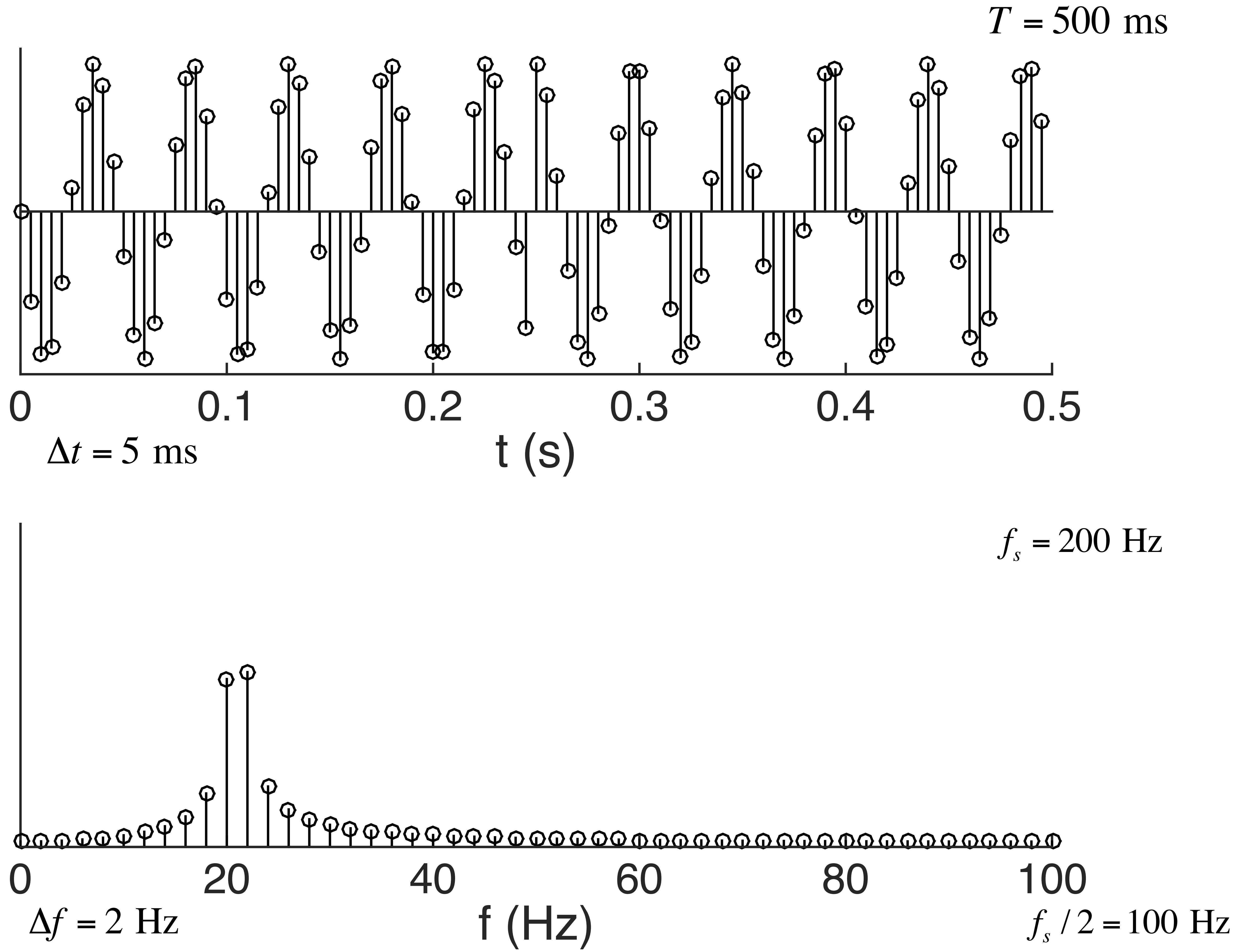

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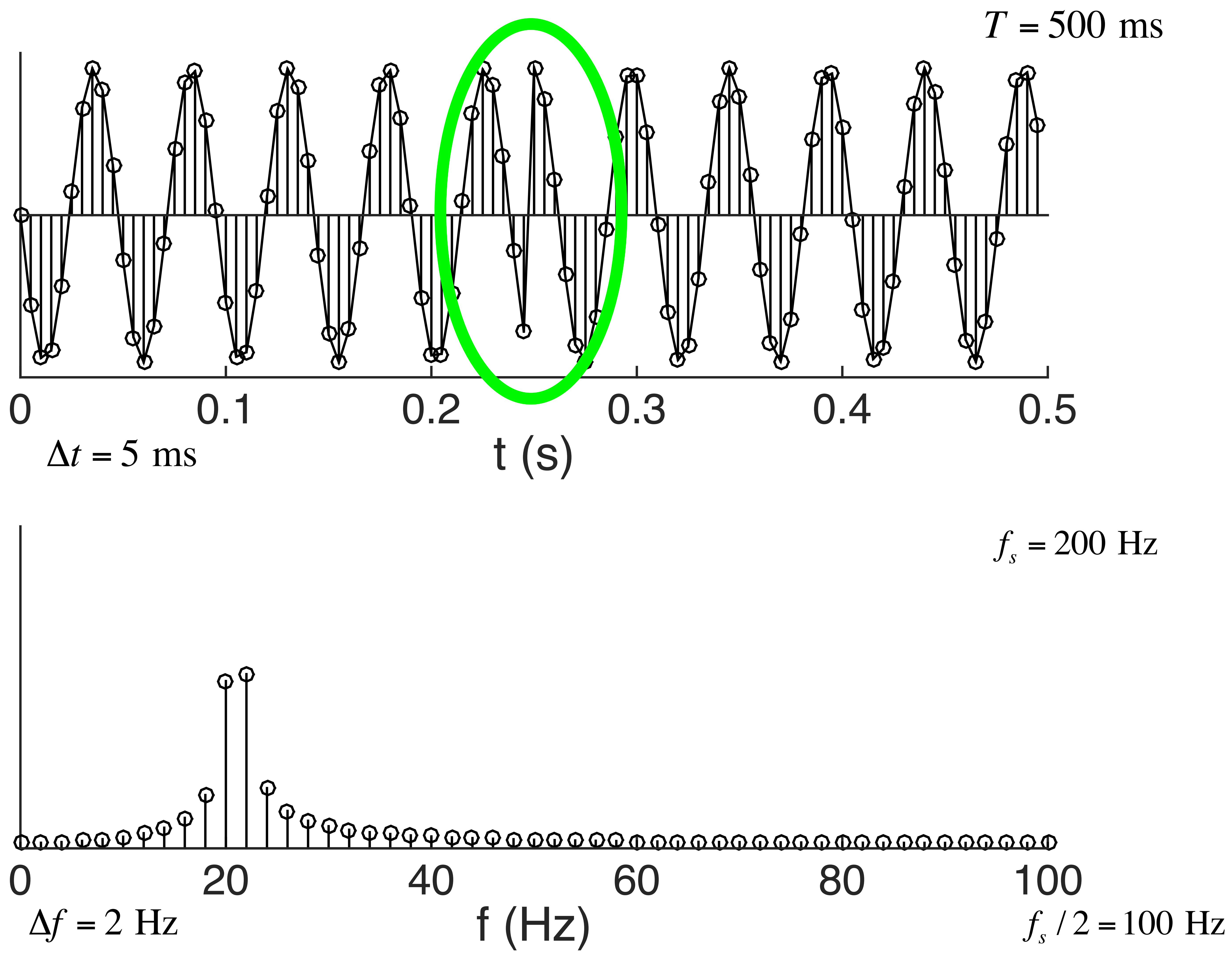

Spectral Leakage

How do we ameliorate the edge “discontinuity”?

Modulate the signal by a window (i.e.,“window” the signal).

$$x[t] = \cos(2\pi f_b t)$$
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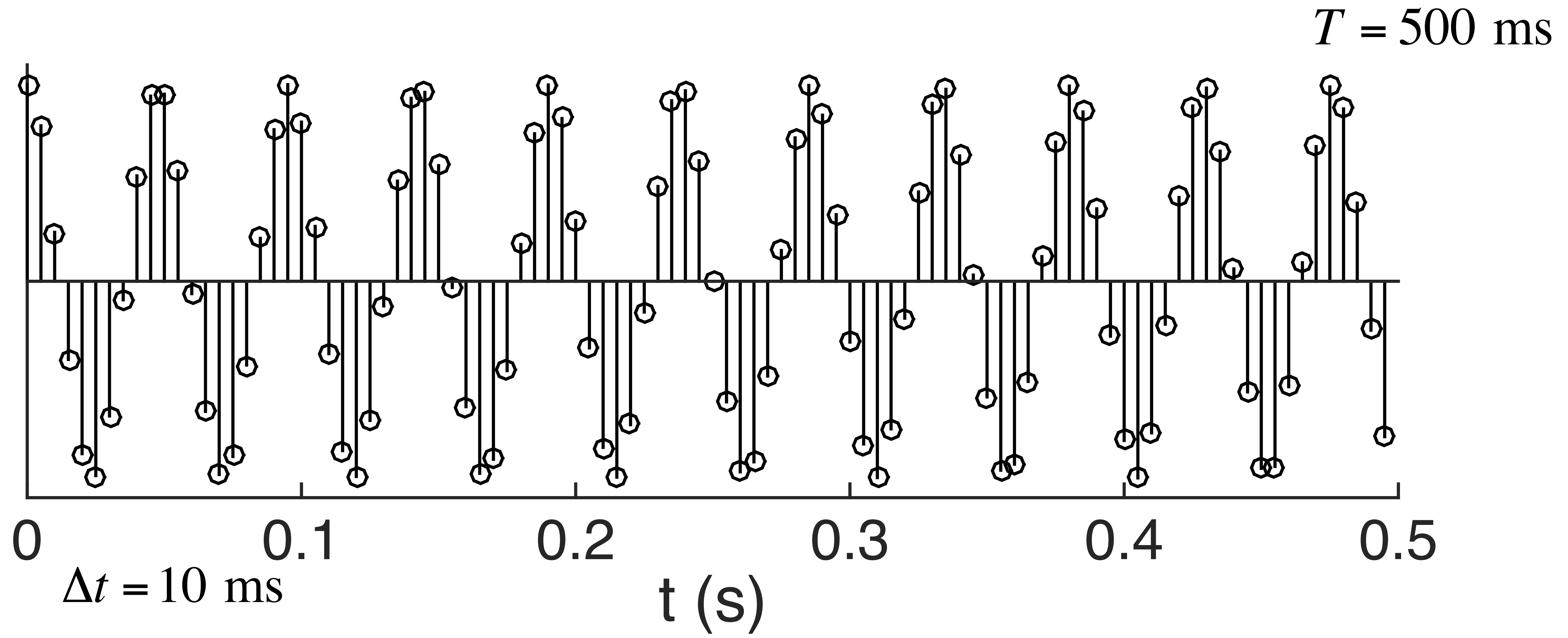
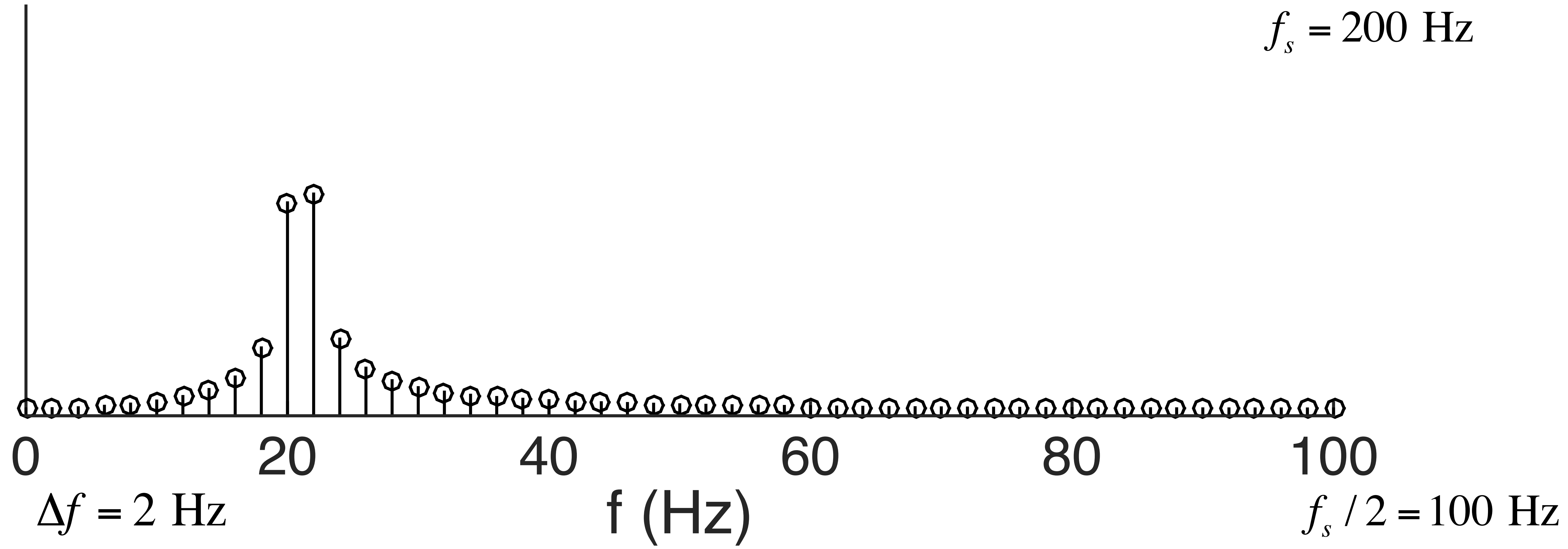
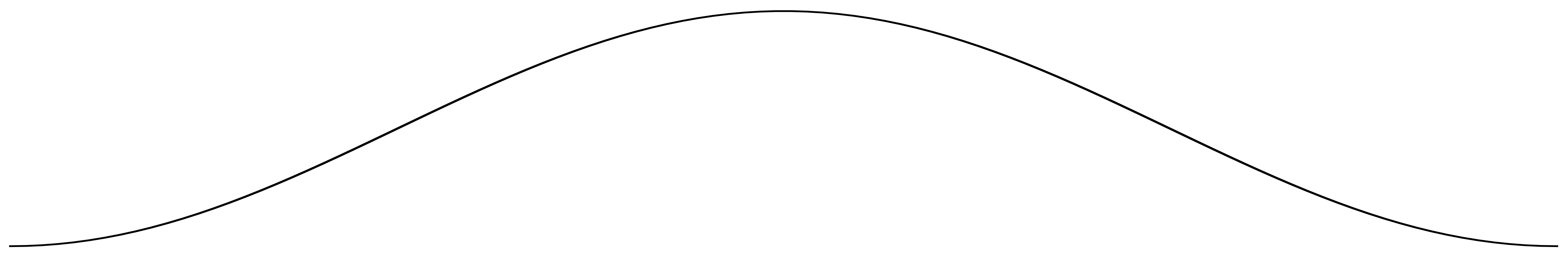




Spectral Leakage

$T = 500$ ms



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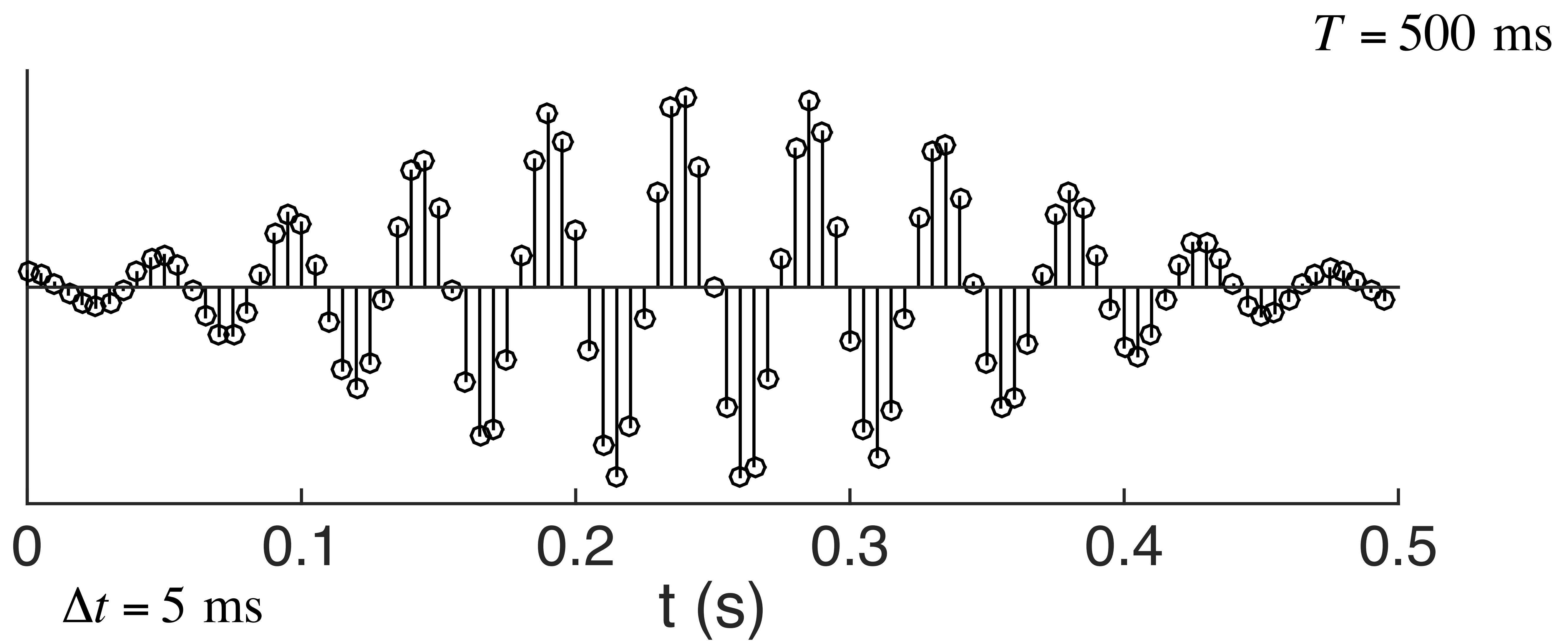

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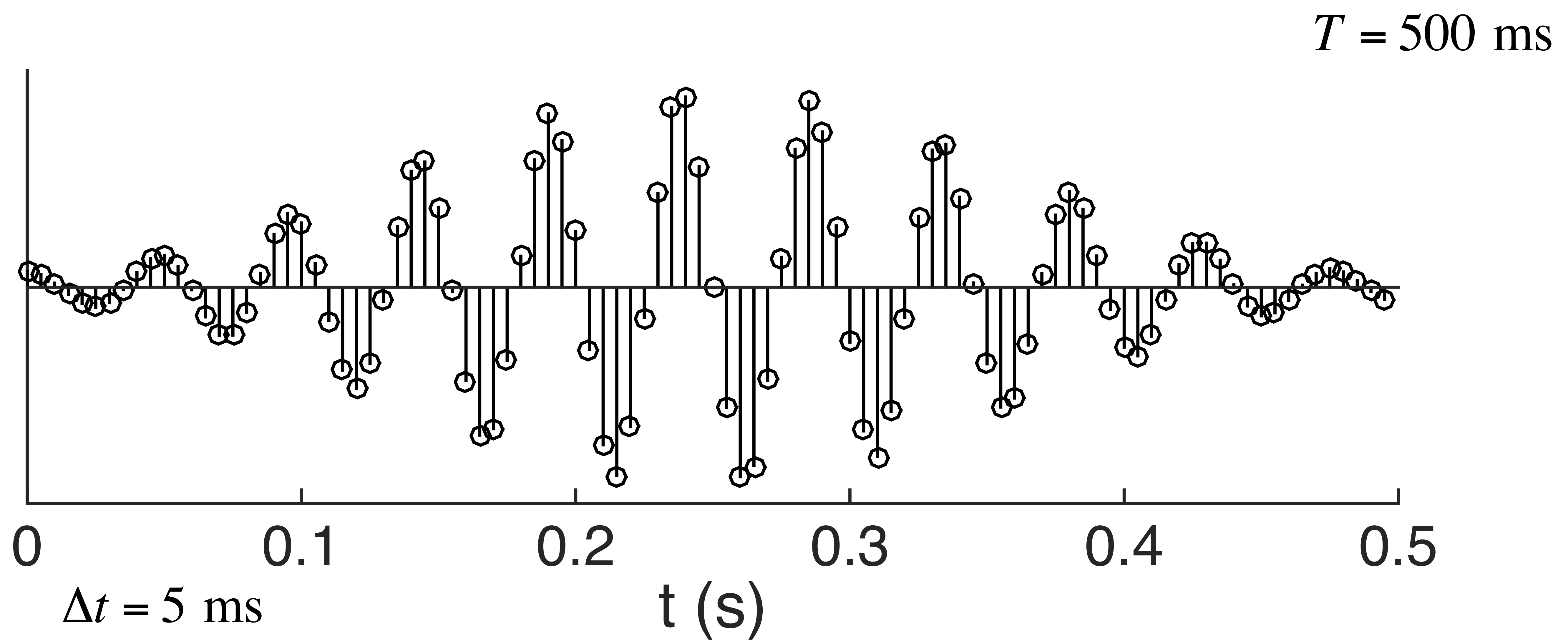

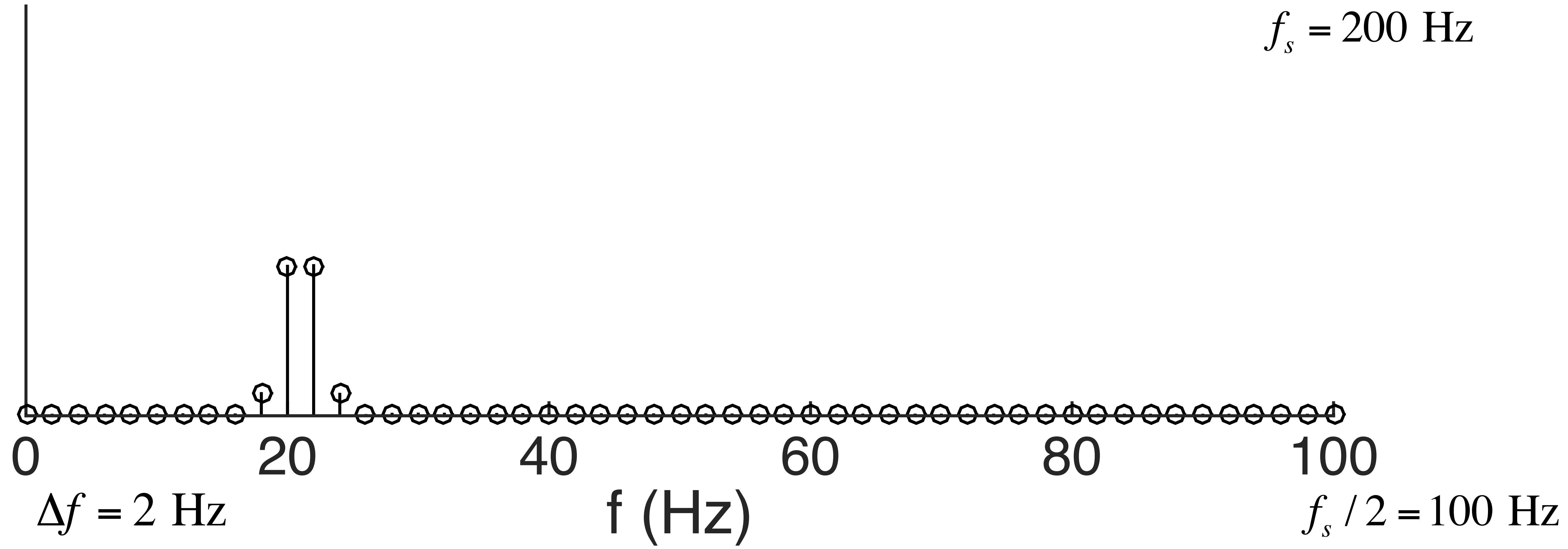

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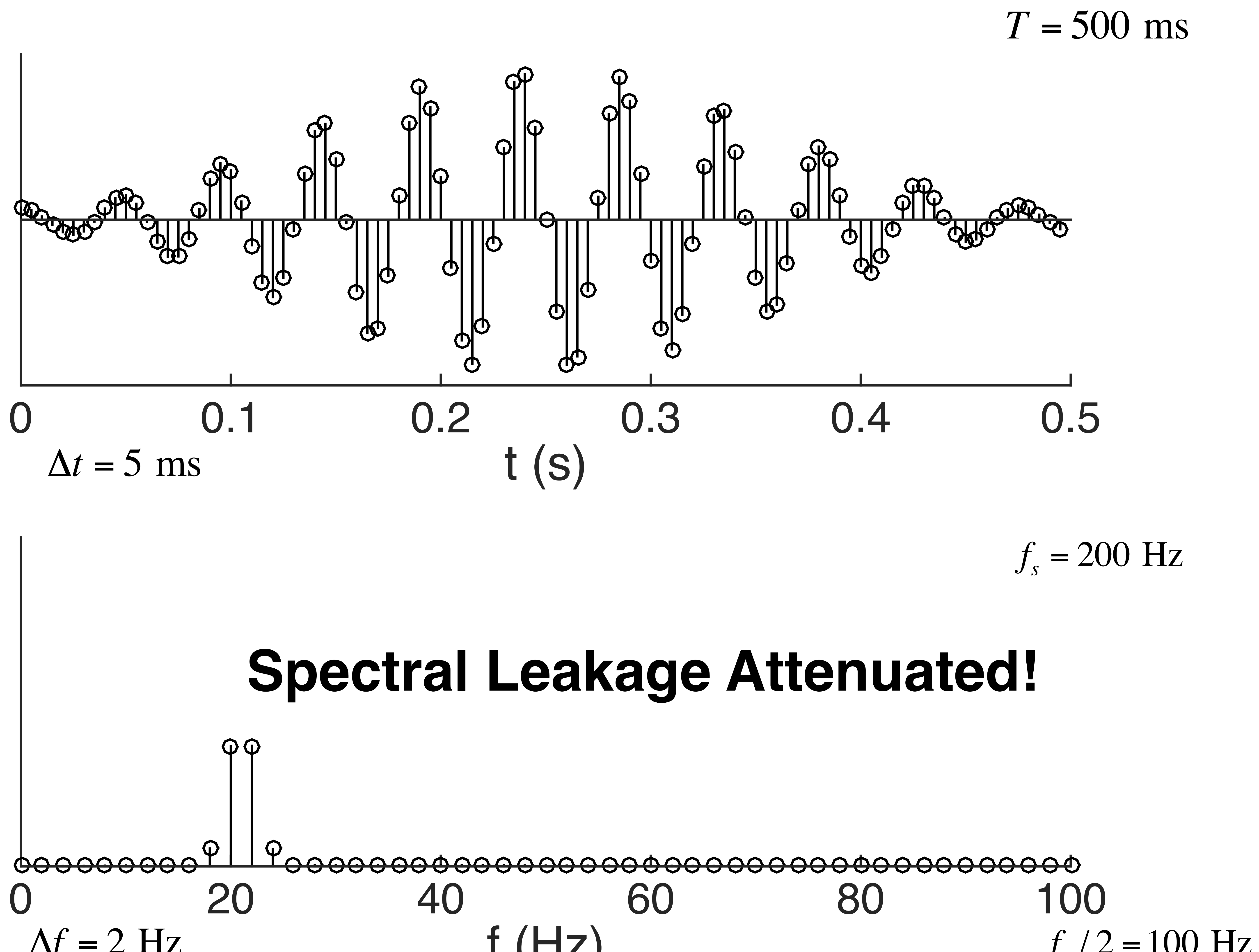


Windowing & Frequency Resolution

- Windowing to attenuate spectral leakage is critical for frequency estimation (spectral power, spectrogram, etc.).

- Achieves this by blurring the frequency resolution (typically by 2x).

- If you require spectral resolution of Δf, you also require a signal duration of not just $1/\Delta f$, but really $2/\Delta f$.

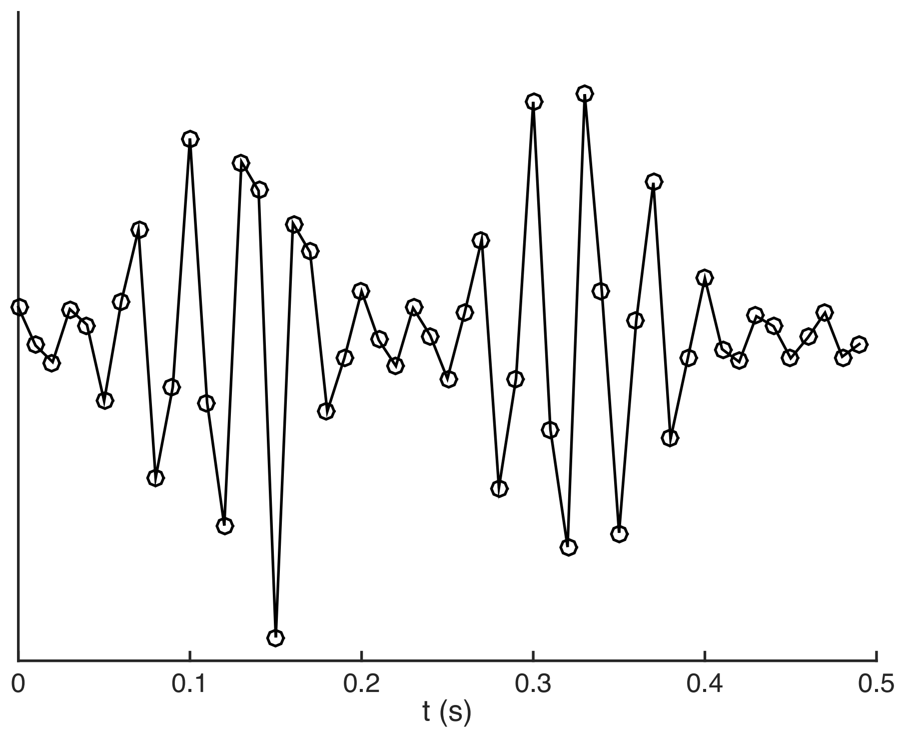
- For example, 1 Hz resolution, without spectral leakage corruption, requires ~2 s signal duration. 2 Hz resolution, without spectral leakage corruption, requires ~4 s signal duration.

Low Passing of Envelopes

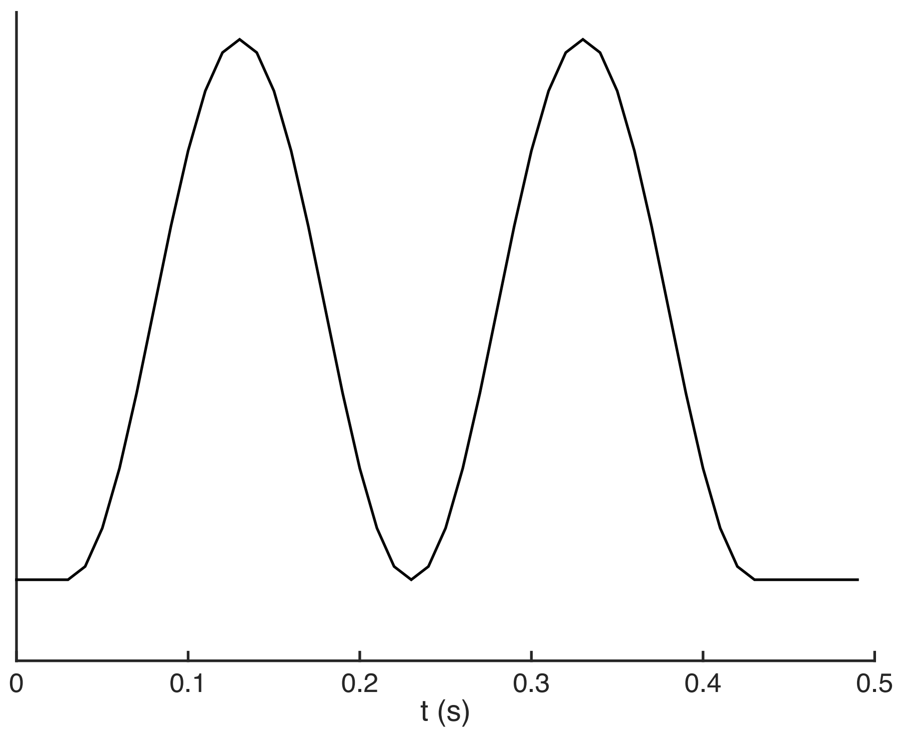
- An envelope is any slow amplitude modulation of a signal
- No single definition of envelope, except that it is slow and positive
- Commonly used definitions
    - Low passed half-wave rectified signal
    - Low passed magnitude of Analytic Signal (“hilbert” in Matlab).
- Note that the low pass filter is *not optional*
    - An envelope is any ***slow*** amplitude modulation of a signal.

Low Passing & Envelopes

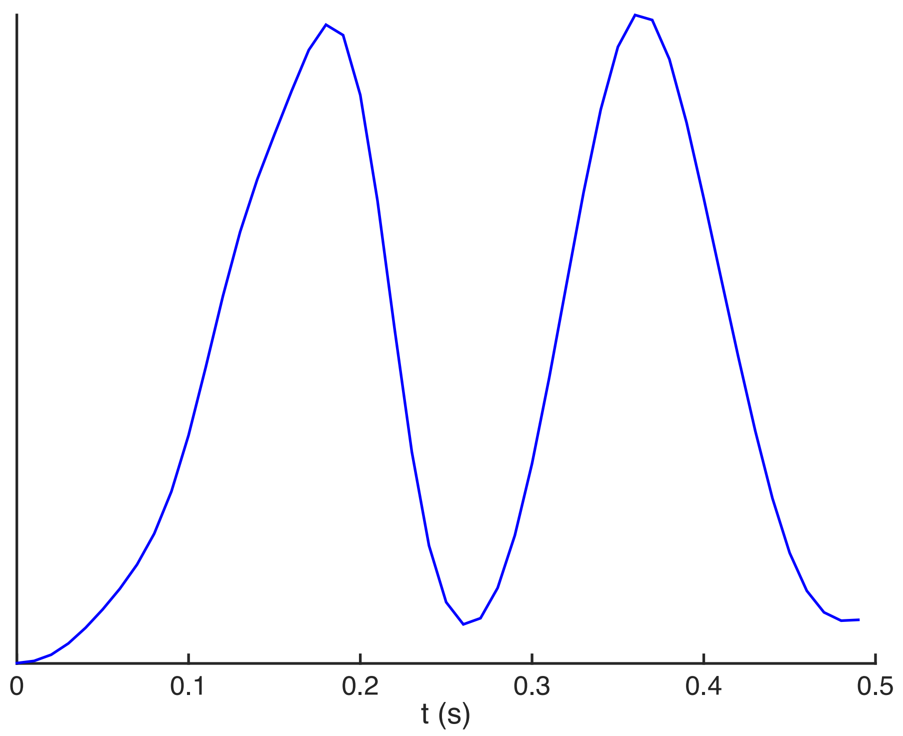
Raw Signal



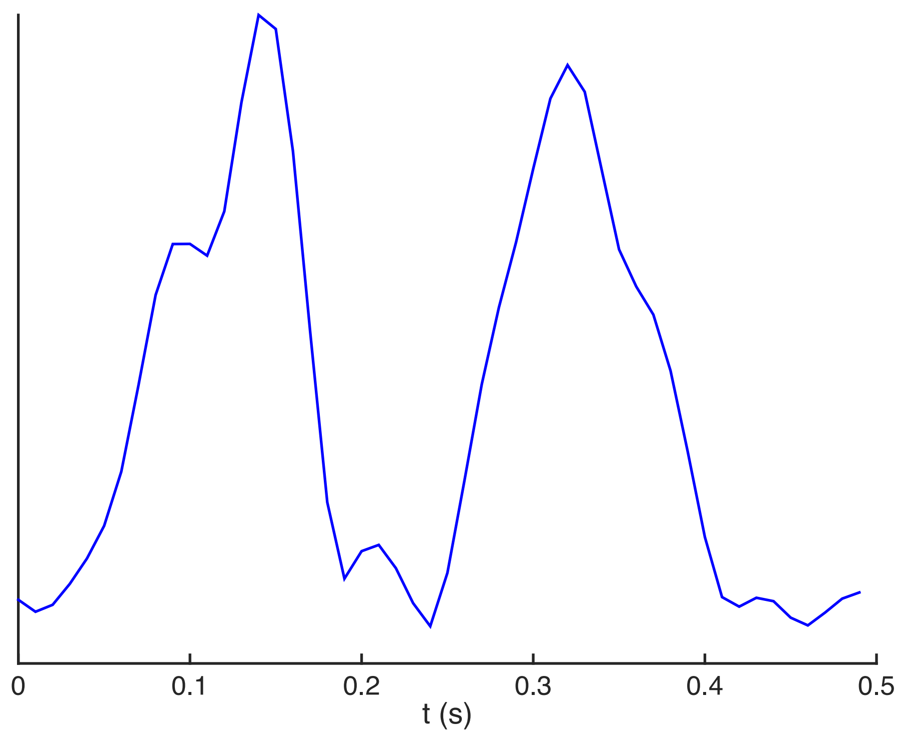
Actual Envelope



Low Passed Analytic Magnitude



Analytic Magnitude



Outline

- Fourier Transform: *Why It's Useful*, and *What it Can/Cannot Do For You*
- Filters: *What They Do*, and *How They Do It*
- Filters: *Why So Many Different Kinds? Which Should I Use and When*?
- Grab Bag:
  - *Use Causal Filters; Windowing is Good*; *Low-Pass your Envelopes*

Conclusions

- Signal Processing is Complicated
- But not Too Complicated
- Mathematical Definitions will always Win/Tie over Intuition
- But Guided Intuition will put on a Strong Show
- Debugging using Guided Intuition faster than using Math